STABILITY TESTING IN REGRESION MODELS

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Abstract

The paper presents stability tests for regression models, that cover most cases of practical interest. Special attention is paid to the computational aspects of the problem so that most formulae given in the paper can be readily implemented with existing econometric software packages. It is argued that stability tests are an important class of tests in applied research as a safeguard against data mining and pretesting.

Keywords

Stability tests, regression models, computational feasibility, data mining.

Resumen

Este trabajo presenta contrastes de estabilidad para modelos de regresión en la mayor parte de los casos empíricos de interés. Se ha puesto especial enfásis en presentar las fórmulas de modo que sean fácilmente aplicables con los programas existentes. El trabajo argumenta que los contrastes de estabilidad son esenciales para aliviar los problemas del 'agotamiento de los datos' y de los contrastes sucesivos de hipótesis.

Palabras clave

Contrastes de estabilidad, modelos de regresión, facilidad de cálculo, agotamiento de los datos.

I.- Introduction

Stability tests have been generally regarded as a means of testing for a structural break. They are certainly so, but their scope is wider from the viewpoint of applied research.

Two questions that arise in applied work and that do not fit very well into the standard statistical framework are the problems of data mining and pretesting. There is a large probability that we find a statistically significant correlation between two variables in a large set of independent variables, and this is the data mining problem. Put it in other words, by trying different variables in an equation we are likely to find significant regressors at the conventional 't' levels, even if they are totally meaningless. The other problem comes from the sequential procedure used in applied work. Let us suppose now that we test a hypothesis by means of a statistic 's_1', and that we make a decision about it. If we are to test yet a second hypothesis with another statistic 's_2', the distribution of s_2 depends on the decision rule laid down for testing the first hypothesis. This is basically the problem of pretesting and its main implication is that conventional significance levels are incorrect.

In principle, there is a simple solution to both problems. If we have enough observations we can split the sample in two (or more) subsamples, and then estimate the model with one, using the second fresh data to test it. Since in practice we do not have samples large enough to follow this procedure, the obvious alternative is to check for stability over a small part of the sample. This is why stability tests are an essential part of applied modelling.

The paper discusses first stability tests in the ordinary least squares context. This is important in itself, and is also useful to provide correction factors derived from exact theory, for asymptotic approximations. Most results in this context are known, and section II tries to give a compact and thorough account of them. Section III presents stability tests for single structural equations estimated by IV, and section IV for simultaneous equations models. Stability testing under these last conditions has not been explored very well in the literature, and the paper presents tests for a variety of practical situations, stressing the computational aspects of the problem. That is, most tests are presented in a form that can be readily implemented with existing software packages. This is specially important for simultaneous equations models.

Instability may arise for one of several reasons. We may have a change at a point in time of either the parameters of the regressors or the variance of the errors. It might also happen that the parameters are themselves a stochastic process. This last possibility is somewhat difficult to implement in practice since the alternative is not defined in a clearcut way. The problem of variance constancy or homoskedasticity, is also important and there is already a vast literature about it. This paper concentrates on the first type of test.

II.- Stability tests in the least squares context

The problem of testing a structural break in a single equation estimated by ordinary least squares (OLS) can be conveniently tackled in a fairly general and unified way following the framework proposed by Fisher [5]. The analytical set up of the problem is given as follows,

$$\varepsilon \sim N\,(O,\ \sigma I)$$
$$MM = M = M',\ M^{+}M^{+} = M^{+} = M^{+'},\ MM^{+} = M^{+} \qquad (2.1)$$

where M and M^{+} are matrices of order T, and 'ε' is a vector of the same dimension. Then it is well known that (see for example [5, 8]),

$$\left(\frac{\varepsilon' M\varepsilon - \varepsilon' M^{+}\varepsilon}{\varepsilon' M^{+}\varepsilon}\right) \frac{\text{tr } M^{+}}{\text{tr}(M-M^{+})} \sim F_{\text{tr}(M-M^{+}),\ \text{tr } M^{+}} \qquad (2.2)$$

The standard regression model can be written as

$$y = XB + \varepsilon \qquad (2.3)$$

where 'x' is a (TxK) matrix of observations on k non stochastic regressors. Any linear restriction on B can be reparametrized so that the new model can be written in terms of an unrestricted vector B^{o} as

$$y = x^{o}B^{o} + \varepsilon \qquad (2.4)$$

and x^{o} is now a linear combination of the columns in X, that is, $x^{o} = xH$. Defining now,

$$M = I - x^{o}(x^{o'}x^{o})^{-1}\,x^{o'}$$
$$M^{+} = I - x\,(x'\,x)^{-1}\,x' \qquad (2.5)$$

we have

$$M\,M^{+} = M^{+} - x^{o}(x^{o'}x^{o})^{-1}\,x^{o'}M^{+}$$
$$= M^{+}$$

since $\quad x^{o'}\;M^{+} = H'\;x'\;M^{+} = H'\;O = \;O$ $\qquad$ (2.6)

Provided $tr(M - M^{+})>0$, the test given in (2.2) is valid, and therefore the exact finite sample distribution of all Wald type of tests can be derived easily. The distribution of the Chow type of test can also be obtained very easily in this framework. Just to show an application, let us consider a somewhat messy case, where it is wished to test the constancy of a subvector of coefficients over three different subperiods. We start by partitioning the regression matrix as

$$x = \begin{bmatrix} z_1 & w_1 & & \\ z_2 & & w_2 & \\ z_3 & & & w_3 \end{bmatrix} \qquad (2.7)$$

where z_i is $(T_i \times p)$, w_i $(T_i \times q)$, and $T = T_1 + T_2 + T_3$. It is convenient to define

$$x^{o} = \begin{bmatrix} z_1 & & w_1 & \\ & z_2 & & w_2 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} z_1 & w_1 & \\ z_2 & & w_2 \end{bmatrix} = \quad x^{o}\;H$$

$$M^{+} = \begin{bmatrix} M^{o} & O \\ O & O \end{bmatrix} \quad ,$$

$$M = I - x\;(x'x)^{-1}x' \quad , \quad M^{o} = I - x^{o}\;(x^{o'}x^{o})^{-1}\;x^{o'} \qquad (2.8)$$

Let us consider the case in which a separate regression can be run on (z_1,w_1) and (z_2,w_2) but not on (z_3,w_3), that is, $(T_1,\;T_2) > (p + q)$, $q < T_3 < (p + q)$. The requirement $T_3 > q$, is needed for identification of the vector associated to w_3. Then from (2.8)

$$x' \, M^{+} = \begin{bmatrix} x_1' \, M^{o}, & 0 \\ 0 \; M^{o} \, , & \end{bmatrix} = 0 \tag{2.9}$$

We also have that

$$tr \, M^{+} = tr \, M^{o} = T_1 + T_2 - 2 \, (p + q)$$

$$tr \, M = T - (p + 3q)$$

$$tr \, (M - M^{+}) = T_3 + (p - q) \tag{2.10}$$

so that the test is easily set up as

$$\frac{n_2}{n_1} \quad \frac{e'e - (e_1' \, e_1 + e_2' \, e_2)}{e_1' \, e_1 + e_2' \, e_2} = F_{(n_1, \, n_2)}$$

$$n_1 = T_3 + p - q$$

$$n_2 = T_1 + T_2 - 2 \, (p + q) \tag{2.11}$$

where $(e_1' \, e_1)$, $(e_2' \, e_2)$ are he unrestricted sum of squares over the first and second subsamples, and $(e' \, e)$ the restricted sum of squares over the whole sample.

From the viewpoint of asymptotic theory, expression (2.1) is useful in the sense that it provides a natural way of deriving 'correction factors' for tests in more complex situations. This 'ad-hoc' procedure, has been found to act in the right direction frequently, in Monte Carlo experiments. Some theoretical justification for the use of correction factors can also be found in [7].

We can also note that the maximum likelihood ratio counterpart of (2.11) is given by

$$T \log (S_R/S_{UR}) \underset{A}{\sim} X^2_{T_3 + p - q} \tag{2.12}$$

where S_R and S_{UR} are respectively the restricted and unrestricted sums of squared residuals over the whole sample. It is in this sense, that both Wald and Chow tests can be regarded as belonging to a wider class, that is, the likelihood ratio tests. Loosely speaking, both are the same tests for different situations (see also [4]).

It is of some interest to derive the Chow test as a prediction test. This property was pointed out by Chow [3], and highlights its relationship to other tests. As it will be seen, the Chow test is a corrected prediction test, in the sense that it takes account of the variance of the estimated parameter vector in the regression. Let us define then,

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\tilde{B} = (x' \ x)^{-1} \ x' \ y$$

$$\hat{B} = (x_1' \ x_1)^{-1} \ x_1' \ y_1$$

$$\tilde{\varepsilon}_i = y_i - x_i \ \tilde{B} , \qquad i = 1,2$$

$$\hat{\varepsilon} = y_1 - x_1 \ \hat{B}$$

$$\hat{\varepsilon}_f = y_2 - x_2 \ \hat{B} \tag{2.13}$$

Since $(x'x) \ \tilde{B} = x' \ y$, we can write

$$(x_1' \ x_1) \ \tilde{B} + (x_2' \ x_2) \ \tilde{B} = x_1' \ y_1 + x_2' \ y_2$$

$$\tilde{B} = \hat{B} + (x_1' \ x_1)^{-1} \ x_2' \ (y_2 - x_2\tilde{B})$$

$$= \hat{B} + (x_1' \ x_1)^{-1} \ x_2' \ \ \tilde{\varepsilon}_2 \tag{2.14}$$

and we get from a well known result (see for example [8]),

$$\tilde{\varepsilon}_1' \ \tilde{\varepsilon}_1 = \hat{\varepsilon}' \ \hat{\varepsilon} + (\tilde{B} - \hat{B}) \ x_1' \ x_1 \ (\tilde{B} - \hat{B})$$

$$= \hat{\varepsilon}'\hat{\varepsilon} + \tilde{\varepsilon}_2'x_2 \ (x_1' \ x_1)^{-1}x_2'\tilde{\varepsilon}_2 \tag{2.15}$$

Adding $\tilde{\varepsilon}_2' \, \tilde{\varepsilon}_2$ in this last expression yields

$$\tilde{\varepsilon}'\tilde{\varepsilon} = \hat{\varepsilon}'\hat{\varepsilon} + \tilde{\varepsilon}_2'\tilde{\varepsilon}_2 + \tilde{\varepsilon}_2' \, x_2(x_1' \, x_1)^{-1} \, x_2' \, \tilde{\varepsilon}_2$$

$$\tilde{\varepsilon}'\tilde{\varepsilon} - \hat{\varepsilon}'\hat{\varepsilon} = \varepsilon_2'(I + x_2(x_1' \, x_1)^{-1}x_2' \,) \, \varepsilon_2$$

$$= \tilde{\varepsilon}_2' \, D \, \tilde{\varepsilon}_2 \qquad (2.16)$$

in a self evident notation. From (14) we can write

$$x_2 \, \tilde{B} = x_2 \, \hat{B} + x_2(x_1' \, x_1)^{-1} \, x_2' \quad \tilde{\varepsilon}_2$$

$$\tilde{\varepsilon}_2 = \hat{\varepsilon}_f - x_2' \, (x_1' \, x_1)^{-1} \, x_2' \, \tilde{\varepsilon}_2$$

$$\hat{\varepsilon}_f = (I + x_2 \, (x_1' \, x_1)^{-1} \, x'_2) \, \tilde{\varepsilon}_2$$

$$= D \, \tilde{\varepsilon}_2 \qquad (2.17)$$

so that we finally have the Chow test written as

$$\left(\frac{T_1-K}{T_2}\right) \left(\frac{\tilde{\varepsilon}'\tilde{\varepsilon}-\hat{\varepsilon}'\hat{\varepsilon}}{\hat{\varepsilon}'\hat{\varepsilon}}\right) = \frac{\hat{\varepsilon}_f D^{-1}\hat{\varepsilon}_f}{\hat{\varepsilon}' \, \hat{\varepsilon}} \left(\frac{T_1-K}{T_2}\right) - F_{(T_2, \, T_1-K)} \qquad (2.18)$$

But now,

$$\hat{\varepsilon}_f = y_2 - x_2 \, \hat{B}$$

$$= \varepsilon_2 - x_2 \, (x_1'x_1)^{-1} \, x_1' \, \varepsilon_1 \qquad (2.19)$$

so that

$$V(\hat{\varepsilon}_f) = \sigma^2 \, (I + x_2 \, (x_1'x_1)^{-1} \, x_2')$$

$$= \sigma^2 \, D \qquad (2.20)$$

that is, the Chow test is a prediction test where the variance of the estimated vector $\hat{B}$, has been taken into account. This also immediately shows that as $T_1 \to \infty$, the test shrinks to a conventional prediction test. Therefore, the Chow test is valid for dynamic models in the asymptotic sense. It is very likely, that in these models, the Chow form of the prediction test, introduces an adequate correction for small T_1.

The asymptotic distribution of the Chow type of test can be derived in another way, that will enable us to deal with autoregressive errors. Let us consider then,

$$\tilde{\varepsilon}'\tilde{\varepsilon} - \hat{\varepsilon}_1' \hat{\varepsilon}_1 = \tilde{\varepsilon}_2'\tilde{\varepsilon}_2 + \tilde{\varepsilon}_1'\tilde{\varepsilon}_1 - \hat{\varepsilon}_2'\hat{\varepsilon}_2 \qquad (2.21)$$

where the superscripts (~) (^), denote estimators with the whole sample and the first T_1 observations respectively, and the subindices $(\ )_1$, $(\ )_2$ denote the set of the first T_1 and the last T_2 observations, respectively.

We have now

$$\tilde{\varepsilon}_1' \tilde{\varepsilon}_1 - \hat{\varepsilon}_1' \hat{\varepsilon}_1 = (\hat{B} - \tilde{B})\ x_1'\ x_1\ (\hat{B} - \tilde{B})$$

$$+ 2\ \hat{\varepsilon}_1'\ x_1\ (\hat{B} - \tilde{B}) \qquad (2.22)$$

Let us suppose now that $B = B(\Theta)$ where $B(.)$ is twice differentiable and the derivatives are bounded. Then, the estimator B, is obtained by solving the non linear equations,

$$0 = \left(\frac{\delta B}{\delta \Theta}\right) x'\ (y - xB(\Theta))$$

$$= \left(\frac{\delta B}{\delta \Theta}\right) \left[\left(\frac{x_1'y_1}{T_1} - \frac{x_1'x_1}{T_1}\ B(\Theta)\right) + \left(\frac{x_2'y_2}{T_1} - \frac{x_2'x_2}{T_1}\ B(\Theta)\right)\right] \qquad (2.23)$$

and the estimate $\hat{B}$, is obtained neglecting the second part in (2.23). Since this last expression is obviously $O(T_1^{-1})$ we have $(\hat{B} - \tilde{B}) = O\ (T_1^{-1})$.

It is also easy to see that under the null,

$$\frac{\hat{\varepsilon}_1' x_1}{T_1} = \frac{\varepsilon_1' x_1}{T_1} + O\ (T_1^{-1/2}) = O\ (T_1^{-1/2}) \qquad (2.24)$$

Considering now the first element in (2.21) we have

$$\tilde{\varepsilon}_2 = \varepsilon_2 + x_2\ (\tilde{B} - B) = \varepsilon_2 + O\ (T^{-1/2})$$

$$\tilde{\varepsilon}_2'\ \tilde{\varepsilon}_2 = \varepsilon_2'\ \varepsilon_2 + O\ (T^{-1/2}) \qquad (2.25)$$

so that we can finally write

$$\tilde{\varepsilon}'\ \tilde{\varepsilon} - \hat{\varepsilon}_1'\ \hat{\varepsilon}_1 = \varepsilon_2'\ \varepsilon_2 + O\ (T^{-1/2})$$

$$\frac{\tilde{\varepsilon}'\ \tilde{\varepsilon} - \hat{\varepsilon}_1'\ \hat{\varepsilon}_1}{\tilde{\sigma}_2} \quad \tilde{A} \quad \chi^2_{T_2} \qquad (2.26)$$

provided plim $\tilde{\sigma}^2 = \sigma^2$. If we are in a linear case, $\hat{\varepsilon}'\ x_1 = 0$, and it is clear that the distribution in (2.26) is then valid for dynamic cases. The autoregressive error case, can be tackled in the framework of non linear restrictions and the above development is therefore immediately applicable.

For reasons that will become apparent later, it is convenient to present a further asymptotic extension of the more general case given in (2.11). Let us define

$$e'e = \tilde{\varepsilon}_3'\ \tilde{\varepsilon}_3 + \tilde{\varepsilon}_2'\ \tilde{\varepsilon}_2 + \check{\varepsilon}_1'\ \tilde{\varepsilon}_1$$

$$B' = (\gamma',\ \delta_1',\ \delta_2')$$

$$\begin{pmatrix} \tilde{\varepsilon}_1 \\ \tilde{\varepsilon}_2 \end{pmatrix} = (Y - x_1 \tilde{B}) \qquad (2.27)$$

where $(\gamma, \delta_1, \delta_2)$ is the vector associated to the 'x_1' matrix in (2.8) and the superindex (~) means that the estimate has been obtained under the restricted hypothesis.

Defining now,

$$U_1 = \begin{bmatrix} z_1 & w_1 & 0 \\ z_2 & 0 & w_2 \\ z_3 & 0 & 0 \end{bmatrix} \quad , \quad U_2 = \begin{bmatrix} 0 \\ 0 \\ w_3 \end{bmatrix} \qquad (2.28)$$

we have

$$\tilde{B} = (u_1' u_1)^{-1} u_1' (y - u_2 \tilde{\delta}_3)$$

$$= \left(\frac{u_1' u_1}{T_1}\right)^{-1} \left(\frac{u_1' y}{T_1} - \frac{u_1' u_2}{T_1} \tilde{\delta}_3\right) \qquad (2.29)$$

But then

$$\frac{u'_1 \; u_2}{T_1} = \begin{bmatrix} z_3' \; w_3/T_1 \\ 0 \end{bmatrix} = O \; (T_1^{-1})$$

$$\frac{u_1' \; y}{T_1} = \frac{x_1' \; y_1}{T_1} + O \; (T_1^{-1}) \qquad (2.30)$$

Therefore, we can define an estimator $\bar{B}$, neglecting the last T_3 observations and have,

$$\bar{B} = (x_1' \; x_1)^{-1} \; x_1' \; y_1$$

$$= \tilde{B} + O \; (T_1^{-1}) \qquad (2.31)$$

so that applying the result quoted in (2.15)

$$\tilde{\varepsilon}_2' \; \tilde{\varepsilon}_2 + \tilde{\varepsilon}_1' \tilde{\varepsilon}_1 - \hat{\varepsilon}_2' \; \hat{\varepsilon}_2 - \hat{\varepsilon}_1' \; \hat{\varepsilon}_1 =$$

$$= (\bar{B} - \hat{B})' \, x^{o'} \, x^{o} \, (\bar{B} - \hat{B}) + O\,(T_1^{-1}) \qquad (2.32)$$

and this last expression is the numerator of a standard Wald test, that is asymptotically distributed as a χ^2_p. It only depends on the errors (ε_1, ε_2) asymptotically and is valid for dynamic models.

Let us consider now $\tilde{\varepsilon}_3' \, \tilde{\varepsilon}_3$. By definition, (2.27)

$$\tilde{\varepsilon}_3 = y_3 - (z_3 \, \tilde{\gamma} + w_3 \, \tilde{\delta}_3) =$$

$$= z_3(\gamma - \tilde{\gamma}) + w_3(\delta_3 - \tilde{\delta}_3) + \varepsilon_3 \qquad (2.33)$$

From standard regression results we get

$$\tilde{\delta}_3' = (w_3'w_3)^{-1} \, w_3' \, (y_3 - z_3 \, \tilde{\gamma}) =$$

$$= \delta_3 + (w_3'w_3)^{-1} \, w_3'(\varepsilon_3 + z_3(\gamma - \tilde{\gamma})) \qquad (2.34)$$

Since $(\gamma - \tilde{\gamma}) = O\,(T_1^{-1/2})$, plugging (2.34) into (2.33) yields

$$\tilde{\varepsilon}_3 = (\varepsilon_3 - w_3(w_3' \, w_3)^{-1} \, w_3 \, \varepsilon_3 = M_w \, \varepsilon_3) + O\,(T^{-1})$$

If w_3 does not include lagged dependent variables, and noting the idempotency of w_3, we get

$$\tilde{\varepsilon}_3' \, \tilde{\varepsilon}_3 \;\tilde{A}\; {}^2\chi_{T_3 - q} \qquad (2.35)$$

This expression does not depend on ($\varepsilon_1, \varepsilon_2$) asymptotically and is therefore independent of that given in (2.31). We can therefore write in the notation of (2.11)

$$(T_1 + T_2) \frac{(e'\ e - (e_1'\ e_1 + e_2'\ e_2))}{e_1'\ e_1 + e_2'\ e_2} \quad \tilde{A} \quad \chi^2_{T_3 + p - q} \qquad (2.36)$$

as $(T_1, T_2) \to \infty$

and this is valid in the case that 'z' includes lagged dependent variables. It would be advisable to use the form given in (2.11) for small samples. If we were to test the stability of the whole vector, that is assuming under the null $\delta_1 = \delta_2 = \delta_3$, the degrees of freedom in (19) would be $(T_3 + p + q)$ and lagged dependent variables would be allowed to enter 'z' and 'w'.

III.- Testing the stability of a single structural equation

Let us take up now the case of testing the stability of a single structural equation estimated by instrumental variables (IV). The notation and assumptions are as follows:

$$y = x\,\alpha + \varepsilon \qquad\qquad \varepsilon \sim (0,\ \sigma)$$

$$\text{plim}\ (x'\varepsilon/T) = 0$$

$$\text{plim}\ (z'\varepsilon/T) = 0$$

$$|\text{plim}\ (z'z/T)| \neq 0$$

$$\text{rank}\ (\text{plim}\ (z'x/T)) = n_1 + k_1 \tag{3.1}$$

where 'x' is the matrix of T observations on the n_1+k_1 regressors and z are the observations on the k predetermined IV. Suppose now that we estimate (3.1) with T_1 observations and it is wished to check the stability of these estimates over the remaining $T-T_1=T_2$ observations. There are two possible situations according to whether 'T_2' is larger than 'k' or not. Let us consider first the case $T_2 > K$.

In an obvious notation we can split the set of observations as

$$x'=(x'_1\ x'_2),\ z'=(z'_1\ z'_2),\ y'=(y'_1\ y'_2) \tag{3.2}$$

and define the IV estimator for the first T_1 observations by

$$\alpha_1 = (x'_1\ \aleph_1\ x_1)^{-1}\ x'_1\ \aleph_1\ y_1$$

$$\aleph_1 = z_1(z'_1\ z_1)^{-1}\ z'_1 \tag{3.3}$$

Since $T_2 > K$, $(z_2' z_2)^{-1}$ is well defined so that we can define the IV estimator for the second subsample similarly to (3.3). It is more or less clear that both estimates are independent since they will depend finally on a different set of errors 'ε'. More formally,

$$\begin{pmatrix} \tilde{\alpha}_1 \\ \tilde{\alpha}_2 \end{pmatrix} = \begin{bmatrix} (x_1' \mathfrak{X}_1 x_1)^{-1} & \\ & (x_2' \mathfrak{X}_2 x_2)^{-1} \end{bmatrix} \begin{bmatrix} (x_1' \mathfrak{X}_1 \ y_1) \\ (x_2' \mathfrak{X}_2 \ y_2) \end{bmatrix}$$

$$= \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{bmatrix} x_1' \mathfrak{X}_1 x_1 & \\ & x_2' \mathfrak{X}_2 \ x_2 \end{bmatrix}^{-1} \begin{bmatrix} x_1' \mathfrak{X}_1 & \\ & x_2' \mathfrak{X}_2 \end{bmatrix} \varepsilon \qquad (3.4)$$

where $\mathfrak{X}_2 = z_2 \ (z'_2 \ z_2)^{-1} \ z'_2$

From the idempotency of $\mathfrak{X}_1$, $\mathfrak{X}_2$, it follows that both estimators are asymptotically independent. Then, we can easily set up a Wald test to check the null hypothesis $\alpha_1 = \alpha_2$, as follows,

$$C_1 = (\tilde{\alpha}_1 - \tilde{\alpha}_2)' [(x_1' \mathfrak{X}_1^{-1} x_1) + (x_2' \mathfrak{X}_2^{-1} x_2) \ ]^{-1} \ (\tilde{\alpha}_1 - \tilde{\alpha}_2) \ / \ \tilde{\sigma}^2$$

$$\underset{A}{\sim} \ \chi^2_{(n_1 + k_1)} \qquad (3.5)$$

where $\tilde{\sigma}^2$ is some consistent estimate of σ^2. In small samples it may be better to use the following approximation

$$\frac{T - n_1 - k_1}{T \ (n_1 + k_1)} \ C_1 - F_{(n_1 + k_1, \ T - n_1 - k_1)} \qquad (3.6)$$

if $\tilde{\sigma}^2$ is defined as $(\varepsilon'\varepsilon/T)$, ε being the IV errors.

In the form given in (3.5), the test is readily applicable with many existing software packages. Hewever, it is still possible to derive a further test, which may be easier to compute. Let us define the matrices

$$x^+ = \begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix}, \quad z^+ = \begin{bmatrix} z_1 & 0 \\ 0 & z_2 \end{bmatrix}, \quad \alpha^+ = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

$$\mathfrak{X}^+ = z^+(z^{+\prime} z^+)^{-1} z^{+\prime}, \quad R = (I, -I) \tag{3.7}$$

so that equation (3.1) can be written without imposing parameter constancy as

$$y = x^+ \alpha^+ + \varepsilon \tag{3.8}$$

The problem can now be thought of as a standard test of the linear restriction $R\,\alpha^+ = 0$, in an IV estimation context. The maximum likelihood ratio test of this restriction under the limited information assumption is know to be asymptotically equivalent to

$$T\left(\frac{\tilde{\varepsilon}'\,\mathfrak{X}^+\,\tilde{\varepsilon}}{\tilde{\varepsilon}'\,\tilde{\varepsilon}} - \frac{\hat{\varepsilon}'\,\mathfrak{X}^+\,\hat{\varepsilon}}{\hat{\varepsilon}'\,\hat{\varepsilon}}\right) \;\tilde{A}\; \chi^2_{(n_1+k_1)} \tag{3.9}$$

where $\tilde{\varepsilon} = y - x^+ \tilde{\alpha}^+$, and $\tilde{\alpha}^+$ is the limited information maximum likelihood estimator (LIML)of α^+. Similarly, $\hat{\varepsilon} = y - x^+ \hat{\alpha}^+$, and $\hat{\alpha}^+$ is the LIML restricted estimator of α^+. Since these estimators have the same asymptotic distribution that the IV estimators, we can alternatively define the errors by

$$\tilde{\varepsilon} = y - x^+ \tilde{\alpha}\,, \quad \tilde{\alpha} = (x^{+\prime}\,\mathfrak{X}^+\,x^+)^{-1}\,x^{+\prime}\,\mathfrak{X}^+\,y$$

$$\hat{\varepsilon} = y - x\,\hat{\alpha}\,, \quad \hat{\alpha} = (x'\,\mathfrak{X}^+\,x)^{-1}\,x'\,\mathfrak{X}\,y \tag{3.10}$$

and (3.9) will retain the same asymptotic distribution. Then, we can implement (3.9) by means of two auxiliary regressions as follows

$$C_2 = T\,[R^2(\tilde{\varepsilon} \mid z^+) - R^2(\hat{\varepsilon} \mid z^+)] \underset{A}{\sim} \chi^2_{(n_1+k_1)} \qquad (3.11)$$

where $R^2(\tilde{\varepsilon} \mid z^+)$ is the R^2 obtained by regressing $\tilde{\varepsilon}$ on z^+, and similarly $R^2(\hat{\varepsilon} \mid z^+)$. The restricted estimator of α^+, $\hat{\alpha}^+$ as it is defined in (3.10) is not the common estimator, although it is asymptotically equivalent. The reason to consider it is that it simplifies the algebra considerably (Other possible IV estimators under structural break are considered in [1]).

If 'α' depends on a vector of 'p' parameters 'Θ', $(p<n_1 + k_1)$, and $\alpha(\Theta)$ is twice differentiable being the derivatives bounded, the maximum likelihood ratio test of (3.9) applies straightforwardly and it may be computed as in (3.11). The test will have now 'p' degrees of freedom. The autoregressive errors situation clearly falls under this category and the test is therefore readily applicable.

If $T_2 < k$, $(z_2' z_2)^{-1}$ is not defined and the obvious alternative is to compute a stability test based on predictive accuracy. Let us define then,

$$f_{T+s} = (\tilde{Y}_{T+s} - Y_{T+s}) = x_{T+s}\,(\alpha - \alpha) - \varepsilon_{T+s} \qquad (3.12)$$

and we immediately have

$$f^2_{T+s} = \varepsilon^2_{T+s} + O\,(T^{-1/2}) \qquad (3.13)$$

so that we can set up an asymptotic test for the prediction errors as

$$C_3 = \sum_{s=1}^{n} f^2_{T+s} / \sigma^2 \underset{A}{\sim} \chi^2_n \qquad (3.14)$$

If $\tilde{\sigma}^2$ is defined by $\tilde{\sigma}^2 = (\tilde{\varepsilon}'\tilde{\varepsilon}/T)$, where $\tilde{\varepsilon}$ are the IV errors, it may be a good idea to correct in small samples the test as in preceding cases.

If $\alpha = \alpha(\Theta)$, (3.12) is identical, and therefore the autoregressive errors case can be dealt with in very much the same way.

It may be of some interest to remark that in this case, the analogue of the Chow test does not have the standard χ^2 distribution with T_2 degrees of freedom. This can be seen as follows: let us write in the notation of (2.21) where the errors are now IV errors,

$$\tilde{\varepsilon}'\tilde{\varepsilon} - \hat{\varepsilon}'_1\hat{\varepsilon}_1 = \tilde{\varepsilon}'_2\tilde{\varepsilon}_2 + \tilde{\varepsilon}'_1\tilde{\varepsilon}_1 - \hat{\varepsilon}'_1\hat{\varepsilon}_1$$

$$\tilde{\varepsilon}_1 = \hat{\varepsilon}_1 + x_1(\hat{\alpha} - \tilde{\alpha})$$

$$\tilde{\varepsilon}'_1\tilde{\varepsilon}_1 = \hat{\varepsilon}'_1\hat{\varepsilon}_1 + (\hat{\alpha} - \tilde{\alpha})\, x'_1 x_1 (\hat{\alpha} - \tilde{\alpha}) + $$

$$+ 2\hat{\varepsilon}'_1 x_1 (\hat{\alpha} - \tilde{\alpha}) \qquad (3.15)$$

But now we have that,

$$\hat{\varepsilon}'_1 x_1 (\hat{\alpha} - \tilde{\alpha}) = \frac{\hat{\varepsilon}'_1 x_1}{T_1} (\hat{\alpha} - \tilde{\alpha})\, T_1 = O(1) \qquad (3.16)$$

since $(\hat{\varepsilon}'_1 x_1 / T_1) \not\to 0$ because of the simultaneity problem.

IV.- Stability tests for simultaneous equations models

The tests presented in previous sections can be readily extended to cover the multiequational model. First of all, in the framework of section II, let us suppose that,

$$y = XB + \varepsilon , \qquad \varepsilon \sim N(O,\Omega) \tag{4.1}$$

and then it is immediate that

$$(\tilde{\varepsilon}' \; \Omega^{-1} \; \tilde{\varepsilon} - \hat{\varepsilon}_1' \; \Omega^{-1} \; \hat{\varepsilon}_1) \sim \chi^2_{T_2} \tag{4.2}$$

where $\tilde{\varepsilon}$ and $\hat{\varepsilon}$ are the generalized least squares errors of model (4.1), and the notation is basically that of section II. We can use (4.2) to obtain an immediate generalization, but before that we need some notation.

Let us define a simultaneous equation model by,

$$BY_t + C\; z_t = \varepsilon_t , \varepsilon_t \sim N(O,\Sigma), \; E(\varepsilon_t \varepsilon_s') = O, \; t \neq s \tag{4.3}$$

where B is squared of order 'n', and there are 'k' predetermined variables z. The reduced form is given by

$$\begin{aligned} Y_t &= -B^{-1} C z_t + B^{-1} \varepsilon_t \\ &= \Pi \; z_t + v_t , \quad V(v_t) = \Omega \end{aligned} \tag{4.4}$$

I define also $z' = (z_1, --, z_T)$ and similarly for other matrices (The notation is basically that of [6]. The superscripts (~), (^) will denote maximum likelihood estimators with the whole sample and the first T_1 observations respectively. The subindex $(\;)_1$ will denote the set of the first T_1 observations, and $(\;)_2$

the last $T_2 (T_1 + T_2 = T)$. The vectorization of a matrix is defined as the vector obtained stacking its rows, and is indicated by vec (A).

Supposing for the time being that 'z' does not include any lagged dependent variable, we can generalize (4.2) immediately and write,

$$(\text{vec } \tilde{V}')'(\Omega^{-1} \otimes I)\ (\text{vec } \tilde{V}') - (\text{vec } \hat{V}_1')'(\Omega^{-1} \otimes I)\ (\text{vec } \hat{V}_1') \sim \chi^2_{nT_2} \qquad (4.5)$$

This expression can be written as

$$\text{tr } \Omega^{-1}(\tilde{V}'\ \tilde{V} - \hat{V}_1'\ \hat{V}_1) = \text{tr } \hat{\Omega}^{-1}(\tilde{V}'\ \tilde{V} - \hat{V}_1'\ \hat{V}_1) + O(T^{-1}) \qquad (4.6)$$

provided $\hat{\Omega}$ is a consistent estimator of Ω. From the derivation in II – (21,26) it is also clear that the previous result can be extended straightforwardly to cover both dynamic and autoregressive errors cases. It is also evident that the test can be cast as a prediction test, that is,

$$\text{tr } \hat{\Omega}^{-1}(\tilde{V}'\tilde{V} - \hat{V}_1'\hat{V}_1) = \text{tr } \hat{\Omega}\ (\bar{V}_f'\ \bar{V}_f) + O(T_1^{-1}) \qquad (4.7)$$

where $\bar{V}_f$ are the forecast errors for the second period given by

$$\bar{V}_f' = Y_2' - z_2'\ \hat{\Pi} \qquad (4.8)$$

and $\hat{\Pi}$ is the maximum likelihood estimator of Π, based on the first T_1 observations.

We also note that denoting by $\bar{E}_f$ the forecast errors for the structural form we get

$$\bar{E}_f{}' = \hat{B}\, y_2{}' + \hat{c}\, z_2{}'$$

$$\hat{B}^{-1}\, \bar{E}_f{}' = y_2{}' - \hat{\Pi}\, z_2{}' = \bar{\upsilon}_f{}' \qquad (4.9)$$

and if $(\hat{\Sigma}, \hat{\Omega}, \hat{B}, \hat{c})$ are maximum likelihood estimators with the first T_1 observations, then

$$\hat{\Omega} = \hat{B}^{-1}\, \hat{\Sigma}\, \hat{B}^{-1\prime} \qquad (4.10)$$

so that,

$$\text{tr}\ \hat{\Omega}^{-1}\bar{\upsilon}_f{}'\bar{\upsilon}_f = \text{tr}\ \hat{\Sigma}^{-1}\ (\bar{E}_f{}'\ \bar{E}_f) \qquad (4.11)$$

that is, the test can be written as a prediction test for, either the structural or reduced form errors.

The test given in (4.6) can be written as

$$\text{tr}\ \hat{\Omega}_1^{-1}(\tilde{\upsilon}'\ \tilde{\upsilon} - \hat{\upsilon}_1'\ \hat{\upsilon}_1) = T_1\ \text{tr}(\tilde{\upsilon}'\ \tilde{\upsilon}\ (\hat{\upsilon}_1'\ \hat{\upsilon}_1)^{-1} - I)$$

$$= T_1 \text{Log}\,|\tilde{\upsilon}'\ \tilde{\upsilon}\ (\hat{\upsilon}_1'\ \hat{\upsilon}_1)^{-1}| + O\ (T^{-1}) \qquad (4.12)$$

where the last step is based upon the fact

$$\text{Log}\,|I+B| = \text{tr}\ B + O\ (T^{-2}),\ \text{if}\ B = O(T^{-1}) \qquad (4.13)$$

If there are enough observations in the second subsample to estimate the structural parameters, the likelihood ratio test for stability is not a prediction test. Let us first look into the question of how large the second sample must be in this case. We need some extra notation first,

$$A = (B = C),\quad x = (y : z)$$

$$Ax' = By' + cz' = E'$$

$$\text{vec}\ A = s - \int \alpha \qquad Q' = (\Pi,\ I_k) \qquad (4.14)$$

where the last expression is just a reparametrization of the restrictions required to identify A. Vectorizing the system we can write

$$y^{+} = x^{+}\,\alpha + \varepsilon^{+}, \qquad \begin{aligned} y^{+} &= (I \otimes X)s \\ X^{+} &= (I \otimes X)\,\int \\ \varepsilon^{+} &= \text{vec } E' \end{aligned} \tag{4.15}$$

The maximum likelihood estimator of 'α' can be written as the solution of the following set of equations,

$$[x^{+\prime}(\Sigma^{-1}\otimes I)\; x^{+}]\;\alpha = \bar{x}^{+\prime}(\Sigma^{-1}\otimes I)\; y^{+} \tag{4.16}$$

where $\bar{x}^{+} = (I\otimes zQ')\int$. Denoting by '$m_i$' the number of unrestricted parameters in one equation we require

$$\text{rank }(\bar{x}_2^{+\prime}(\Sigma^{-1}\otimes I)\; x_2^{+}) = \sum_{i=1}^{n} m_i \tag{4.17}$$

for (4.16) to have a unique solution. Writing now

$$x^{+} = \begin{bmatrix} x_1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & x_n \end{bmatrix} \tag{4.18}$$

where 'x_i' is the set of observations on the variables entering the i^{th} equation, (4.17) implies $T>m_i$, for all i. Equation (4.16) can also be written as

$$[\bar{x}^{+\prime}\;(\Sigma^{-1}\otimes I)]\;(y^{+} - x^{+}\,\alpha) = 0 \tag{4.19}$$

so that if $T<m_i$, for all i we can make $y^{+} - x^{+}\alpha = 0$, and there is no unique solution for α^{+}. The reduced form errors will be trivially zero, too. We require that $T> n$, since otherwise $\hat{\Sigma}$ is singular and the maximum likelihood estimator is not well defined. The number of predetermined variables, k, may be larger than T. Going back to the case we are considering, the likelihood can then be written as

$$LK_R = cte - \frac{T}{2} \; L \left| \Omega \right| - \frac{1}{2} \; tr \; \Omega^{-1} (V_1' V_1) \qquad (4.20)$$

and the likelihood ratio test for parameter constancy becomes

$$-2 \; (LK_R - LK_{UR}) = tr \; \Omega^{-1} (V'V - V_1' V_1) \qquad (4.21)$$

where LK_R, LK_{UR} are the restricted and unrestricted versions of the maximum likelihood respectively. Then (4.6) is in fact a likelihood ratio test. If we concentrate out Ω_1 the test becomes simply,

$$T \; \log \left| \tilde{v}' \; \tilde{v} \; (\hat{v}_1' \; \hat{v}_1)^{-1} \right| \qquad (4.22)$$

If the parameters in the second sample can be estimated (4.6) is not the appropriate test. We remark again that the conditions $T_2 > n$, $T_2 > n_i + k_i$, for all 'i', are required for the estimator in the second subsample to be well defined. This case can be tackled in the following way. We define first in an obvious notation

$$y' = (y_1', \; y_2'), \qquad y^{o\,'} = \begin{bmatrix} y_1' & 0 \\ 0 & y_2' \end{bmatrix} \; , \; B^o = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$$

$$B_1 \; y_1' + c_1 \; z_1' = E_1'$$

$$B_2 y_2' + c_2 \; z_2' = E_2' \qquad (4.23)$$

We can write the system then as

$$B^o \; y^{o\,'} + c^o \; z^{o\,'} = E'$$

and apply a conventional maximum likelihood estimation procedure to this unrestricted system. The likelihood ratio test can therefore be readily calculated with existing software.

If for some equations $T_2 > m_i$ and for others $T_2 < m_i$ we are in an intermediate situation. Then (4.17) is not met. Considering an equation of the first set,

$$y_{2i} - x_{2i}\,\hat{\alpha}_{2i} \neq 0 \tag{4.24}$$

since $T_2 > m_i$ and therefore, there is no way we can make the errors zero, unless the observations are dependent, which by assumption, are not. The estimator for the second subsample is not uniquely defined, but the maximum likelihood ratio test does not reduce to (4.21) either. This situation is somewhat peculiar, and in terms of finite sample power, it may be that the test given in (4.21) is good enough for most practical purposes.

From the developments in (II - (27,36)) it is clear that we can define stability tests for several periods in the simultaneous equation context too. For example, if we breakdown the sample into three periods so that $T_1 + T_2 + T_3 = T$, and supposing that there are enough observations to estimate the model in the first two, we have

$$\mathrm{tr}\ \Omega^{-1}(\tilde{v}'\,\tilde{v} - \hat{v}_1'\,\hat{v}_1 - \hat{v}_2'\,\hat{v}_2) \ \underset{A}{\sim}\ \chi^2_p$$

$$p = nT_3 + \sum_{i=1}^{n} m_i \tag{4.25}$$

When testing stability by means of predictive accuracy it is implicitely assumed that the errors are normally distributed. It is therefore advisable to test for this hypothesis. In a scalar case, we know that under normality,

$$E\,u^3 = 0$$

$$E\,u^4 = 3\,E\,u^2 \tag{4.26}$$

so that a natural test for normality is to check the proximity to zero of the following quantities appropiately rescaled, (see [2]).

$$\hat{\gamma}_1 = \Sigma\,\hat{u}_t^3\,/T$$

$$\hat{\gamma}_2 = (\Sigma\,\hat{u}_t^4\,/\,T) - 3\,(\Sigma\,\hat{u}_t^2\,/\,T)^2 \tag{4.27}$$

Then,

$$C = \left(\frac{\hat{\gamma}_1^2}{6\sigma^6} + \frac{\hat{\gamma}_2^2}{24\sigma^8}\right) \;\tilde{A}\; \chi^2_{(2)} \tag{4.28}$$

Similarly, if we have a vector $\varepsilon_t' = (\varepsilon_{1t},\ldots,\varepsilon_{nt})$ $v(\varepsilon_t) = \Sigma$, we can transform to get independence by

$$u_t = \Sigma^{-1/2}\,\varepsilon_t \tag{4.29}$$

and define 'n' independent tests of normality as in (4.28) that can be combined to yield

$$\sum_{i=1}^{n} c_i \;\tilde{A}\; x^2_{2n} \tag{4.30}$$

where c_i is (4.28) for every error.

The stability test based on predictive ability as given in (4.6,12) cannot generally be implemented with standard packages. However, a simple transformation makes this possible. Let us then write the log of the likelihood function for the system set out at the begining of this section as

$$LK = -\frac{nT}{2} \text{ Log } 2\pi - \frac{T}{2} \text{ Log } \left|\Omega\right| - \frac{1}{2} \text{ tr } \Omega^{-1} \upsilon' \upsilon \qquad (4.31)$$

Concentrating out Ω, we get

$$\Omega = (\upsilon' \ \upsilon / \ T)$$

$$LK^* = -\frac{nT}{2} (1 + \log 2\pi) - \frac{T}{2} \text{ Log } \left|\Omega\right| \qquad (4.32)$$

and similarly for the first T_1 observations. Let us consider now

$$-2(LK^* - LK_1^*) = nT_2(1+\log 2\pi) + T\log\left|\Omega\right| - T_1\log\left|\Omega_1\right|$$

$$= T_2 \left[n(1 + \log 2\pi) + \log\left|\Omega\right|\right] + T_1 \log\left|\Omega\right|\left|\Omega_1^{-1}\right|$$

$$= -\frac{2T_2}{T} LK^* + T_1 \log\left|\Omega \ \Omega_1^{-1}\right|$$

$$= -\frac{2T_2}{T} LK^* + T_1 \log\left|\tilde{\upsilon}'\tilde{\upsilon} \ (\hat{\upsilon}_1'\hat{\upsilon}_1)^{-1}\right| + T_1 \log \left(\frac{T_1}{T}\right)^n \qquad (4.33)$$

The last term in this expression becomes

$$T_1 \log \left(\frac{T_1}{T}\right)^n = n \ T_1 \log \frac{T_1}{T}$$

$$\text{Log } \frac{T_1}{T} = \log \left(1 - \frac{T_2}{T}\right) = -\frac{T_2}{T} + O\left(T^{-2}\right) \qquad (4.32)$$

so that finally

$$T_1 \log \left(\frac{T_1}{T}\right)^n = -\frac{nT_2T_1}{T} + O\,(T^{-1}) \qquad (4.33)$$

$$= -\, n\, T_2 + O\,(T^{-1})$$

since $T_1/T = 1 + O\,(T^{-1})$

From (4.33) we get now

$$-2(LK^* - LK_1^*) + (nT + 2LK^*)\,\frac{T_2}{T} = T_1 \log \left|\Omega\, \Omega_1^{-1}\right| \qquad (4.34)$$

and therefore

$$-2\,\left[(1-\frac{T_2}{T})\, LK^* - LK_1^*\,\right] + nT_2 \;\underset{A}{\sim}\; \chi^2_{nT_2} \qquad (4.35)$$

The point of this last expression is of course that it can be readily evaluated with most existing software econometric packages since they generally give the value of the likelihood as a standard result. The advantage of (4.35) is therefore that it is fully operational.

V.- Conclusions

Extensive testing of estimated models is the only way of validating the product of applied econometrics research. One essential type of tests are the stability tests. This is because they provide a safeguard against the well known problems of data mining and pretesting.

The paper has intended to provide formulae for stability tests in a wide variety of practical situations, that can be readily implemented with existing software packages. Some of the results presented in the paper are scattered in the literature, and some are new, particularly those referring to simultaneous equations models.

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