

AN APPLICATION OF MODEL-BASED SIGNAL EXTRACTION

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1. INTRODUCTION

Seasonal adjustment is a routine activity in which institutions involved in policy-making and short-term monitoring of the economy indulge heavily. This creates an important demand for methods which are easy to apply to a large number of series, involving little input from the part of the user. With a few exceptions, X11 is the program which presently satisfies that demand.

Often, X11 is used as a black box: the series enters one way and the seasonally adjusted one exits another, with little attention being paid to what goes on inside the box. Moreover, the lack of an underlying model makes it difficult to analyse results.

In connection with another important "routine" activity (forecasting) the decade of the seventies witnessed the proliferation of ARIMA models. Broadly, this was the result of two factors: first, the work of Box and Jenkins, which permitted their large scale use, and, second, the fact that ARIMA models captured well the main features of many series. Since these features are naturally related to the presence of trend, seasonal and noise components, the possibility of using ARIMA models to estimate these components (or signals) was appealing. Over the last years, ARIMA-based signal extraction has been the subject of important research (see, for example, [2], [3], [5], [7], [13]). There are however few reported applications and ARIMA-based adjustment is still far from widespread use.

The purpose of this paper is to look somewhat carefully into a real world application, where the overall trend/seasonal/irregular decomposition (and not simply seasonal adjustment) is of interest. The application involves the monthly foreign trade series of the Spanish economy. These are highly erratic series with a stochastic structure which is, as we shall see, not too distant nor too close to the one for which X11 is thought to be roughly appropriate.

In the discussion two points of general interest emerge. First, the sensitivity of the model-based decomposition to changes in the ARIMA specification (which do not violate the series structure) is analysed. This has implications for the ARIMA model selection criteria, when signal extraction is to be performed. Second, we shall see how the ARIMA-based method provides a framework for systematic analysis of the results and for comparison of different adjustment methods. In fact, in our application, comparison of X11 to the model-based method favours heavily the latter. The results suggest that ARIMA-based signal extraction has come of age and is ready for large-scale real world application.

2. THE EXPORTS SERIES: SENSITIVITY OF THE DECOMPOSITION TO THE ARIMA SPECIFICATION:

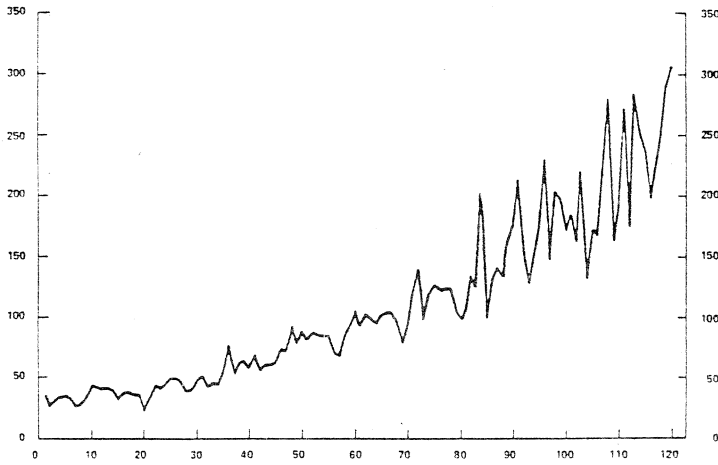
2.1. SEASONAL ADJUSTMENT WITH X11 ARIMA

In order to somewhat smooth their monthly fluctuations, at the Bank of Spain the series were seasonally adjusted with X11, in its multiplicative version, run once-a-year. In our comparison, two modifications to X11 were introduced. First, since X11 can be seen as its default option, we use X11 ARIMA (hereafter, X11A), with an ARIMA specification that shall be given later. (The results, however, were practically unaffected when the Automatic option is used.) Second, adjustment was done concurrently, since it reduced revisions by approximately 40%.

For reasons given in [10], the period selected was January 74-December 83 (T=120). The original and seasonally adjusted series and the estimated factors are given in Figures 1, 2 and 3. Seasonality seems to evolve somewhat rapidly, still no residual seasonality could be detected in the adjusted series. However, from an applied point of view, these present two important problems:

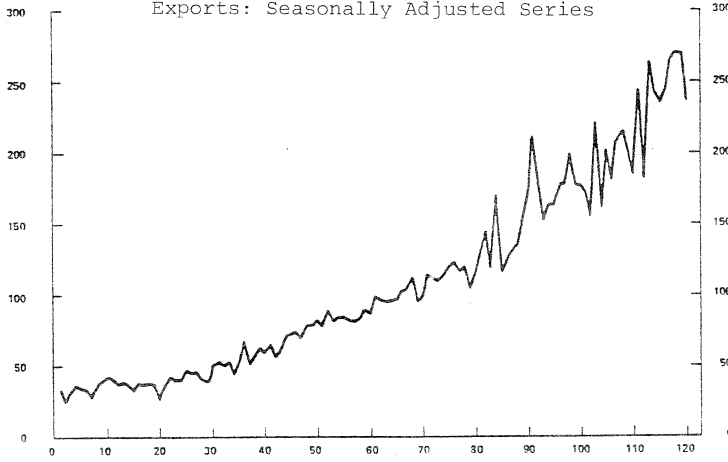
Exports: Original Series

Figure 1



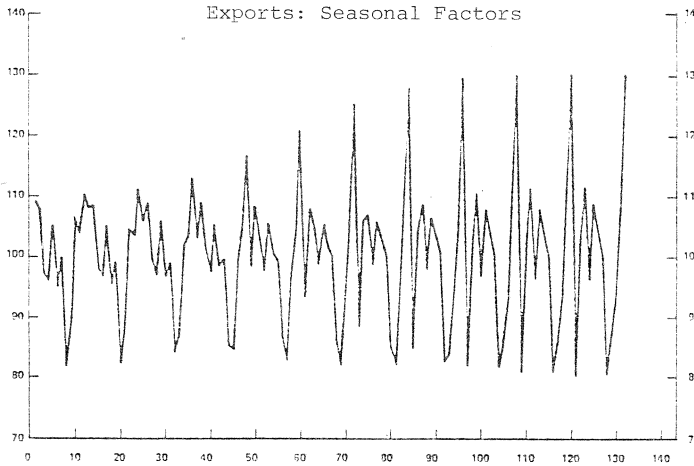
Exports: Seasonally Adjusted Series

Figure 2



Exports: Seasonal Factors

Figure 3



a) Although less so than the original series, the seasonally adjusted one behaves quite erratically. The first two columns in Table 1 display the annualized rates of growth of the original and concurrently adjusted series: the manic depressive behavior is clearly present.

b) The adjusted series is subject to very large revisions. Since -assuming that we are using adequate filters- the best estimate is the final estimate, if revisions are large, the concurrent measure will be a poor estimate. The third column of Table 1 displays the annualized rates of growth of the seasonally adjusted series for 1979 after 4 additional years of data have become available. (In fact, the revision induced by the fourth year is negligible.) What was thought in January 79 a very small growth (well below the average) turned out to be very large. In February, a preliminary increase of 74% was finally changed to a 37% decrease. The rest of the months also present important differences between the concurrent and final measures. Obviously, the concurrent estimate is close to worthless.

Perhaps it could be possible to obtain a more satisfactory seasonally adjusted series using a different method. It is well-known (see [6]) that X11 is appropriate for series with an Autocorrelation Function (ACF) associated (broadly) with an ARIMA structure of the type:

$$\nabla \nabla_{12} \log x_t = \theta(B) a_t \quad , \quad (1)$$

where a_t is white-noise (w.n.), $\theta(B)$ is a relatively long polynomial (which implies $\rho_{12} \approx .35$). Perhaps the export series does not fit into that type.

2.2. STRUCTURE OF THE SERIES

Let x_t denote the export series. The ACF of $z_t = \nabla_{12} \log x_t$ (Figure 4) shows no indication of nonstationarity and the

Table 1

Exports: Monthly Rates of Growth (1979)

	Original	S.A. X11	
		Concurrent	Final
Jan.	-79.5	9.1	374.1
Feb.	302.9	73.9	-30.8
Mar.	-41.0	30.4	-17.4
Abr.	-44.0	24.1	17.7
May.	133.7	-11.2	5.2
Jun.	45.4	205.9	115.6
Jul.	-13.6	-22.7	7.7
Aug.	-59.0	81.0	160.0
Sep.	-92.3	-81.8	-86.6
Oct.	730.8	33.0	51.4
Nov.	2223.1	687.6	549.8
Dic.	475.0	11.0	-30.6

Table 2

Exports: Estimation Results

Model	ϕ	θ	θ_{12}	SE(a_t)	Q_{24}
A	.182 (.094)	-	.635 (.077)	.1421	20.2
B	-	-.159 (.094)	.626 (.076)	.1424	21.6
C	.824 (.167)	.688 (.217)	.625 (.078)	.1417	20.1
D	-	.843 (.052)	.641 (.075)	.1433	20.1

In parenthesis: SE of estimates.

Q_{24} : Ljung-Box statistics.

Figure 4

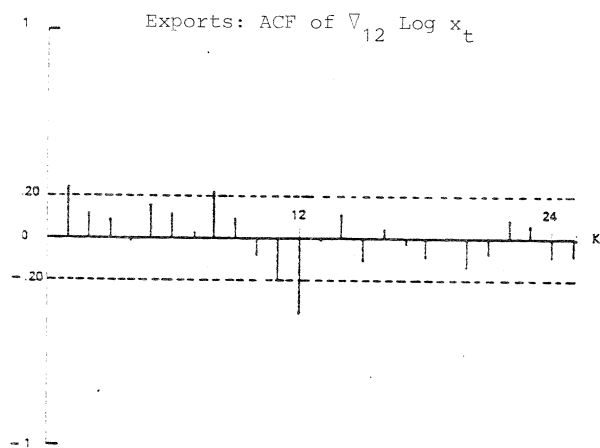
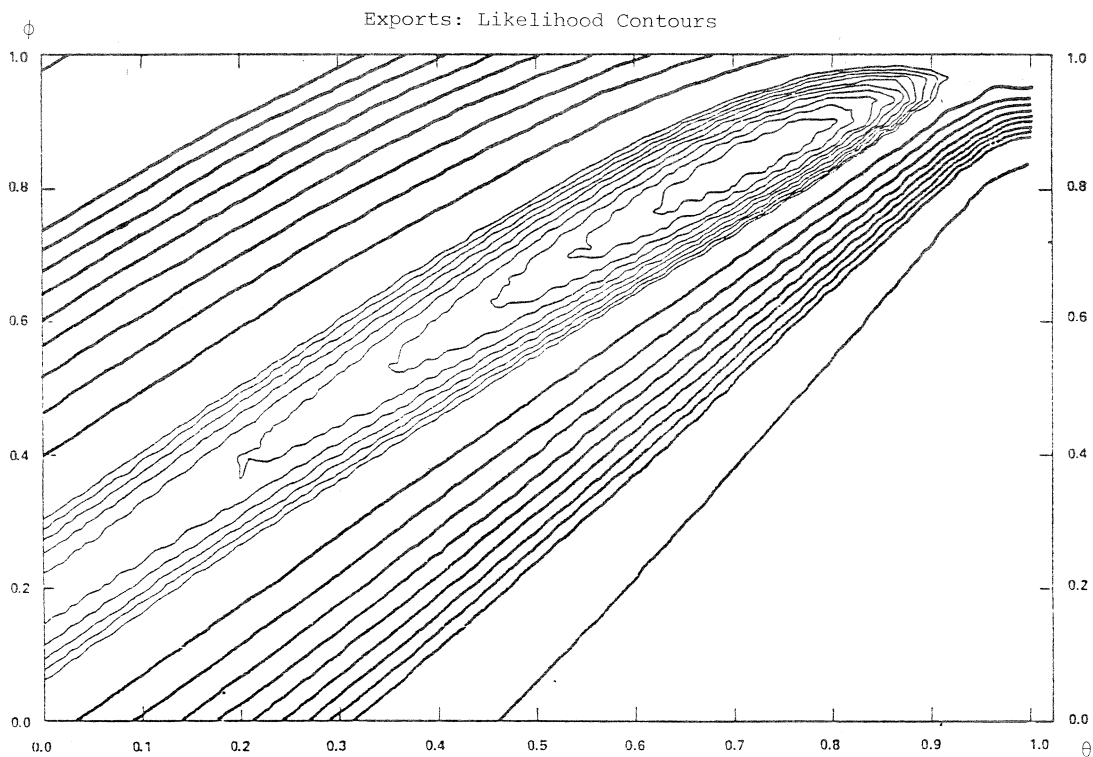


Figure 5



variance of z_t is smaller than that of any other differencing. (For example, $\text{Var } z_t = .0286$, while $\text{Var } (\nabla \nabla_{12} \log x_t) = .0413$.) Aside from a lag-12 autocorrelation which can be captured with the MA factor $(1-\theta_{12} B^{12})$, the only non-zero autocorrelation of z_t is $\rho_1 = .23$, a relatively small value. This small low-order autocorrelation could be captured with an AR(1) factor, an MA(1) factor or with an ARMA(1,1) for which $(\phi-\theta)$ is small. Thus, letting

$$b_t = (1-\theta_{12} B^{12})a_t \quad ,$$

where a_t is w.n., the following specifications provided similar fits, all of them acceptable:

$$(A) \quad (1-\phi B)z_t = b_t + c$$

$$(B) \quad z_t = (1-\theta B)b_t + c$$

$$(C) \quad (1-\phi B)z_t = (1-\theta)b_t + c$$

If a regular AR(2) was used, $\phi_2 \approx 0$; if the fitted model was simply $z_t = b_t + c$, the residuals displayed significant first-order autocorrelation. Finally, the model:

$$(D) \quad \nabla z_t = (1-\theta B)b_t \quad ,$$

the so-called "Airline model" (see [4]), gave results rather close to those of (A), (B) and (C). This is not surprising, since the likelihood function is very flat in the direction $\phi = \theta + .2$ (see Figure 5). The estimation results are summarized in Table 2. The residuals for the four models can be taken as white-noise. Their standard error vary from 0.1417 (model C) to .1433 (model D). However, the correlation between the estimated parameters ϕ and θ in (C) is .95. Thus (C) represents a non-parsimonious model, and a reasonable starting choice could be model (A), that is:

$$(1-.182B)\nabla_{12} \log x_t = (1-.635 B^{12})a_t + .175 \quad . \quad (2)$$

This was the model used to extend the series when running X11 ARIMA.

2.3 MODEL BASED-SEASONAL ADJUSTMENT

Since (1) is not quite like (2), perhaps a model-based approach using (2) would provide more reasonable seasonally adjusted series. Burman has made available a very efficient program which, briefly, adjusts the series in the following manner:

Let $h_z(x)$ denote the spectrum of the ARIMA model, where $x = \cos \omega$, and unit roots are allowed in the AR polynomial (see [1].) Then $h_z(x) = U(x)/V(x)$, where $U(x)$ and $V(x)$ are associated with the MA and AR parts of the model, respectively. The function $h_z(x)$ can be expressed as the partial fractions decomposition:

$$h_z(x) = M_p(x)/V_p(x) + M_s(x)/V_s(x) + Q(x), \quad (3)$$

where $V_p(x)$ and $V_s(x)$ are associated with the trend and seasonal roots of the AR polynomial, respectively, and $Q(x)$ is a constant (for "bottom heavy" or "balanced" models) or a low order polynomial (for "top heavy" models, see [5].) The first and second fraction provide the trend and seasonal spectrums, $h_p(x)$ and $h_s(x)$, and $Q(x)$ will be assigned to the irregular. The decomposition is made unique by setting $\min. h_p(x) = \min. h_s(x) = 0$, in which case the variance of the irregular is maximized. Finally, the seasonal component is estimated with the filter $h_s(x)/h_z(x)$.

For the case of model (A), it can be seen that the corresponding models for the components are of the type:

$$(1-\phi B) \nabla \log p_t = \alpha_p(B) c_t + k$$

$$U(B) \log s_t = \alpha_p(B) b_t$$

$$\log u_t \sim \text{w.n.}$$

where $U(B) = 1 + B + \dots + B^{11}$, $\alpha_p(B)$ and $\alpha_s(B)$ are polynomials of order 2 and 11, respectively, and c_t , b_t and u_t are independent white-noises. (Since the series has a multiplicative structure, the additive decomposition in the model based method applies to the logs of the series.)

We adjusted the export series with Burman's program using model (2). The results were very similar to those obtained with X11A. The first two columns in Table 3 display the ACF of $y_t = \nabla \log x_t^a$ (where x_t^a denotes the adjusted series) for X11 and the model-based approach. Aside from the negative ρ_1 -partly induced by the fact that the same month enters with opposite signs in two consecutive values of y_t - nothing much seems to remain; they both behave quite erratically. As for the precision of the concurrent estimate, the standard deviation of the revision error of y_t (multiplied by 12 and expressed in percent points) is 68.3 for the model-based adjustment and 70.0 for X11A.

Thus the model-based method does not improve the behavior of the seasonally adjusted series. After all, if the irregular component is large, it should be contained in the seasonally adjusted series, making them highly erratic. Also, if forecast errors are large and seasonality changes fast, the concurrent estimate will be unreliable. Hence the two problems associated with the seasonally adjusted series do not seem to be due to insufficiencies of the seasonal adjustment methods, but to characteristics of the series. Perhaps we should try to find a measure which does not behave so erratically and which can be estimated concurrently with more precision.

Table 3

Exports: ACF of $V\log x_t^e$

Lag	X11A	A	C	D
1	-.46	-.45	-.44	-.44
2	-.02	-.02	-.03	-.03
3	-.04	-.03	-.02	-.03
4	-.09	-.07	-.07	-.06
5	.12	.09	.09	.09
6	.04	.07	.06	.06
7	-.17	-.22	-.22	-.22
8	.19	.22	.22	.22
9	-.03	-.01	-.01	-.01
10	.02	.00	.00	.00
11	-.06	-.03	-.03	-.03
12	-.15	-.20	-.20	-.19
13	.08	.12	.13	.12

Table 4

Exports: ACF of $V\log p_t$

Lag	X11A	A	C	D
1	.83	.31	.82	.94
2	.44	-.45	.48	.82
3	.01	-.35	.24	.70
4	-.27	-.04	.14	.60
5	-.31	.11	.13	.51
6	-.18	.00	.13	.42
7	-.01	.03	.12	.33
8	.06	.20	.07	.24
9	-.03	.23	-.05	.12
10	-.22	-.08	-.24	.00
11	-.39	-.39	-.38	-.10
12	-.42	-.29	-.36	-.15
13	-.30	.14	-.22	-.17

2.4. TREND ESTIMATION: PRELIMINARY RESULTS

If the series is the sum of a trend, seasonal and irregular component:

$$z_t = p_t + s_t + u_t \quad , \quad (4)$$

where the latter is white-noise, by removing the irregular, the erratic behavior should decrease. Hence we shall consider estimation of the trend.

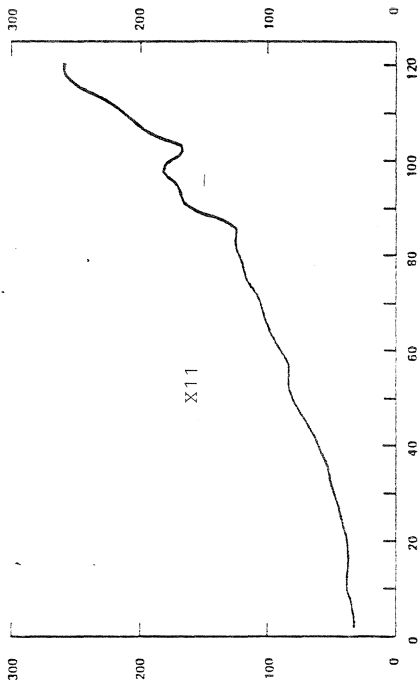
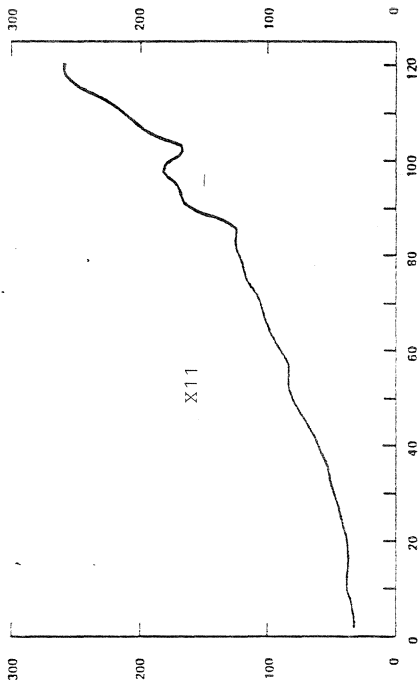
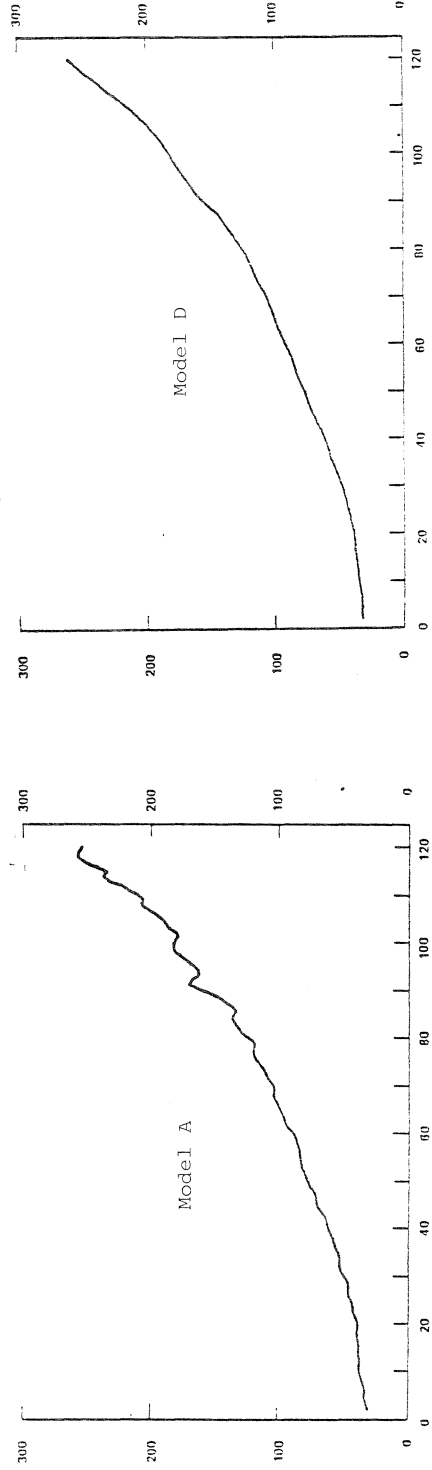
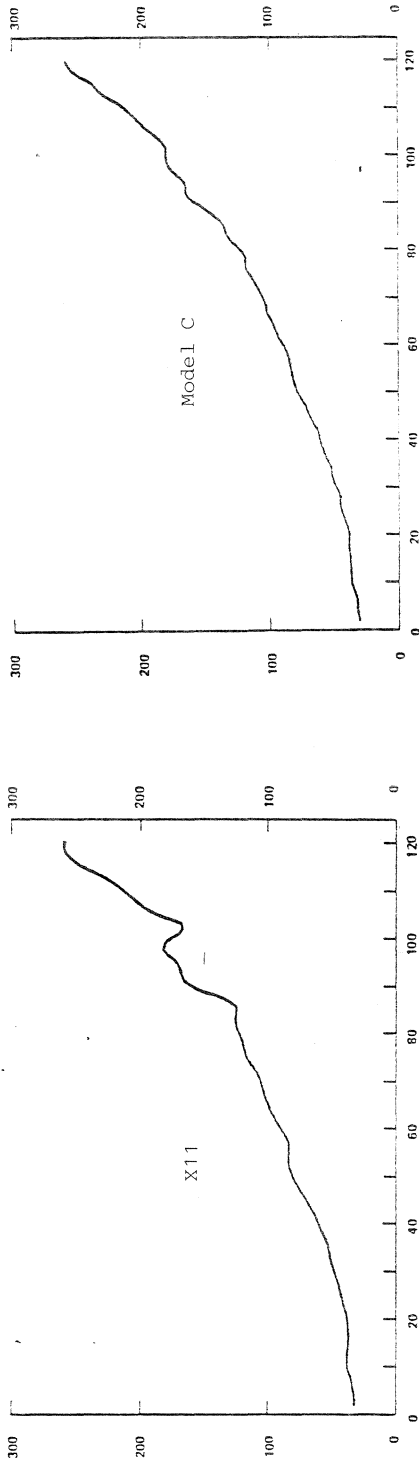
A standard trend estimation procedure is to use moving averages, derived from LS local approximation to polynomials in time (see, for example, [9].) The trend estimate provided by X11, which shall be denoted p_t^x , implies such a procedure. A sequence of two filters is passed twice: the first filter is a symmetric [2x12] moving average and the second a Henderson moving-average (see [14].)

The top left part of figure 6 displays p_t^x ; the first column of Table 4 the ACF of its monthly rate of growth. For the second half of the period, p_t^x appears to be somewhat bumpy. Also the large negative value of ρ_{12} would be associated with a 2-year period effect, whose presence is undesirable in a trend. Finally, the standard deviation of the revision error in the (annualized) monthly trend's rate of growth is equal to 8.6 percent points.

Similarly to X11A, Burman's program provides an estimate of the trend, computed through the filter $h_p(x)/h_z(x)$, where the h-functions were described in Section 2.3. Using model A (i.e. equation (2)), the estimated trend, p_t^A , is displayed in Figure 6. The ACF of its monthly rate of growth appears in the 2nd column of Table 4. The trend behaves rather erratically with too much short-term variation. Also the standard deviation of the revision error in its monthly rate of growth equals 13.6 percent points.

Figure 6

Export: Trend Estimates



Comparing p_t^x and p_t^A , the former seems preferable. In both cases the difference between the seasonally adjusted series and the trend (i.e., the irregular component) could be accepted as white noise. The Q-statistic for the first 12 autocorrelations were 14.58 (X11A) and 21.86 (model-based). The difference is mostly due to a relatively large negative value of ρ_{12} for the model-based irregular ($\rho_{12} = -.20$). Also, the variance of the X11A irregular is larger.

2.5. SERIES COMPATIBLE MODELS

For the case of the seasonally adjusted series, both approaches yielded similar results. We concluded that the problems were related to the properties of the series. However, when estimating the trend, the two methods differ substantially. Perhaps the poor performance of the model-based approach is due to inadequacies in the model. Thus we are interested in determining how sensitive the estimated components are to changes in the ARIMA specification.

Of course, it would not make sense to use an ARIMA model not in agreement with the time series in question. But, as we saw in Section 2.2, there are several ARIMA specifications that are approximately equally compatible with the series. This is a not unusual: often, the limitations of the sample do not permit to assert that one and only one specification is acceptable. We shall refer to this (loosely defined) group of models which are compatible with a given series as the group of "series compatible" models. This group may include slightly non-parsimonious models, with very large autocorrelation between parameter estimates, or with some parameter whose estimate is not quite significant. The models in this group will basically satisfy three conditions:

- a) they provide clean residual ACF,
- b) the residual variances are of a similar (approximately minimum) size, and
- c) they provide similar forecasts.

2.6. SENSITIVITY OF THE DECOMPOSITION TO CHANGES IN ARIMA SPECIFICATION.

In section 2.2 we discussed the estimation results of several models. Since our objective is to remove white-noise irregular, we shall not consider B, which -being "top heavy"- yields an autocorrelated irregular (see section 2.3.). A, C and D produced white-noise residuals which were in fact quite close (Figure 7). The residual variance for D was slightly larger, though the difference was in the order of only 1%. As for the forecasts, Figure 8 displays the 12-period ahead forecast functions. They are practically indistinguishable. In fact, the standard deviation of the one-period ahead forecast error for 1983 was slightly smaller for models C and D, although the difference was in all cases smaller than 2%. Since the three models provide similar fits and forecasts, they are "series compatible". Does this imply that the way they decompose into trend, seasonal and irregular is also similar?

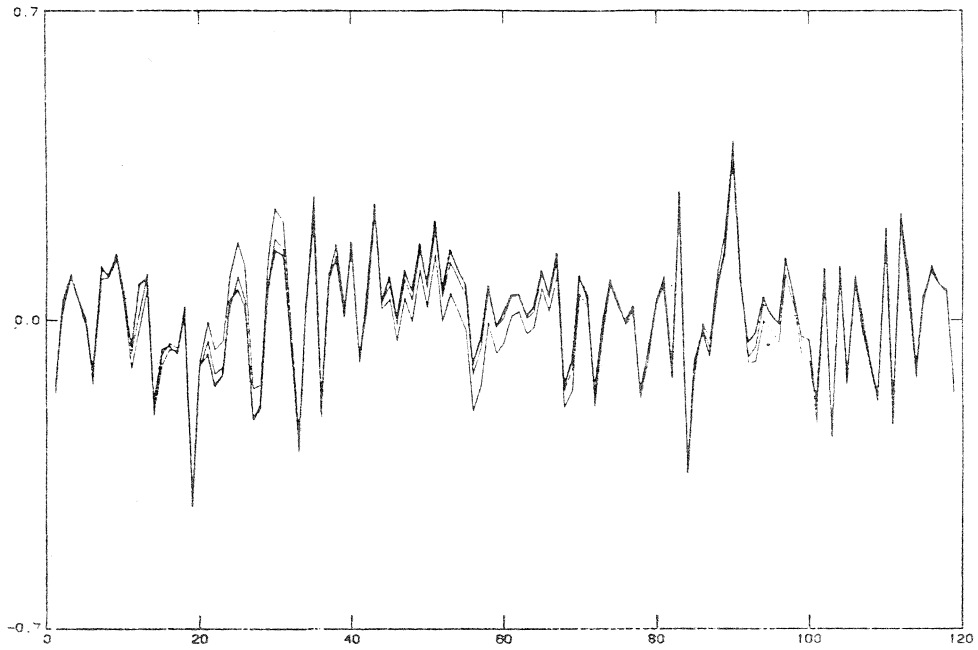
If X11 ARIMA is used, since the ARIMA model only influences the last years of the estimation period, and since the differences among the forecasts are very small, the decomposition is practically unaffected. We compare next the model-based trend-seasonal-irregular decomposition of A, C and D.

a) Seasonally Adjusted Series

Figure 9 displays the three spectrums $h_s(\cos\omega)$ for the range $0 \leq \omega \leq \pi$, and Table 3 (last three columns) shows the ACF of $\nabla \log x_t^a$. The seasonally adjusted series are very much alike.

Exports: Residuals for Models A, C, D

Figure 7



Exports: Forecast Function for Models A, C, D

Figure 8

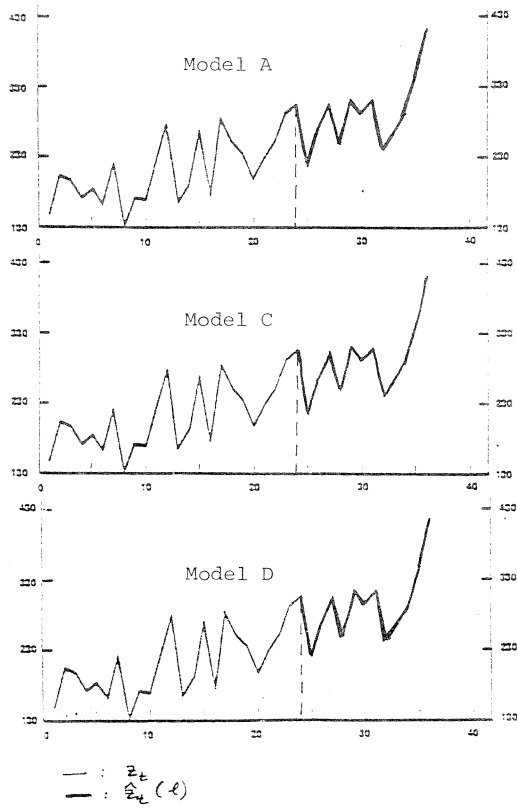


Figure 9

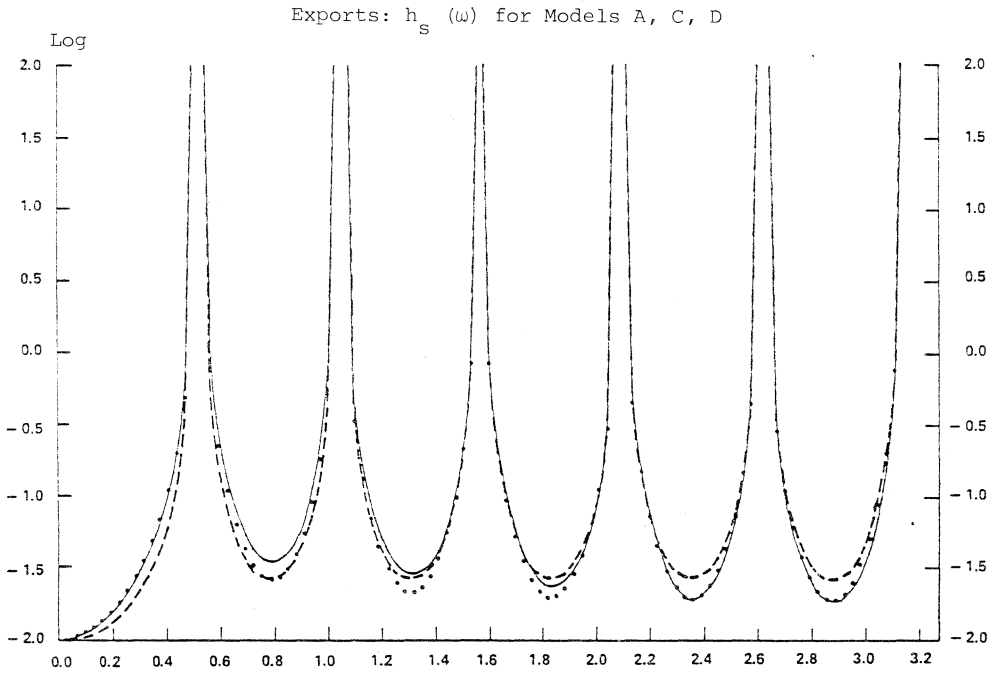
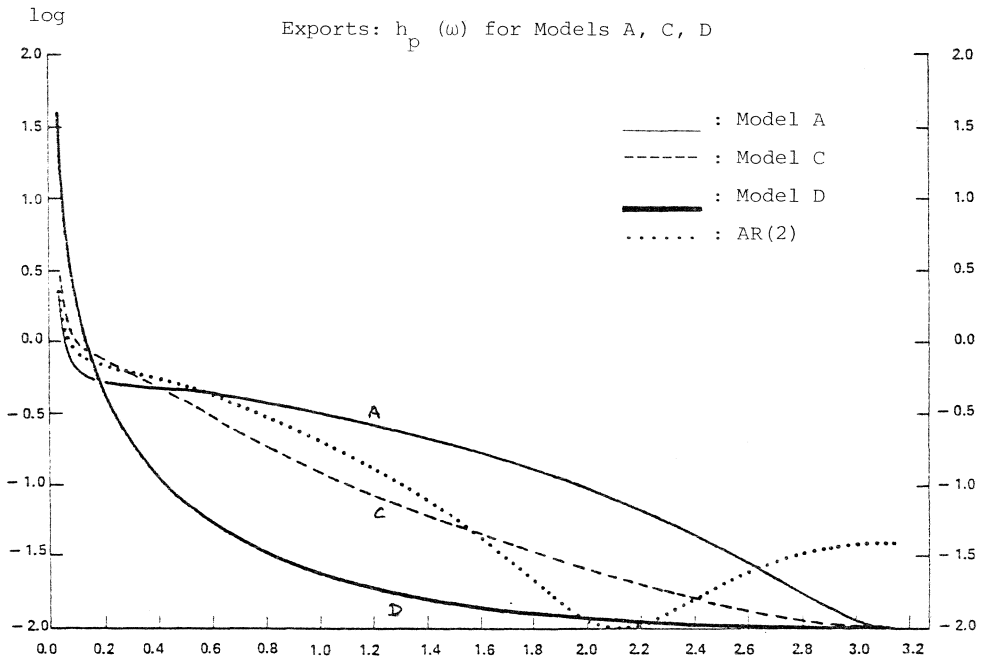


Figure 10



This, after all, is not surprising, since the seasonal factors in the multiplicative ARIMA specification are in all cases approximately the same (an IMA (1,1)₁₂ structure, with $\theta_{12} \approx .6$.)

b) Trend

The second, third and fourth column of Table 4 present the ACF of $\nabla \log p_t$; they differ markedly. The three estimated trends are shown in Figure 6: model A produces the most erratic one, while model D produces the smoothest one.

We mentioned earlier that the size of the revision in the concurrent estimate is of applied interest, since it is an indication of how reliable measures of the present evolution can be. Revisions were computed for the annualized monthly rate of growth of p_t for the years 1979 and 1980, after three years of additional data have become available. In all cases, the means could be assumed zero and the standard deviations were the following (expressed in percent points):

Table 5

Model	A	C	D	X11A
σ	13.58	3.76	2.45	8.60

Thus the trends obtained for C and D represent a large improvement in the precision of the concurrent estimate, when compared with those obtained for A and X11A.

Comparison of $h_p(\cos \omega)$ -Figure 10- is particularly revealing. Model A contains a much larger contribution of frequencies not associated with a trend than D (C stands in between). The effect of the ARIMA specification on the trend spectrum is illustrated with a 4th example: If a second AR parameter is added to A, the estimated AR(2) polynomial is:

$$\phi(B) = 1 - .180B - .079B^2 \quad (5)$$

(.095) (.095)

where the numbers below are the corresponding standard errors. Thus $\phi_2 \approx 0$. Reestimating the model setting $\phi_2 = 0$ yields, as we saw, $(1 - .182B)$. If the AR(2) specification is nevertheless used, the residuals and forecasts are practically unaffected. However $h_p(x)$ becomes the dotted line in Figure 10. It differs substantially from that of model A. The difference between them becomes apparent if (5) is factorized, which gives:

$$\phi(B) = (1 - .380B)(1 + .205B)$$

The first factor implies a root larger than .182, hence it accounts for the steeper decline in $h_p(x)$ for the AR(2) model. Also, the second factor implies a peak in the spectrum for $\omega = \pi$, as seen in the figure.

c) Irregular

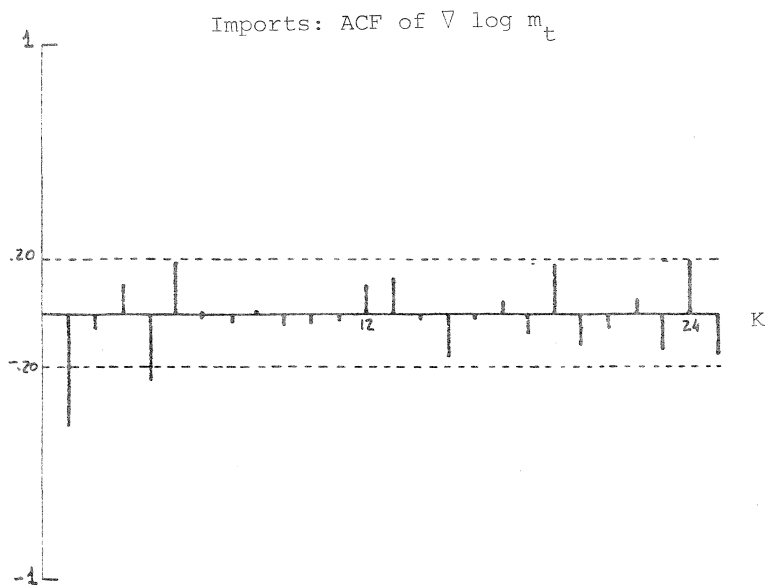
Table 6 displays the ACF of the three estimated irregulars (in logs). Except for a negative seasonal autocorrelation, they are close to that of white-noise. Since the irregular is computed as the residual, after the seasonal and trend have been estimated and removed from the series, its spectrum will display dips for seasonal and low-frequencies. This can explain the negative values of ρ_{12} and ρ_1 . The standard deviation of the three irregulars are .0719 for model A, .0837 for model C, and .0857 for model D.

Table 6

Exports: ACF of Irregular (log u_t)

Lag	A	C	D	X11A(1)
1	-.09	-.08	-.02	-.10
2	.07	-.11	-.09	-.19
3	.03	-.11	-.11	-.22
4	.00	-.08	-.09	-.19
5	.13	.09	.07	.06
6	.10	.05	.03	.06
7	-.12	-.11	-.11	-.06
8	.23	.22	.20	.27
9	.01	.06	.03	.11
10	-.01	-.05	-.08	-.04
11	-.08	-.17	-.21	-.19
12	-.20	-.25	-.26	-.31
13	.08	.10	.07	.14
Q ₂₄	36.1	40.7	37.7	43.3

Figure 11



d) The Preferred Decomposition

Of the three models, D removes more irregular variation and provides a smoother trend. Since the seasonality removed was pretty much the same in the three cases, D is the model which provides a more satisfactory decomposition. It seems also preferable to X11A; in particular, the trend is also smoother and has smaller revisions (see Table 5).

However, the model-based extraction consisted only of linear filters, while X11A was used under the standard option for outliers. If only the linear filters are used (and no outlier treatment is performed), the trend obtained with X11A becomes smoother. Still, it is not as smooth as p_t^D and is subject to larger revisions ($\sigma=4.04$). Moreover, the irregular deteriorated strongly, as shown by its ACF, displayed in the last column of Table 6.

Therefore, the model-based method (using Burman's program) applied to the specification D clearly provides the best decomposition among the ones we have considered.

e) ψ -weights versus AR factors

One could think that the sensitivity of the components to the overall ARIMA specification may not be a property of ARIMA models, but rather a property of the particular decomposition used.

Let the models for the components be, in general,
 $\phi_p(B) p_t = \alpha_p(B) c_t$ and $\phi_s(B) s_t = \alpha_s(B) b_t$,
where the trend and seasonal AR operators do not share a common root (a most sensible assumption), and u_t is white-noise. Then, from (4), it is easily seen that the AR operator in the ARIMA expression for z_t is: $\phi(B) = \phi_p(B) \phi_s(B)$. Thus, given $\phi(B)$, the allocation of its root to $\phi_p(B)$ and $\phi_s(B)$ is, in all our examples, unique: a

root of the type $1-\phi B$ (with $0 < \phi < 1$) will be assigned to the trend; the roots of $U(B)$ to the seasonal. Hence, given the overall model, the only unknown parameters of the components' models are those in $\alpha_p(B)$, $\alpha_c(B)$ and the variances of the components' innovations. It can be seen that fixing σ_u^2 then all parameters are uniquely determined. In fact, from results in [7], maximizing σ_u^2 (subject to the constraint that σ_c^2 and σ_b^2 are nonnegative) identifies uniquely the components' models. The decomposition obtained in this case is called "canonical", and its properties and virtues are discussed in [2], [7] and [11]. In particular, the components of any other admissible decomposition (with $\sigma_u^2 \geq 0$) can be expressed as the canonical components plus superimposed noise.

Hence, given the overall ARIMA model for z_t and requiring u_t to be white-noise as large as possible, for the models we have discussed, no other (sensible) decomposition of the type $z_t = p_t + s_t + u_t$ seems possible.

The sensitivity of the decomposition to changes in the ARIMA specification -within the set of series-compatible models- can be heuristically explained in the following manner: Let the ARIMA model for z_t be: $\phi(B)z_t = \theta(B) a_t$, which can also be expressed as: $z_t = \psi(B) a_t$. The series-compatible models could be taken to be the ones with similar ψ -weights. However, expressing $\psi(B)$ as $\theta(B)/\phi(B)$, similar $\psi(B)$'s may imply considerably different $\phi(B)$ polynomials (models A and C provide a clear illustration.) Since the AR roots of $\phi(B)$ dominate the spectrum of p_t and s_t , it follows that series compatible models may yield fairly different decompositions.

2.7. A COMMENT ON ARIMA SPECIFICATION

In applied analysis, whatever the method used for tentative identification, the selection of an ARIMA model depends ultimately on

two factors: the quality of the pre-whitening achieved and the accuracy of forecasts. This simply reflects the fact that pre-whitening and forecasting have been the most important applications of ARIMA models.

In our discussion, we have seen that models which differ little in terms of fits or forecasts may provide remarkably different decompositions. In particular, we focussed on changes in the "regular" structure which yielded, in all cases, reasonable specifications to account for the small low-order autocorrelation present in y_t . The differences in the estimated components were particularly noticeable for the trend. Therefore, when signal extraction (likely, on top of forecasting) is to be performed, the usual criteria for ARIMA model selection may be inappropriate. Attention should be paid to the way the model "decomposes". Useful (and easy to compute) tools in this respect are, for example, the frequency domain representation of the seasonal and trend models, which are implied by the overall ARIMA, and the ACF of the estimated irregular.

A reasonable model selection strategy would, from the set of series-compatible models, pick up the one which yields the best decomposition. In our application, this led us to an "Airline model" specification, which is known to decompose nicely (see [7]).

This way of proceeding somewhat combines the two alternative approaches to signal extraction. One of them (exemplified by X11) is to use "ad hoc" filters for the signal, with desirable properties, at the risk of violating the series stochastic structure. The other approach models the series first, and from this derives the implied filters, which may then present undesirable properties. What we have seen is that the limitations imposed by the sample information, and the relatively large sensitivity of the model-based decomposition to changes in the ARIMA specification, may provide an ample room for signal improvement, within the set of series-compatible models.

3. THE IMPORTS SERIES: COMPARISON WITH X11

3.1. STRUCTURE OF THE SERIES

Similarly to the exports series, the imports one (which shall be denoted m_t) is subject to highly erratic monthly swings. There is however an important difference between the two: while the transformation $\nabla_{12} \log x_t$ seemed close to stationarity for the exports series, in the case of imports the ACF of $y_t = \nabla \log m_t$ (Figure 11) does not indicate the need for any additional differencing. In particular, $\rho_{12} = .09$ so that little seasonality seems to be present. However, although small, the values of ρ_{12k} ($k=1,2,3,4$) are all positive, suggesting directly the ACF of an $IMA(1,1)_{12}$, with a positive large value for θ_{12} .

Somewhat symmetrically to the discussion in the previous section, using a multiplicative ARIMA model and fixing the regular part as an $IMA(1,1)$, the small seasonal autocorrelation can be captured with an $ARMA(1,1)_{12}$. The likelihood function of the associated ϕ_{12} and θ_{12} parameters is shown in Figure 12. Even more so than in the export case, the Airline model suggest itself as a reasonable specification. (Notice that the choice of different values of ϕ_{12} will also affect the trend, since $(1-\phi_{12} B^{12})$ will imply different peaks in the spectrum for $\omega = 0$. The differences however are likely to be small. Since the trend root in $(1-\phi_{12} B^{12})$ is $(1-\Phi B)$, where $\Phi = \phi_{12}^{1/12}$, even when ϕ_{12} is small, Φ will be close to 1.)

The estimated model was:

$$\nabla \nabla_{12} \log m_t = (1-.732 B)(1-.893 B^{12}) a_t, \quad (6)$$

with the standard error of both parameter estimates close to .06.

Figure 12

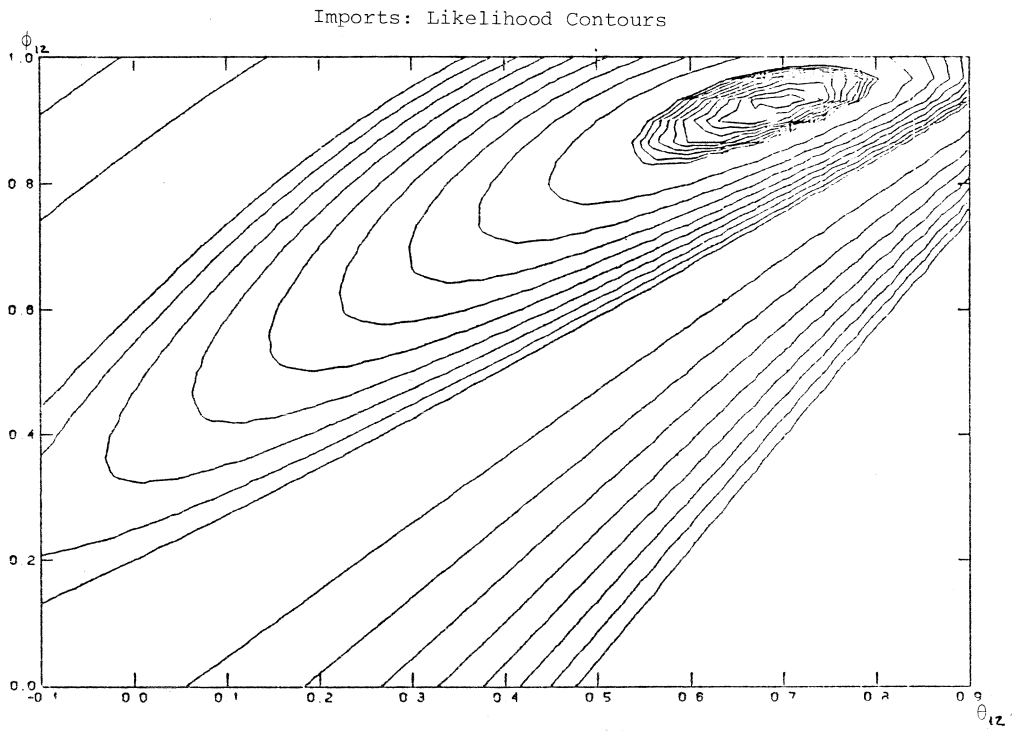
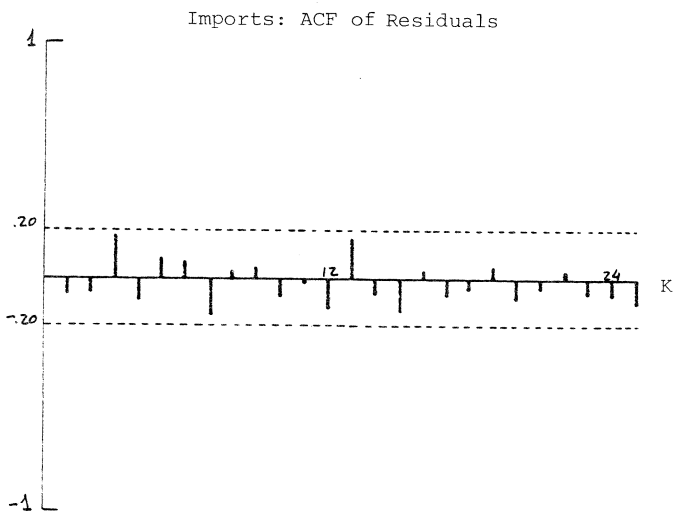


Figure 13



The residuals are clean (see Figure 13) and, for example, $Q(24) = 23.0$. Since θ_{12} is close to 1, one could think of using seasonal dummy variables. Regression models with such variables showed however few significant coefficients for the seasonal dummies and considerable variation within the sample period (see [10]).

Thus comparison of the exports and imports series indicates that the latter has a less stable trend and a more stable seasonal. Moreover, seasonality in the imports series is of relatively less importance. It is of interest that the Airline specification is flexible enough to capture the two different behaviors.

3.2. ESTIMATION OF COMPONENTS WITH X11.

We consider the seasonally adjusted series (m_t^a), trend (p_t) and irregular (u_t) estimated with the multiplicative version of X11A, with the ARIMA model given by (6). Since we are interested in linear filters, no outlier modifications were performed.

Part A of Table 7 gives the main characteristics of the original series and of the components (in logs). (For the former, the seasonally adjusted series and the trend, a $\sqrt{2}$ transformation was used, necessary to achieve stationarity.) It is seen how seasonal adjustment preserves the low-order autocorrelation of the original series. As for the seasonal autocorrelation, the adjusted series, the trend and the irregular display a relatively large negative value of ρ_{12} . In particular, for the adjusted series, $\rho_{12} = -.47$. Even if optimal seasonal extraction induces negative values for ρ_{12} , the former value seems undesirably high. It implies a non-negligible two-year periodic effect and suggests the possibility of overadjustment. (The large negative value of ρ_{12} for the adjusted series was not affected by varying the length of the Henderson moving-average.)

Table 7

Imports: Characteristics of the Components

	Series	Variance	ρ_1	ρ_2	ρ_{12}	Q_{12}
	Original	$(.68) 10^{-1}$	-.63	.07	.02	66.6
A (X11 linear)	Seasonally Adjusted	$(.45) 10^{-1}$	-.65	.06	-.47	113.0
	Trend	$(.10) 10^{-4}$.92	.73	-.33	376.9
	Irregular	$(.57) 10^{-2}$	-.29	-.19	-.37	46.4
B (Model- based)	Seasonally Adjusted	$(.52) 10^{-1}$	-.62	.02	-.28	85.4
	Trend	$(.16) 10^{-4}$.50	-.04	-.16	50.6
	Irregular	$(.69) 10^{-2}$	-.19	-.15	-.15	22.4
B (X11 default)	Seasonally Adjusted	$(.47) 10^{-1}$	-.65	.07	-.36	92.4
	Trend	$(.42) 10^{-4}$.72	-.20	-.38	165.4
	Irregular	$(.63) 10^{-2}$	-.24	-.14	-.25	29.6

Finally, the irregular is not close to white-noise and, for example, $Q(12) = 46.4$. Since the irregular is computed as what is left after trend and seasonality have been removed, some negative autocorrelation for lags 1 and 12 could be expected. Nevertheless, the values $\rho_1 = -.29$ and $\rho_{12} = -.37$ seem again large. Since our purpose was to remove white-noise variation, it would be desirable, after all, to actually remove something close to it.

3.3. ARIMA-BASED SIGNAL EXTRACTION

In the ARIMA-based approach, the Airline model yields the following specifications for the components:

$$\begin{aligned} \nabla^2 \log p_t &= \alpha(B) c_t, \\ U(B) \log s_t &= \beta(B) b_t, \end{aligned}$$

and $\log u_t$ white-noise. The polynomials $\alpha(B)$ and $\beta(B)$ are of order 2 and 11, respectively. Since $\log m_t^a = \log p_t + \log u_t$ and the components are orthogonal, $\log m_t^a$ is also an IMA(2,2) model. Figures 14 and 15 exhibit the frequency domain representation of the trend and seasonal models.

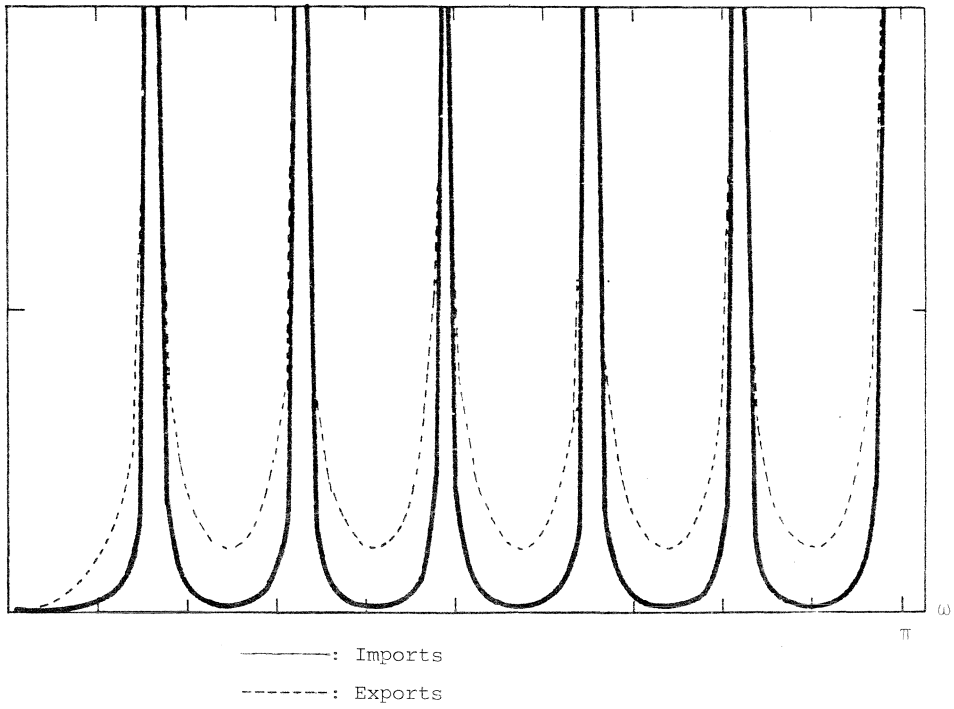
Part B of Table 7 summarizes the characteristics of the estimated components. As with X11A, the low order autocorrelation of the series is not much affected by seasonal adjustment. Also, the lag-12 autocorrelation for the adjusted series, the trend and the irregular are all three negative, but of much smaller (hence more acceptable) absolute value. Comparison of the variances of the seasonally adjusted series shows that X11 treats more of the series variation as seasonality.

In agreement with the theoretical decomposition, both $\nabla^2 \log m_t^a$ and $\nabla^2 \log p_t$ behave as a low order MA process. Since,

(in
logs)

Imports and Exports: $h_s(\omega)$

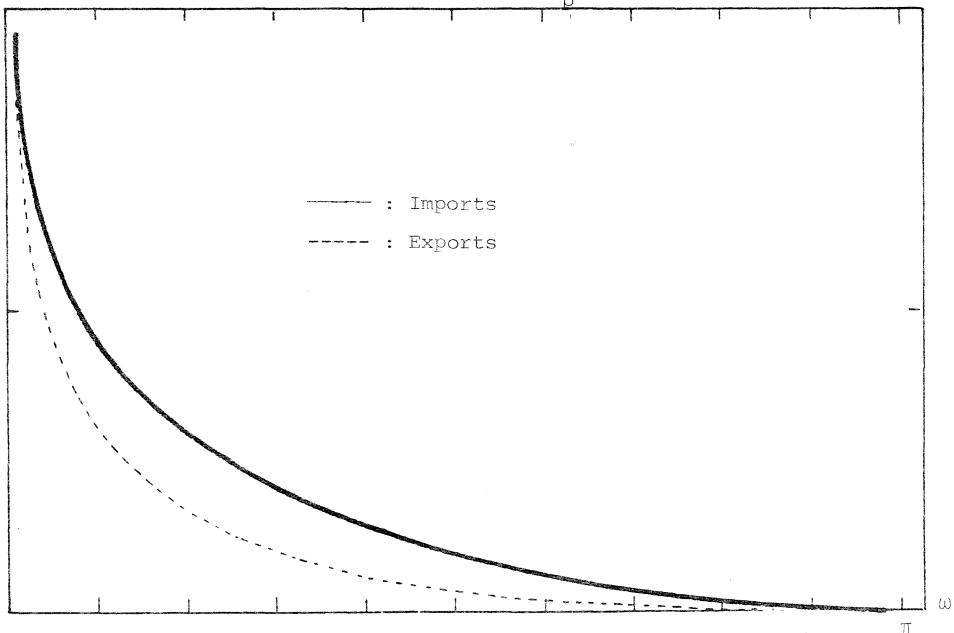
Figure 14



(in
logs)

Imports and Exports: $h_p(\omega)$

Figure 15



in the trend case, $\rho_1 = .5$ and ρ_2 is close to zero, $B = -1$ will (approximately) be a root of $\alpha(B)$, which implies a zero in the spectrum for $\omega = \pi$. Hence the estimated trend is a "canonical" trend, with no superimposed noise.

Finally, the estimated irregular is reasonably white-noise, with $Q(12) = 22.4$. The values of ρ_1 and ρ_{12} are negative, as should be expected, but of moderate size. Notice that, on top of being closer to white-noise, the model-based irregular has a larger variance than the X11 one.

Therefore, the import series illustrates a case in which X11 overadjusts when little seasonality is present in the series. Some of the trend and (especially) irregular variation is treated as seasonal. On the contrary, the ARIMA-based method appears to properly handle what seasonality is present in the series, and provides more satisfactory components' estimates.

One final comment: we have centered the discussion on linear filters. Part C of Table 7 summarizes the properties of the estimated components when X11A is used with the outlier default option. The evidence of overadjustment decreases; still, the ARIMA-based method seems preferable. (Despite their erraticity, none of the series seems to be much affected by outliers. For the export series, 5 residuals and 5 irregular are larger in absolute value than the corresponding values of 2σ , and one of each barely equals 3σ . In the imports case, 6 residuals and 5 irregulars exceed -in absolute value- the 2σ limit, with none reaching 3σ . Since $T=120$, these are reasonably in agreement with the tails of the Normal distribution. A simple and more general Normality test, particularly appropriate for the detection of outliers, is given by the ACF of the squared residuals (see [12]). For exports, the associated value of Q_{12} is 10.8 and for imports, $Q_{12}=12.7$. Hence both series appear to behave as linear processes. Of course, when appropriate, outlier treatment can also improve the model-based method, see [8].)

Thus the ARIMA-based decomposition of the export and import series, using in both cases the Airline model specification, captures well (in particular, better than X11A) the underlying components in both series. These are displayed in Figures 16 and 17 and Table 8 summarizes the standard deviation of the revisions in the concurrent estimates of the seasonally adjusted series and the trend (both measured as V_{log} , multiplied by 12 and expressed in percent points.)

Table 8

Standard Deviation of
the Revisions in Concurrent Estimates

	Seasonally Adjusted Series	Trend
Exports	68.3	2.45
Imports	55.5	3.53

Figure 16

Exports: Componentes (in logs)

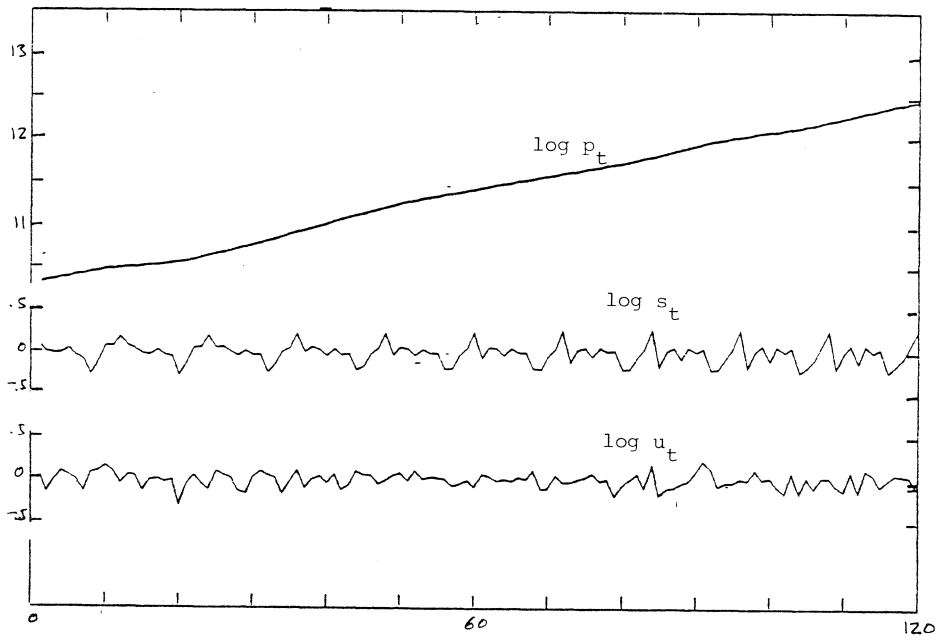
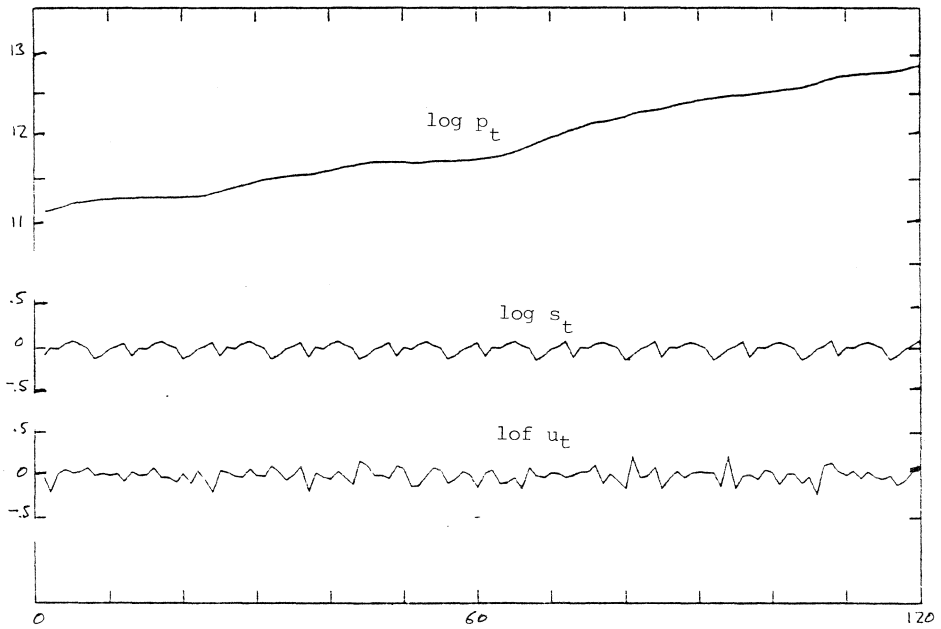


Figure 17

Imports: Componentes (in logs)



4. THE BALANCE OF TRADE SERIES

An important foreign trade series is the balance between imports and exports. Since its trend, seasonal and irregular variations are also of interest, and since these have to be consistent with the corresponding ones in the imports and exports series, we shall look at the properties of the balance of trade components, obtained indirectly from the ARIMA-based components estimated for the latter two series. Having used multiplicative adjustment, we shall consider the difference in logs, hence the ratio $r_t = m_t/x_t$.

When $\log m_t$ and $\log x_t$ follow Airline type models, it is easily seen that $\nabla\nabla_{12} \log r_t$ is an MA(q) where q may be larger than 13 if there is lagged crosscorrelation between the two univariate innovations. As seen in figure 18 for lags different from zero, there is only some small seasonal crosscorrelation. Thus $\log r_t$ will not depart much from an Airline structure. (Also the potential gain from a multivariate method seems minor.)

Letting r_t^a denote the seasonally adjusted r_t -series, and p_t and u_t its trend and irregular components respectively, Table 9 displays the main features of $\nabla^2 \log r_t$, $\nabla^2 \log r_t^a$, $\nabla^2 \log p_t$ and $\log u_t$, when indirect ARIMA-based estimation is used. The components in the balance of trade series behave quite properly. First, seasonal adjustment does not alter the low-order autocorrelation of the series. Second, the seasonal autocorrelation of the adjusted series, trend and irregular is negative (as should be expected) though of moderate size. Third, both $\nabla^2 \log r_t^a$ and $\nabla^2 \log p_t$ are low-order MA's, in agreement with the model, and the trend presents (approximately) a zero for $\omega = \pi$, behaving thus as a canonical trend. Finally, the irregular appears to be white-noise, with $Q(12) = 11.6$ (and small negative values for ρ_1 and ρ_{12} .)

From an applied point of view it is of interest that if the balance series is computed as the difference -instead of as the ratio- the previous results are practically unaffected.

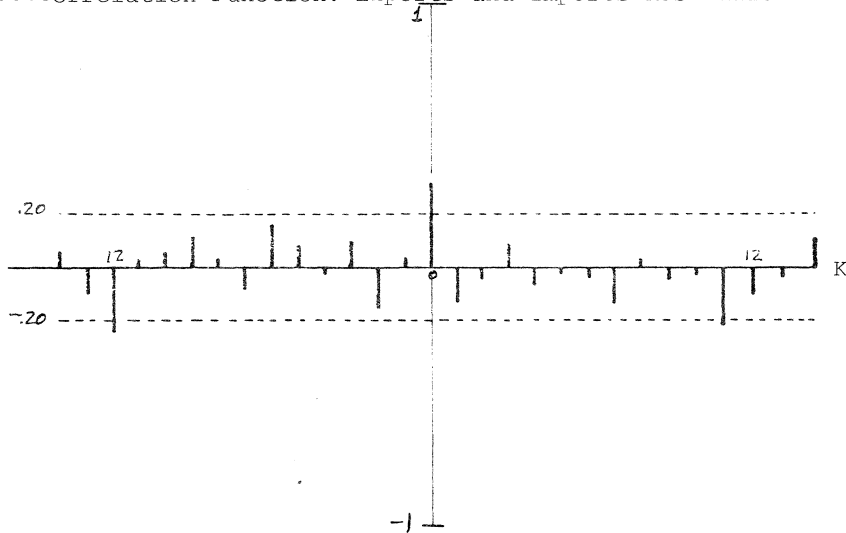
Table 9

Balance series: Characteristics of the Components

Series	Variance	ρ_1	ρ_2	ρ_{12}	Q_{12}
Original	.102	-.67	.20	.18	69.6
Seasonally Adjusted	.062	-.69	.25	-.17	78.1
Trend	$(.15) 10^{-4}$.54	-.03	-.03	50.1
Irregular	$(.94) 10^{-2}$	-.07	-.00	-.05	11.6

Crosscorrelation Function: Exports and Imports Residuals

Figure 18



5. SUMMARY AND CONCLUSIONS

Foreign trade is supposed to play an important role in the evolution of the Spanish economy, and the monthly series of exports and imports are closely watched by policy-makers and the media. Due to their wild oscillations, they are difficult to interpret and inevitably induce a manic-depressive behavior in the followers. In order to get a better view of their underlying evolution, both series are seasonally adjusted at the Bank of Spain, by running X11 once a year (the standard treatment of many hundred series.) The adjusted series however offer little improvement.

This paper analyses monthly monitoring of the foreign trade series. Since our objective is to provide a practical solution to a real world "routine" operation, we confine our attention to methods which can be enforced routinely with the same level of simplicity as X11. Specifically we consider X11 ARIMA and ARIMA-based signal extraction.

Due to the characteristics of the two series, the concurrently seasonally adjusted measures are highly erratic and subject (mainly in the exports case) to large revisions. In order to obtain a smoother measurement we extract, besides seasonality, white-noise irregular variation. Hence we consider the overall decomposition: trend/seasonal/irregular, where the latter is white-noise. Since they represent measurement error, the size of the revisions in the concurrent estimate of the components is also of interest.

Both series display a large irregular. The exports one contains a relatively important, not very stable, seasonal and a relatively stable trend. On the contrary, imports contain little, fairly stable, seasonality and a less stable trend (compare Figures 14 and 15 and Figures 16 and 17). Both fit however into the Airline model specification. Considering the ARIMA interpretation of X11 the two series are -so to speak- not too distant, nor too close to that structure.

Section 2 of the paper illustrates an interesting empirical property of ARIMA models, when used in signal extraction. Models which differ little in terms of fits or forecasts ("series compatible models") may provide remarkably different decompositions. Thus when signal extraction is to be performed, the usual criteria for ARIMA model selection are inappropriate. Simple and useful additional tools are, for example, the frequency domain representation of the seasonal and trend models implied by the overall ARIMA, and the ACF of the estimated irregular.

It is then seen how (if sensibly used) ARIMA-based signal extraction can improve substantially over X11 results. In the case of exports, this translates into a better separation of trend and irregular. For the import series, it affects mostly the seasonal and irregular.

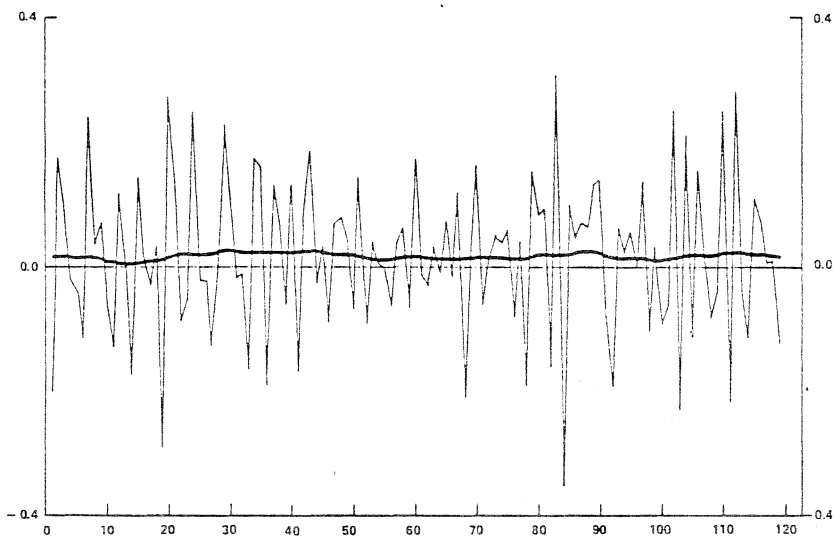
A convenient feature of the ARIMA-based method is that it offers a proper framework to analyse results. Section 3 illustrates this point. For a given overall model, the implied models for the components can be derived. Thus, on the one hand, the properties of these models can be checked against our a priori notions of what are desirable trend-seasonal-irregular properties. On the other hand, it is possible to see whether the estimated components are reasonably close to their theoretical models. (Of course, by permitting systematic analysis, the method also provides the basis for improvement.)

Finally, section 4 analyses indirect estimation of the components for a third series of interest: the balance between imports and exports. The ARIMA-based method is seen to perform very well.

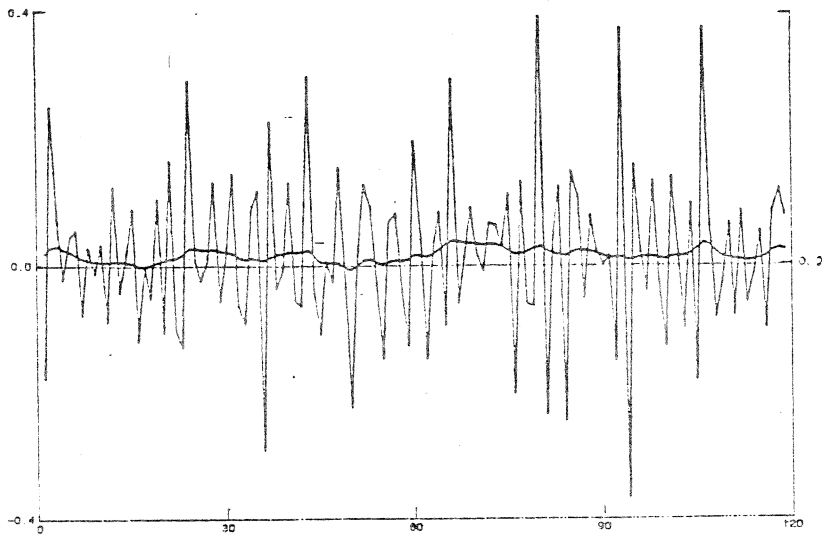
The three-component ARIMA-based method is presently used in monthly monitoring of the foreign trade series at the Bank of Spain. It has reduced indeed the manic-depressive moods (see Figure 19) and

Figure 19

Exports: Rates of Growth of Seasonally Adjusted Series and of Trend



Imports: Rates of Growth of Seasonally Adjusted Series and of Trend



increased the precision of the measurements. Since the Airline model is the default option in Burman's program, the operational simplicity is the same as that of X11 ARIMA in its Automatic option. Moreover, the better results obtained with the model-based method cast some doubt as to the validity of the assertion in [6] that "a series obeying ... $y_t = \theta(B)a_t$ for some polynomial $\theta(B)$ might be fairly accurately analyzed by the census program". A more general conclusion is that ARIMA-based signal extraction has come of age and is ready for large-scale real world applications.

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