A NOTE ON REVISIONS IN ARIMA-BASED SIGNAL EXTRACTION

Agustín Maravall

Banco de España. Servicio de Estudios
Documento de Trabajo n.º 8421

El Banco de España al publicar esta serie pretende facilitar la difusión de estudios de interés que contribuyan al mejor conocimiento de la economía española.

Los análisis, opiniones y conclusiones de estas investigaciones representan las ideas de los autores, con las que no necesariamente coincide el Banco de España.

ISBN: 84-505-0846-0

Depósito legal: M. 43460 - 1984

Talleres Gráficos del Banco de España

Abstract

Revisions for different admisible decompositions in ARIMA-based Signal Extraction are analysed. It is seen how the canonical decomposition maximizes the variance of the revision in the preliminary estimate of the signal. As shown in an example, the result may extend to more general decompositions.

Keywords: ARIMA models; Signal Extraction; Revisions; Canonical Decomposition.

1. Background

Let z_t be the outcome of the ARIMA process:

$$\phi(B)\ z_t = \theta(B)a_t\ , \tag{1.1}$$

and we wish to decompose z_t into:

$$z_t = p_t + u_t\ , \tag{1.2}$$

where p_t is the signal and u_t the noise. The signal follows the process (with the same AR polynomial):

$$\phi(B)\ p_t = \alpha(B)\ b_t$$

where b_t and u_t are mutually independent white-noises. The MA polynomial $\alpha(B)$ and the variances σ_b^2 and σ_u^2, are determined from the system of equations which results from equating the covariances in both sides of the identity:

$$\theta(B)a_t = \alpha(B)\ b_t + \phi(B)\ u_t \tag{1.3}$$

It is well-known that, in general, if a solution exists, it will not be unique. Of all the admissible decompositions, the one that has a maximum noise variance is termed "canonical". Its use has been recommended in the work of Box, Bell, Burman, Hillmer, Pierce and Tiao, among others (see, for example, the references in Bell-Hillmer, 1984.) The model for the canonical signal is non-invertible, and an important property of the canonical decomposition is the following (see Hillmer-Tiao, 1982):

Let p_t^*, u_t^* be the canonical components. Any other admissible decomposition (p_t, u_t) satisfies:

$$p_t = p_t^* + e_t \tag{1.4}$$

where e_t is white-noise, orthogonal to p_t^*.

Since the components are solely defined in terms of their statistical properties, there seems to be no point in removing noise from the noise component and adding it to the signal. Thus the canonical decomposition is a sensible choice. The purpose of this note is to look at one property of the canonical decomposition which is of applied interest.

Let $\psi(B) = \theta(B)/\phi(B)$ and $\psi_p(B) = \alpha(B)/\phi(B)$. Conditional on $[z_t, -\infty<t<\infty]$, the optimal estimate of p_t is given by:

$$\hat{p}_t = k_p \frac{\psi_p(B)\ \psi_p(F)}{\psi(B)\ \psi(F)} z_t = \nu(B)\ z_t \ , \tag{1.5}$$

where $k_p = \sigma_b^2/\sigma_a^2$ and $F = B^{-1}$. The filter $\nu(B)$ is symmetric, centered in t, and may extend from $-\infty$ to $+\infty$. At time t, since observations are available up to and including z_t, the "end" part of the filter $(\nu_{t+1}\ z_{t+1} + \nu_{t+2}\ z_{t+2} + \ldots)$ cannot be applied. As shown in Cleveland-Tiao (1976), an optimal procedure is to replace unknown future observations with their conditional expectations at t. As time goes by, forecasts are either updated or replaced by new observations, which induces revisions in the estimate of p_t. Some properties of these revisions have been analysed by Pierce (1980). Write the final estimate as:

$$\hat{p}_t = \nu_o\ z_t + \Sigma\ \nu_j(z_{t-j} + z_{t+j}) \ ,$$

where (for the rest of the paper) the summation signs extend from j=1 to j=∞. The concurrent estimate will be given by:

$$\hat{p}_t^o = \nu_o z_t + \Sigma \nu_j (z_{t-j} + \hat{z}_t(j)) \ .$$

so that the revision is:

$$d_t = \hat{p}_t - \hat{p}_t^o = \Sigma \nu_j e_t(j) \ , \quad (1.6)$$

where $e_t(j) = z_{t+j} - \hat{z}_t(j)$.Hence the revision d_t is a weighted average of forecast errors. Revisions are of applied interest: since the final estimate is the best estimate, if revisions are large, the preliminary or concurrent estimate (the one used in monitoring and policy making) will be unreliable. In practice, thus, large revisions are (understandably) feared.

2. Signal Extraction

Consider first the decomposition of an IMA(1,1) into signal plus noise (see Box-Hillmer-Tiao, 1979).Equation (1.1) becomes:

$$\nabla z_t = (1-\theta B)a_t \tag{2.1}$$

and an admissible component for p_t is of the form:

$$\nabla p_t = (1-\alpha B)b_t$$

with $-1\leq\alpha\leq\theta$ (so that $\sigma_u^2\geq 0$). The canonical decomposition implie $\alpha=-1$ (hence the pseudospectrum of p_t has a zero for $\omega = \pi$.)

For any admissible decomposition, it is easily seen that:

$$\nu(B) = k_p \frac{(1-\alpha B)(1-\alpha F)}{(1-\theta B)(1-\theta F)}$$

and, since (1.3) implies

$$(1-\theta B)(1-\theta F)\sigma_a^2 = (1-\alpha B)(1-\alpha F)\sigma_b^2 + (1-B)(1-F)\sigma_u^2 \ ,$$

it can also be expressed as:

$$\nu(B) = 1-k \frac{(1-B)(1-F)}{(1-\theta B)(1-\theta F)} \ ,$$

where $k = \sigma_u^2/\sigma_a^2$. Let $H_o = 2/(1+\theta)$ and $H_1 = (\theta-1)/(1+\theta)$; the weights of $\nu(B)$ are found to be:

$$\nu_o = 1-k\ H_o\ ;\ \nu_j = -k \cdot \theta^{j-1}\ H_1\ (j\geq 1).$$

thus, in (1.6),

$$d_t = -k\ H_1\ \Sigma\ \theta^{j-1}\ e_t(j)\ .$$

Since, for (2.1),

$$e_t(j) = a_{t+j} + (1-\theta)\ \Sigma_{i=1}^{j-1}\ a_{t+i}\ .$$

it is finally found:

$$d_t = -k(1-\theta)\ \Sigma\ \theta^{j-1}\ a_{t+j}\ .$$

or, letting $k_o = -k(1-\theta)$,

$$d_t = k_o\ y_t\ .$$

where y_t is the AR(1) process:

$$(1-\theta F)y_t = a_{t+1}.$$

Thus, except for a scale factor, the series of revision errors is the same for all admissible decompositions. Also, considering that, after simplifying, it is obtained that

$$\text{Var}\ (d_t) = \frac{\sigma_u^4}{\sigma_a^2}\ \frac{1-\theta}{1+\theta}\ ,$$

since σ_a^2 and θ are the same for all admissible decomposition, the maximum revision variance is obtained when the noise variance is maximized. Thus the canonical decomposition maximizes the variance of the revision.

This result generalizes to any signal plus noise decomposition of an ARIMA model. Let the ARIMA model be (1.1), and we wish to decompose z_t as in (1.2), with u_t white-noise. Since:

$$\psi(B)\ \psi(F)\ \sigma_a^2 = \psi_p(B)\ \psi_p(F)\ \sigma_b^2 + \sigma_u^2 ,$$

the filter $\nu(B)$ in (1.5) is equal to:

$$\nu(B) = 1-k\ \frac{\phi(B)\ \phi(F)}{\theta(B)\ \theta(F)}$$

Hence, from (1.6),

$$d_t = -k\ \Sigma\ \zeta_j\ e_t(j) ,$$

where $\zeta(B) = \psi(B)^{-1}$, and the coefficients ζ_j are fixed, once the overall model is known. Since the same is true for $e_t(j)$ and σ_a^2 ,

$$\text{Var}\ (d_t) = \sigma_u^4\ M,$$

where M is the same for all admissible decompositions. Therefore, in the general signal plus noise decomposition the canonical solution maximizes the variance of the revision. (Notice that, again, the revision error series is the same for all admissible decomposition, aside from a scale factor.)

3. The Case of More than Two Components

The previous result may extend to the case of more than two components. As an example, consider the simplest ARIMA model which accepts a non-trivial trend/seasonal/irregular decomposition (see Pierce-Maravall, 1984):

$$\nabla_2 \, z_t = a_t \, . \qquad (3.1$$

The admissible components are given by:

$$\left.\begin{array}{l} (1-B)\, p_t = (1-\alpha B)\, b_t \\ (1+B)\, s_t = (1-\beta B)\, c_t \\ \qquad u_t \sim \text{white noise.} \end{array}\right\}$$

with

$$\sigma_b^2 = \sigma_a^2/4(1-\alpha)^2$$

$$\sigma_c^2 = \sigma_a^2/4(1+\beta)^2$$

$$\sigma_u^2 = \sigma_a^2 \, [-\alpha/4(1+\alpha)^2+\beta/4(1+\beta)^2] \, .$$

Since it has to be that $\sigma_u^2 \geq 0$, the admissible parameter region has to satisfy:

$$\alpha(1+\beta)^2 - \beta(1-\alpha)^2 \leq 0 \, ,$$

and is represented in Figure 1. (It can be seen that, always, $\alpha \leq 3-2\sqrt{2}$, $\beta \geq -(3-2\sqrt{2})$.) The canonical components imply $\alpha = -1$ and $\beta = 1$, so that the pseudospectra of p_t and s_t have a zero for $\omega = \pi$ and $\omega = 0$ respectively (Figure 2).

The estimate of p_t is found to be.

$$\hat{p}_t = \frac{1}{4(1-\alpha)^2} [2(1-\alpha+\alpha^2)z_t + (1-\alpha)^2(z_{t-1}+z_{t+1}) - $$

$$-\alpha(z_{t-2}+z_{t+2})] .$$

The concurrent estimate $\hat{p}^o_t$ will be obtained by replacing z_{t+1} and z_{t+2} by $\hat{z}_t(1)$ and $\hat{z}_t(2)$ in the above expression. Since for (3.1), $e_t(1) = a_{t+1}$ and $e_t(2) = a_{t+2}$, after simplifying:

$$d_t = \hat{p}_t - \hat{p}^o_t = \frac{1}{4} [a_{t+1} - \frac{\alpha}{(1-\alpha)^2} a_{t+2}].$$

$$\text{Var } (d_t) = \frac{\sigma_a^2}{16} [1 + \frac{\alpha^2}{(1-\alpha)^4}]$$

Again, for $-1 \leq \alpha \leq 3-2\sqrt{2}$ (the admissible region for α), the variance of d_t is maximized for $\alpha = -1$, in which case:

$$\text{Var } (d_t) = \frac{\sigma_a^2}{16} [1 + \frac{1}{16}]$$

(see Figure 3). If δ_t denotes the revision in the concurrent estimate of s_t, $\delta_t = \hat{s}_t - \hat{s}^o_t$, then it is straightforward to verify that

$$\delta_t = \frac{\sigma_a^2}{16} [-a_{t+1} + \frac{\beta}{(1+\beta)^2} a_{t+2}] .$$

hence

$$\mathrm{Var}\ (\delta_t) = \frac{\sigma_a^2}{16}\ [1 + \frac{\beta^2}{(1+\beta)^4}] \quad ,$$

which, for the admissible region ($-3+2\sqrt{2} \le \beta \le 1$), reaches a maximum for $\beta = 1$. The canonical components, therefore, maximize the variance of the revisions in the concurrent estimates of p_t and s_t (and hence, of the concurrently seasonally adjusted series.)

4. Summary and Conclusion

We have seen that, for ARIMA-based signal extraction, the canonical decomposition (which maximizes the variance of the noise) implies maximum revision variances. Since any admissible component is equal to the canonical one plus orthogonal white-noise, the result states that the cleaner the component, the larger the revision of its estimate. It may sound puzzling that the smoother a signal, the larger the revision. However, it is obvious that the estimate of the signal which minimizes the revision is $\hat{p}_t = z_t$, when no noise is extracted and the signal is noisiest.

Although this maximum revision property of the canonical decomposition is somewhat inconvenient, it does not make sense to reduce revisions by improperly labelling as signal, what is purely white-noise variation.

Figure . 1

Admissible Parameter Region

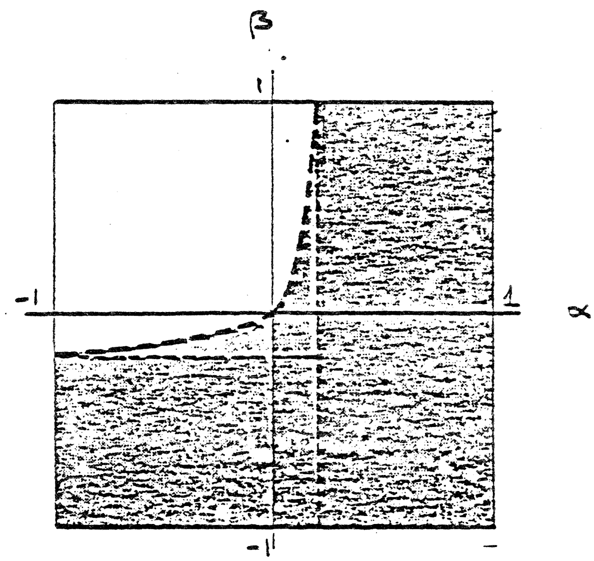


Figure 2

Canonical Decomposition of $\nabla_2 z_t = a_t$

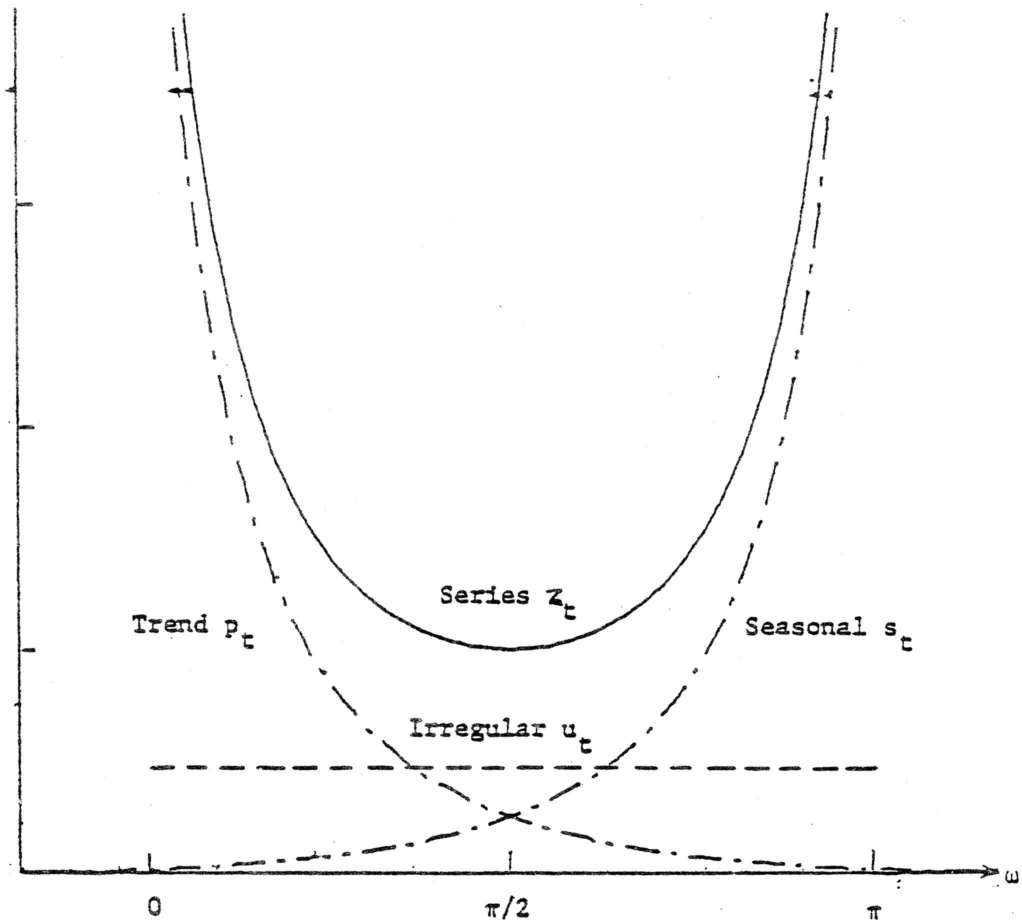
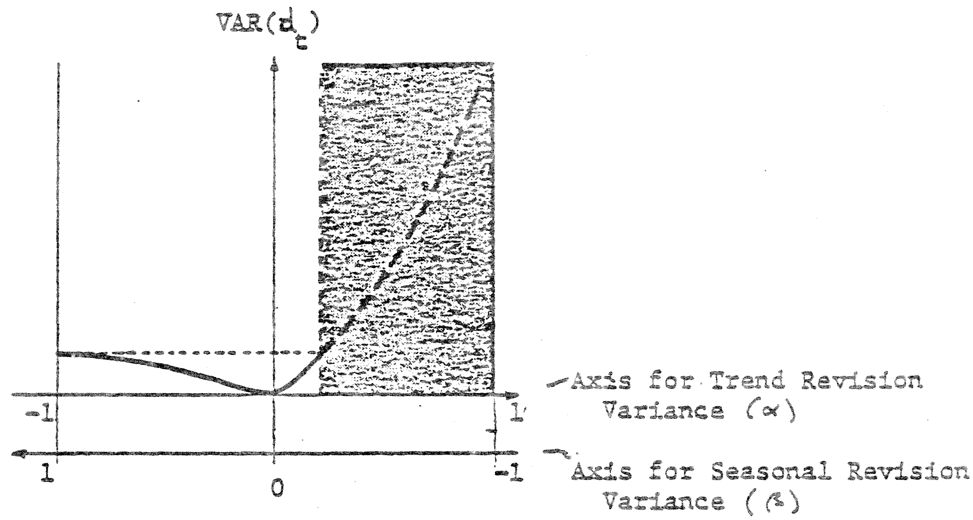


Figure 3

Variance of Revisions in
Concurrent Estimates



REFERENCES

Bell, W.R. and Hillmer, S.C. (1984), "Issues Involved with the Seasonal Adjustment of Economic Time Series," Journal of Business and Economic Statistics, 2,4.

Box, G.E.P., and Hillmer, S.C., and Tiao, G.C. (1979), "Analysis and Modelling of Seasonal Time Series," in Seasonal Analysis of Economic Time Series, ed. Arnold Zellner, Washington, DC: U.S. Department of Commerce, Bureau of the Census, 309-334.

Cleveland, W.P. and Tiao, G.C. (1976), "Decomposition of Seasonal Time Series: A Model for the Census X-11 Program", Journal of the American Statistical Association, 71, 581-587.

Hillmer, S.C. and Tiao, G.C. (1982), "An ARIMA Model-Based Approach to Seasonal Adjustment", Journal of the American Statistical Association, 77, 63-70.

Pierce, D.A., (1980), "Data Revisions with Moving Average Seasonal Adjustment Procedures", Journal of Econometrics, 14, pp. 95-114.

Pierce, D.A. and Maravall, A. (1984), "Advances in Model-Based Seasonal Adjustment", 1984 Proceedings of the Business and Economic Statistics Section, American Statistical Association.

DOCUMENTOS DE TRABAJO:

7801 **Vicente Poveda y Ricardo Sanz**: Análisis de regresión: algunas consideraciones útiles para el trabajo empírico (*).
7802 **Julio Rodríguez López**: El PIB trimestral de España, 1958-1975. Avance de cifras y comentarios (*). (Publicadas nuevas versiones en Documentos de Trabajo núms. 8211 y 8301).
7803 **Antoni Espasa**: El paro registrado no agrícola 1964-1976: un ejercicio de análisis estadístico univariante de series económicas (*). (Publicado en Estudios Económicos n.° 15).
7804 **Pedro Martínez Méndez y Raimundo Poveda Anadón**: Propuestas para una reforma del sistema financiero.
7805 **Gonzalo Gil**: Política monetaria y sistema financiero. Respuestas al cuestionario de la CEE sobre el sistema financiero español (*). Reeditado con el número 8001.
7806 **Ricardo Sanz**: Modelización del índice de producción industrial y su relación con el consumo de energía eléctrica.
7807 **Luis Angel Rojo y Gonzalo Gil**: España y la CEE. Aspectos monetarios y financieros (*).
7901 **Antoni Espasa**: Modelos ARIMA univariantes, con análisis de intervención para las series de agregados monetarios (saldos medios mensuales) M_3 y M_2.
7902 **Ricardo Sanz**: Comportamiento del público ante el efectivo (*).
7903 **Nicolás Sánchez-Albornoz**: Los precios del vino en España, 1861-1890. Volumen I: Crítica de la fuente.
7904 **Nicolás Sánchez-Albornoz**: Los precios del vino en España, 1861-1890. Volumen II: Series provinciales.
7905 **Antoni Espasa**: Un modelo diario para la serie de depósitos en la Banca: primeros resultados y estimación de los efectos de las huelgas de febrero de 1979.
7906 **Agustín Maravall**: Sobre la identificación de series temporales multivariantes.
7907 **Pedro Martínez Méndez**: Los tipos de interés del Mercado Interbancario.
7908 **Traducción de E. Giménez-Arnau**: Board of Governors of the Federal Reserve System-Regulations AA–D–K–L–N–O–Q (*).
7909 **Agustín Maravall**: Effects of alternative seasonal adjustment procedures on monetary policy.
8001 **Gonzalo Gil**: Política monetaria y sistema financiero. Respuestas al cuestionario de la CEE sobre el sistema financiero español (*).
8002 **Traducción de E. Giménez-Arnau**: Empresas propietarias del Banco. Bank Holding Company Act-Regulation «Y» (*).
8003 **David A. Pierce, Darrel W. Parke, and William P. Cleveland, Federal Reserve Board and Agustín Maravall, Bank of Spain**: Uncertainty in the monetary aggregates: Sources, measurement and policy effects.
8004 **Gonzalo Gil**: Sistema financiero español (*). (Publicada una versión actualizada en Estudios Económicos n.° 29).
8005 **Pedro Martínez Méndez**: Monetary control by control of the monetary base: The Spanish experience (la versión al español se ha publicado como Estudio Económico n.° 20).
8101 **Agustín Maravall, Bank of Spain and David A. Pierce, Federal Reserve Board**: Errors in preliminary money stock data and monetary aggregate targeting.
8102 **Antoni Espasa**: La estimación de los componentes tendencial y cíclico de los indicadores económicos.
8103 **Agustín Maravall**: Factores estacionales de los componentes de M_3. Proyecciones para 1981 y revisiones, 1977-1980.
8104 **Servicio de Estudios**: Normas relativas a las operaciones bancarias internacionales en España.
8105 **Antoni Espasa**: Comentarios a la modelización univariante de un conjunto de series de la economía española.
8201 **Antoni Espasa**: El comportamiento de series económicas: Movimientos atípicos y relaciones a corto y largo plazo.
8202 **Pedro Martínez Méndez e Ignacio Garrido**: Rendimientos y costes financieros en el Mercado Bursátil de Letras.

8203 **José Manuel Olarra y Pedro Martínez Méndez**: La Deuda Pública y la Ley General Presupuestaria.
8204 **Agustín Maravall**: On the political economy of seasonal adjustment and the use of univariate time-series methods.
8205 **Agustín Maravall**: An application of nonlinear time series forecasting.
8206 **Ricardo Sanz**: Evaluación del impacto inflacionista de las alzas salariales sobre la economía española en base a las tablas input-output.
8207 **Ricardo Sanz y Julio Segura**: Requerimientos energéticos y efectos del alza del precio del petróleo en la economía española.
8208 **Ricardo Sanz**: Elasticidades de los precios españoles ante alzas de diferentes inputs.
8209 **Juan José Dolado**: Equivalencia de los tests del multiplicador de Lagrange y F de exclusión de parámetros en el caso de contrastación de perturbaciones heterocedásticas.
8210 **Ricardo Sanz**: Desagregación temporal de series económicas (*).
8211 **Julio Rodríguez y Ricardo Sanz**: Trimestralización del producto interior bruto por ramas de actividad. (Véase Documento de Trabajo n.° 8301).
8212 **Servicio de Estudios. Estadística**: Mercado de valores: Administraciones Públicas. Series históricas (1962-1981).
8213 **Antoni Espasa**: Una estimación de los cambios en la tendencia del PIB no agrícola, 1964-1981.
8214 **Antoni Espasa**: Problemas y enfoques en la predicción de los tipos de interés.
8215 **Juan José Dolado**: Modelización de la demanda de efectivo en España (1967-1980).
8216 **Juan José Dolado**: Contrastación de hipótesis no anidadas en el caso de la demanda de dinero en España.
8301 **Ricardo Sanz**: Trimestralización del PIB por ramas de actividad series revisadas
8302 **Cuestionario OCDE. Servicio de Estudios. Estadística**. Cuadro de flujos financieros de la economía española (1971-1981) (*).
8303 **José María Bonilla Herrera** y **Juan José Camio de Allo**: El comercio mundial y el comercio exterior de España en el período 1970-1981: Algunos rasgos básicos.
8304 **Eloísa Ortega**: Indice de precios al consumo e índice de precios percibidos.
8305 **Servicio de Estudios. Estadística**: Mercado de Valores: Instituciones financieras. Renta fija. Series históricas (1962-1982).
8306 **Antoni Espasa**: Deterministic and stochastic seasonality: an univariate study of the Spanish Industrial Production Index.
8307 **Agustín Maravall**: Identificación de modelos dinámicos con errores en las variables.
8308 **Agustín Maravall, Bank of Spain and David A. Pierce, Federal Reserve Board**: The transmission of data noise into policy noise in monetary control.
8309 **Agustín Maravall**: Depresión, euforia y el tratamiento de series maníaco-depresivas: el caso de las exportaciones españolas.
8310 **Antoni Espasa**: An econometric study of a monthly indicator of economic activity.
8311 **Juan José Dolado**: Neutralidad monetaria y expectativas racionales: Alguna evidencia en el caso de España.
8312 **Ricardo Sanz**: Análisis cíclicos. Aplicación al ciclo industrial español.
8313 **Ricardo Sanz**: Temporal disaggregation methods of economic time series.
8314 **Ramón Galián Jiménez**: La función de autocorrelación extendida: Su utilización en la construcción de modelos para series temporales económicas.
8401 **Antoni Espasa y María Luisa Rojo**: La descomposición del indicador mensual de cartera de pedidos en función de sus variantes explicativas.
8402 **Antoni Espasa**: A quantitative study of the rate of change in Spanish employment.
8403 **Servicio de Producción y Demanda Interna**: Trimestralización del PIB por ramas de actividad, 1975-1982.
8404 **Agustín Maravall**: Notas sobre la extracción de una señal en un modelo ARIMA.
8405 **Agustín Maravall**: Análisis de las series de comercio exterior —I—.
8406 **Ignacio Mauleón**: Aproximaciones a la distribución finita de criterios Ji- cuadrado: una nota introductoria.
8407 **Agustín Maravall**: Model-based treatment of a manic-depressive series.
8408 **Agustín Maravall**: On issues involved with the seasonal adjustment of time series.

* *Las publicaciones señaladas con un asterisco se encuentran agotadas.*

Información: Banco de España, Servicio de Publicaciones. Alcalá, 50. 28014 Madrid.