TESTING THE RATIONAL EXPECTATIONS MODEL

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ABSTRACT

This paper presents a simple test of the future expectations model that can be implemented easily with existing software. The test is implemented by means of an instrumental variable criterion. A revision of the solutions of rational expectations models is provided, in order to give some basis for the choice of instruments. The test is used to check the ability of output expectations to stabilize the investment equation in Spain, previously obtained by the author.

CONTENTS

I. INTRODUCTION (*)

Let us suppose that we have an explanatory model for a variable, where its future expected values are on the right hand side. A simple example is given by,

$$Y_t = a_o + a_1 \; E(Y_{t+1}/I_{t-1}) + \varepsilon_t \tag{1.1}$$

Similarly, we could have future expected values of other variables as explanatory. This type of model has aroused considerable interest in the literature but poses considerable problems if one attempts to estimate it, so that a natural question to ask is how to test it, before actually trying to estimate it. The main advantage of the Lagrange type of test lies precisely here, that is, the test can be conducted under the null, so that estimation of the general model is not required. Drawing attention to this fact and deriving a simple test, based on Lagrange principles, to check the validity of the model (1.1), is the main contribution of this paper. The test proposed in the paper is implemented by means of an instrumental variable criterion. This in turn, requires an analysis of the solutions of the rational expectation model, in order to pick the right set of instruments Section II addresses this question while section III discusses the test. A detailed application to the case of investment in Spain, is presented in section IV. The advantage of the test is, obviously, that it can be computed easily with standard packages.

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II. SOLUTIONS TO RATIONAL EXPECTATIONS MODELS: A REVIEW

It is convenient to revise first the main solutions of the model described in the previous section. This has an interest in itself, and will provide a natural introduction to the test proposed in the next section. The discussion will be introduced through a simple example that allows a quick understanding of the essentials of the problem while retaining enough generality.

We start by considering a simple model given by

$$y_t = a\, y_{t-1} + b\, y_{t+1}^{t-1} + z_t \qquad (2.1)$$

$$z_t = \bar{z}_t + u_t \qquad (2.2)$$

where

$$y_{t+s}^{t-1} = E\,(y_{t+s} \mid I_{t-1})$$

$$\bar{z}_t = E\, z_t \qquad (2.3)$$

where I_{t-1} is the amount of information available at time (t-1), and 'u_t' is a white noise error process. Since y_{t+1}^{t-1} is not directly observable, the model (2.1,2) is incomplete and cannot be estimated either. Then, we have to find a solution to the model. In order to do that, we shall treat the stochastic and non-stochastic components separatedly . The non-stochastic element of the model is given by $\bar{z}_t$ and it gathers all possible kinds of deterministic elements. For example, seasonal dummies, and preannounced monetary targets are embodied in $\bar{z}_t$, and therefore belong to I_{t-s}, s > 0.

Taking now unconditional expectations of (2.1) we get,

$$\bar{y}_t = a\,\bar{y}_{t-1} + b\,\bar{y}_{t+1} + \bar{z}_t \tag{2.4}$$

where $\bar{y}_t = E\,y_t$, and similarly for other dates. We can substract (2.4) from (2.1) and get

$$v_t = a\,v_{t-1} + b\,v^{t-1}_{t+1} + u_t$$

$$y_t = \bar{y}_t + v_t \tag{2.5}$$

We consider first the solutions of the stochastic element, v_t. We shall concentrate on stable solutions. This can be justified on the grounds that real economic behaviour is not explosive in general (see also Brock (1974)). Let us consider then the representation

$$v_t = \sum_{s=0}^{\infty} \delta_s\,w_{t-s}\,, \qquad \delta_o = 1 \tag{2.6}$$

$$\lim \sum_{s=0}^{\infty} \delta_s^2 < \infty$$

where 'w_t' is a white noise error process. This type of representation can always be claimed to be valid for a stationary stochastic process, following Wold's theorem (see also Muth (1961), and Taylor (1977)). In order to find the solution for the δ's we just substitute (2.6) into (2.5), and get after trivial manipulations,

$$w_t - u_t + \sum_{r=1}^{\infty} (\delta_r - \delta_{r+1}\,b - \delta_{r-1}\,a)\,w_{t-r} = 0 \tag{2.7}$$

and since w_t, and u_t are serially uncorrelated we get the following expressions,

$$w_t = u_t$$

$$\delta_r - \delta_{r+1} b - \delta_{r-1} a = 0, \; r > 1 \qquad (2.8)$$

so that all the δ's are given, once 'δ_1' is determined. In other words, there are infinite solutions corresponding to the infinite possible values of δ_1, and this implies that the model (2.1,2) is incomplete in general. If one of the roots of the equation $x^2 b - x + a = 0$, is unstable, the solution for 'δ_s' can be made stable by adjusting 'δ_1' so that the unstable root is eliminated. Only in this case we get a unique stable solution for 'v_t'.

We can look at the problem now in an alternative way. First, one notes from (2.6) that

$$v_{t+1} - v_{t+1}^{t-1} = u_{t+1} + \delta_1 u_t \qquad (2.9)$$

and substitution into (2.5) yields after rearranging

$$v_{t+1} = \frac{1}{b} v_t - \frac{a}{b} v_{t-1} + u_{t+1} + (\delta_1 - \frac{1}{b}) u_t \qquad (2.10)$$

This last expression can generally be written as follows

$$H(L) v_t = D(L) u_t \qquad (2.11)$$

and for stability, we require in general that the roots of H (L) = 0 are outside the unit circle. This will ensure that the δ's defined in (2.8) will meet condition (2.6).

If both roots are stationary (real or imaginary), there are infinite stable solutions corresponding to the values of the arbitrary parameter 'δ_1'. If both roots are real, and one is stable (λ_1) and the other is not, (λ_2), the stationary solution is unique and we require D (λ_2) = 0. That is, we require that the unstable root be a common factor of both polynomials H (.) and D (.), so that it can be deleted (strictly speaking, the representation (2.6) is not valid for an unstable model and therefore the argument is not completely correct. However, one can easily show more formally, that the result is valid (Broze, Gourieroux, and Szafarz (1984)). If both roots are unstable (real or imaginary) there is no stationary solution to the model.

When there are infinite solutions, this fact poses considerable problems, specially in policy analysis of optimal rules. One way to achieve uniqueness, is to select the solution with smallest variance (Taylor (1977)). This requirement will impose further conditions on the δ's, and will determine them uniquely. The motivation for this solution, is that variability is 'bad' in the sense that it creates uncertainty, and therefore, rational agents will avoid introducing unnecessary noise into their expectations. For example we get for the model of this section from (2.6) and (2.8),

$$\sigma_v^2 = \sigma_u^2 \left(\sum_{s=0}^{\infty} \delta_s^2 \right)$$

$$= \sigma_u^2 \, f \, (a, b, \delta_1) \qquad (2.12)$$

One could minimize now this last expression with respect to 'δ_1', from which its value would be uniquely determined. It is of some interest to point out that in some cases, this solution is precisely the 'forward solution' (see Appendix A). For example, if $a = 0$ in (2.1), the forward solution is just $y_t = u_t$, so that $(\delta_s = 0, s > 0)$, and it is then evident, that this is the minimum variance solution. But in general, this will not be so, since the purely forward solution does not exist, as son as there are lagged values of the dependent variable in the right hand side. This is because, then, one of the unstable roots in (2.11) cannot be eliminated. As far as the discussion of this paper is concerned, that is, the derivation of a test for model (2.1), this point does not affect much the solution. It would lead to a more efficient test, but then, a special program to compute it would be required.

We consider now the solution for the deterministic element of y_t, $\bar{y}_t$ (see (2.4)). A general solution will be of the following type,

$$\bar{y}_t = \sum_{s=-\infty}^{\infty} \Psi_s \bar{z}_{t-s} \qquad (2.13)$$

and will depend on both roots of H (.). If both roots are stable it is easy to see that $\Psi_s = 0$ for all $s<1$ (backward solution). If both are unstable, $\Psi_s = 0$ for all $s>0$ (forward solution), and if one root is unstable and the other stable, $\Psi_s \neq 0$ for all s. A consequence of this fact is that adjustments to external shocks take place inmediately under the forward solution and only gradually in the backward looking solution. For example, under the forward solution, the model reaches its long run equilibrium immediately after a permanent change in the level of z_t has occured whereas this equilibrium is only approached slowly in the backward solution.

The model (2.1) and its related results can be generalizaed without too much complication. We could assume for example that 'u_t' is a general ARMA process of the type

$$u_t = (\emptyset_1 (L) / \emptyset_2 (L)) \varepsilon_t \tag{2.14}$$

where the roots of $\emptyset_2 (L) = 0$ are outside the unit circle. Then, the reasoning leading to (2.4, 5) is unaltered. The equations (2.7, 8) that define the coefficients 'δ' would be more complicated, but the main results are similar. In particular, the multiplicity of solutions remains, and if one of the roots of H (L) is unstable (λ_2) we require that

$$1 + (\delta_1 - (1/b) \emptyset_1(\lambda_2) / \emptyset_2 (\lambda_2)) \lambda_2 = 0 \tag{2.15}$$

so that the root is common to the autoregressive and the moving average component, of 'v_t', and can therefore be deleted. (Note that in (2.9) we have to substitute 'u_t' by 'ε_t' in this case). We can also add an error 'e_t' to model (2.1) that is

$$y_t = ay_{t-1} + b\, y^{t-1}_{t+1} + z_t + e_t \tag{2.16}$$

and assume that 'u_t' is defined by (2.14). Since the sum of two moving averages is another moving average process under very general conditions, this case can be reduced to the preceding model.

More generally we can write a univariate process including expectations as follows,

$$\alpha (L)\, y_t = \sum_{h=1}^{r} \sum_{k=0}^{s} d_{kh}\, y^{t-k}_{t-k+h} + z_t \tag{2.17}$$

The stability condition and the number of solutions in this model are closely related topics like in model (2.1) and can be treated in a very similar fashion. Assuming the representation (2.6) to hold, we can write,

$$y_t - y_t^{t-s} = \sum_{r=0}^{s-1} \delta_r \, \varepsilon_{t-r} \tag{2.18}$$

and substituting in (2.17) and grouping terms we get

$$\alpha^+(L) y_t = z_t + D\,(L)\,\varepsilon_t \tag{2.19}$$

where $\alpha^+(L)$ is of degree $(r + \max(s,q))$, q being the degree of $\alpha(L)$. For stability, we require that the roots of $\alpha^+(L) = 0$ lie outside the unit circle. If there are one or more unstable roots, we require them to be common factors, that is, they must also be a root of $(\phi_1(L)/\phi_2(L)) + D(L)$. The number of solutions (2.19) will be in general infinite, depending on a certain number of parameters of D(.).

It can be shown that the number of free parameters is the maximum lead in the expectations, that is, 'r' (Broze, Gourieroux and Szafarz (1984)). Therefore, the maximum number of roots of $\alpha^+(L)$ that can be deleted is 'r'. If the number of unstable roots is bigger, there is no stationary solution to the model.

We can think of (2.17) as a system of 'n' equations. Then, $\alpha(L)$ is a matrix polynomial and the coefficients 'd_{kh}' are squared matrices of orden 'n'. In this case, it is more general to write $(C\ z_t)$ instead of just 'z_t', where C is an (n x k) matrix and 'z_t' a vector of orden 'k'.

This is because each equation may include an arbitrary set of z's, that may differ across equations. For stability, we requiere now that the non zero roots of $|\alpha^{+}(L)| = 0$ are outside the unit circle. If there are unstable roots, as before, we have to adjust the free parameters so that they can be deleted.

III. Testing the future expectations model

We start this section considering again the model (2.1) where 'u_t' is defined by (2.14) and there is an error 'e_t' added to the model as in (2.16). Making use of (2.9) we can substitute and get the following expression

$$y_t = a\, y_{t-1} + b\, y_{t+1} + z_t + e_t - b\, (\varepsilon_{t+1} + \delta_1 \varepsilon_t) \qquad (3.1)$$

which in turn can be rewritten as follows,

$$y_t = a\, y_{t-1} + b\, y_{t+1} + z_t + (w_{t+1} + \lambda w_t) \qquad (3.2)$$

where $E(y_{t+1}\ w_{t+1}) \neq 0$, and '$w_t$ is a white noise error. We also note that $E(z_t w_t) \neq 0$. This equation can be estimated now by a non linear instrumental variable method, or else, we can try other approaches reviewed in Appendix B. But a natural question to ask now, is whether $b = 0$, so that the model is of a standard type, and therefore, no special procedure for estimation is required. The way to proceed then, is to test the adequacy of the rational expectations model (2.1), without actually estimating it. A nice feature of the Lagrange type of test is precisely that it only requires estimation under the null, so that this is the type of test that we seek.

Under the null hypothesis that $b = 0$, we also have that $\lambda = 0$. Then, the idea of the test proposed below, is first to set up an instrumental variable criterion adequate to estimate (3.2), and then test the joint hypothesis $b = \lambda = 0$, by means of a Lagrange type of test. (Under the null one can also note that it would not be necessary to use instruments, since the regressors would not be correlated with the errors anymore. But in general, this would not be an optimal procedure and the power might be low). The selection of the set of instruments deserves some discussion. If we assume that the following condition holds,

$$\lim_{T\to\infty} \sum_{t=1}^{T} (\bar{z}_t \, \bar{z}_{t-s})/T \not\to 0 \tag{3.2}$$

then all $\bar{z}_{t+s}$, $s(-\infty, +\infty)$, will be valid instruments for 'y_t' and 'z_t'. Then, there is no problem of shortage of instruments. Assumption (3.2) is fairly mild but it might fail in which case one has to look to the possible different solutions of the model case by case. For example, in the case of the model that we are discussing, under H_o, 'y_t' depends on z_{t-s}, ($s = 0, 1 \ldots$). Under the alternative and if the autoregressive roots are stable 'y_t' depends on $\bar{z}_{t-s}$, ($s = 1, 2 \ldots$), and if the solution is a combination of forward and backward expectations 'y_t' depends on $\bar{z}_{t-s}$, ($s = 0, \pm 1, \pm 2, \ldots$) (see the discussion under (2.13)).

If there are lags of the dependent variable appearing on the right hand side of the equation (i.e. α (L) $\neq$ 1 in (2.17)), then further lagged values of 'y_t' will be valid instruments for its future values. Finally z_{t-s-m}, ($m = 1, 2, \ldots$) are also valid instruments (see (2.17) for the definition of 's'). In spite of the wide list of instruments, there may be cases where they are not sufficient. For example, let us consider the following model,

$$y_t = b \, y_{t+1}^{t-1} + c \, y_{t+2}^{t-1} + d \, z_t$$

$$z_t = \varepsilon_t + \theta \, \varepsilon_{t-1} \tag{3.3}$$

The first equation yields after substitution,

$$y_t = b\ y_{t+1} + c\ y_{t+2} + d\ z_t - d\ c\ \varepsilon_{t+2} -$$
$$- d\ (b + c\ \delta_1)\ \varepsilon_{t-1} - d\ (b\ \delta_1 + c\ \delta_2)\ \varepsilon_{t-2} \qquad (3.4)$$

Under the null hypothesis that b = c = 0, we have $y_t = z_t$, so that z_t and z_{t-1} are valid instruments. But since z_t is an MA(1), z_{t-s}, $s > 1$, are not valid instruments. We need three instruments but we only have two, so that the instrumental variable estimator is not well defined.

But apart from some odd situations, we can expect to have enough instruments in general. We need now a criterion to test the joint hypothesis $b = \lambda = 0$ in the model of (3.2). Since testing against autocorrelation in the errors, produced by a moving average process, yields the same type of formulae than testing against an autoregressive process (see for example Pagan A. and Hall A. (1983)). We can set up the test for the hypothesis that the correct model does not include expected future variables as it is given next. A slight change of notation may be convenient first, to standardize it. Then, we consider the model

$$y_t = a\ y_{t-1} + b\ y^{t-1}_{t+1} + cx_t + \varepsilon_t$$

$$x_t = \bar{x}_t + \psi\ (L)\ u_t \qquad (3.5)$$

where u_t, and ε_t are independent, and $\psi(L)$ is a stationary filter. After substitution we get

$$y_t = a\ y_{t-1} + b\ y_{t+1} + c\ x_t + (w + \lambda w_t) \qquad (3.6)$$

and we consider testing $b = \lambda = 0$. We can set up the test, then as follows,

$$C = \tilde{\varepsilon}' \, X \, w^{+} \, (w^{+\prime} \, X \, w^{+})^{-1} \, w^{+\prime} \, X \, \tilde{\varepsilon} / \tilde{\sigma}^2 \underset{A}{\sim} \chi^2(2) \qquad (3.7)$$

where,

$$w^{+} = (\tilde{\varepsilon}_{-1}, \, w)$$

$$w = (x_t, \, y_{t-1}, \, y_{t+1})$$

$$X = z \, (z'z)^{-1} \, z'$$

$$z_t = (\bar{x}_{t+1}, \, \bar{x}_t, \, x_{t-1}, \, y_{t-1}, \ldots) \qquad (3.8)$$

and $\tilde{\varepsilon}$ are the errors obtained after estimating (3.6) by instrumental variables imposing b = 0 (A generalization to the case of further expected future variables is straightforward). The list of instruments is a sensible choice according to the analysis of section II, but is otherwise arbitrary. We also note that 'x_t' does not belong to the list of instruments, so that this first regression does not reduce to OLS.

A possible reformulation of (3.7) is the following,

$$T \, R^2(\tilde{\varepsilon} \, / \, \hat{w}^{+}) \underset{A}{\sim} \chi^2 \, (2) \qquad (3.9)$$

where,

$$\hat{w}^{+} = X \, w^{+} \qquad (3.10)$$

If there are k regressors in w, computation of (3.9) requires running (k+2) regressions. This may be tedious if k is larger than, say 3 or 4, and then, the following procedure is simpler. It is not too difficult to check that (3.7) can be written as,

$$C = T \left(\frac{\varepsilon_f' \, \hat{X} \, \varepsilon_f}{\varepsilon_f' \, \varepsilon_f} \right) \left(\frac{\varepsilon_f' \, \varepsilon_f}{\tilde{\varepsilon}' \, \tilde{\varepsilon}} \right)$$

$$= T \, R^2 \, (\varepsilon_f / z) \, (\varepsilon_f' \, \varepsilon_f \, / \, \tilde{\varepsilon}' \, \tilde{\varepsilon}) \qquad (3.11)$$

where 'ε_f' is the fit obtained regressing $\tilde{\varepsilon}$ on ($\tilde{\varepsilon}_{-1}$, w) by IV. This last expression is a function of quantities that can be obtained easily with any regression package. The procedure involves then three steps:

1°) Estimate (3.6) by IV, imposing $b = \lambda = 0$, and get the estimated errors, $\tilde{\varepsilon}$.

2°) Regress $\tilde{\varepsilon}$ on ($\tilde{\varepsilon}_{-1}$, w) by instrumental variables and get the fitted values ε_f (The quantities ($\varepsilon'_f \, \varepsilon_f$), ($\tilde{\varepsilon}' \, \tilde{\varepsilon}$) are ussually easy to get).

3°) Regress 'ε_f' on z by OLS.

The test can be computed now as it is given in (3.11), and the set of instruments must be the same in the three steps.

Although it is difficult to give a correction factor, it may be a good idea to substitute T by (T-m) in finite samples, 'm' being the number of variables in 'w^+'. This prevents the test from high probabilities of rejections, simply because too many instruments are taken. A heuristic justification for this factor may at least be partly based on Mauleón (1986). If we had a different dependent variable, say 'y_t^+', the test would be entirely similar. The only difference lies then in the selection of instruments, since the model for 'y_t' would not be completely specified by (3.5).

IV. AN APPLICATION TO THE ANALYSIS OF INVESTMENT

The future expectations model, has been applied in the literature to the analysis of the demand for labour (Nickell (1980), Sargent (1978)). A logical extension of the model is the application to the demand for capital, and some pioneering studies already exist (Kokkelenberg and Bischoff (1985), Schiantarelli (1983)). In the Spanish case, this has a further interest, because the investment function appears to be unstable. This may be due to omitted variables, and therefore, it is interesting to test all reasonable hypothesis.

The empirical results for the investment model presented in Mauleón (1985), can be summarized as follows,

$$I = \alpha_o u + \alpha_1 (E + E_{-1}) + \varepsilon_1$$

$$E = \delta_o y + \varepsilon_2$$

$$y = x\,\beta_o + \beta_1 I + \varepsilon_3 \qquad (4.1)$$

where all variables are measured as growth rates except the rate of capacity utilization U, and the long term rate of interest that is embodied in x. This last function also includes two step dummies (D1, D2). Output is denoted by y, and E is the income share of capital.

Adding the output expectations for period (t+1) made in (t-1), the model may be writen as,

$$I = \alpha_o u + \delta_o \alpha_1 (y + y_{-1}) + a\, y_{t+1}^{t-1} + e_1^*$$

$$y = x\,\beta_o + \beta_1 I + \varepsilon_2^* \qquad (4.2)$$

from which we get the reduced form for y_t as

$$y_t = x_t' \pi_o + \pi_1 y_{t-1} + \pi_2 y_{t+1}^{t-1} + v_t \qquad (4.3)$$

This expression is required to make a selection of instruments for 'y_{t+1}'. According to the analysis of the preceding section, we can now select the instruments among the set

$$(D_{1t+1}, D_{1t}, D_{2t+1}, D_{2t}, E_{t-1}, U_{t-1}, \ldots)$$

If we had y_t^{t-1} as an explanatory variable of investment, the problem can be treated in the conventional simultaneous equation model framework (Indeed, this may be given as an explanation for the existence of simultaneity, Mauleón (1984)). Since the investment study was conducted with yearly data, one could argue that a year is a long enough time unit so that y_{t+1}^{t} might be the right variable to include in the investment equation. Then we would have,

$$y_{t+1} - y_{t+1}^{t} = w_{t+1} \qquad (4.4)$$

where w_t is a combination of 'v_t' and the innovation in x_t. Then it is easy to check that 'x_t' would be a valid instrument now, and it would not be necessary to instrument u_t in (4.1) either. Adding further expectations into the future,

$$(y_{t+2}^{t-1}, y_{t+3}^{t-1}, \ldots .)$$

would require extra instruments that could be easily obtained, following the analysis of the previous section.

Since assuming that the expectational variable that enters the investment equation is 'y^t_{t+1}' rather than 'y^{t-1}_{t+1}' did not change the results much, the regressions reported below are those corresponding to y^t_{t+1}. This is because with a small sample, it is convenient to economize on the number of instrumental variables. The longest expectation horizon considered has been y^t_{t+2}, since it seems unlikely that output con be predicted three or more years ahead with any accuracy in Spain. The regressions have been run on the stable sample, and with all available observations, to check if the lack of stability in the last part could be explained. The test of the expectational model has been computed as in (3.11) and is denoted below, by C. Some problems related to the choice of instruments are discussed in Appendix C. The instrumental variable estimation of the equation containing output expectations is also given, although it is not required to compute the test of (3.11). The results are as follows:

$$\Delta I = \underset{(3.2)}{.016}\,\Delta u + \underset{(4.2)}{1.2}\,\Delta(E+E_{-1}) - \underset{(1.4)}{.7}\Delta y_{+1} + e$$

$R_2 = .7,\ \tilde{\sigma} = .048,\ T = 15(1966-1980),\ Dw = 1.4$

Instruments: ΔE_{-1}, D1, D2, $D1_{+1}$, Δu

$$C = 1.4 \underset{A}{\sim} \chi^2(2) \text{ 95 \% confidence interval } (0,6.0) \qquad (4.5)$$

$$\Delta I = \underset{(2.4)}{.014}\Delta u + \underset{(3.3)}{1.2}\,\Delta(E+E_{-1}) - \underset{(1.2)}{.8}\,\Delta y_{+1} + e$$

$R^2 = .52,\ \tilde{\sigma} = .06,\ T = 17(1966-1982),\ Dw = 1.3$

Instruments: as in (4.5)

$$C = 1.8 \underset{A}{\sim} \chi^2(2);\ 95\ \%\ \text{confidence interval}\ (0,6.0) \qquad (4.6)$$

$$\underset{}{\Delta I} = \underset{(1.7)}{.013\ \Delta u} + \underset{(4.)}{1.4\ \Delta(E + E_{-1})} - \underset{(.1)}{.11\ \Delta y_{+1}} - \underset{(.7)}{.9\Delta y_{+2}} + e$$

$R^2 = .62,\ \tilde{\sigma} = .06,\ T = 14\ (1966\text{-}1979),\ Dw = 1.7$

Instruments: as in (4.5), plus $D1_{+2}$

$$C = 3.8 \underset{A}{\sim} \chi^2(4);\ 95\ \%\ \text{confidence interval}\ (0,9.5) \qquad (4.7)$$

$$\Delta I = \underset{(1.6)}{.011\Delta u} + \underset{(3.9)}{1.4\ \Delta\ (E + E_1)} + \underset{(.14)}{.17\ \Delta y_{+1}} - \underset{(.9)}{1.1\Delta y_{+2}} + e$$

$R_2 = .6,\ \tilde{\sigma} = .056,\ T = 18(1966\text{-}1981),\ Dw = 1.8$

Instruments: as in (4.7)

$$C = 4.3 \underset{A}{\sim} \chi^2\ (4);\ 95\ \%\ \text{confidence interval}\ (0,9.5) \qquad (4.8)$$

The conclusion to be drawn from these results is unambiguous: future output expectations are unable to explain the unstable behaviour of investment. This impression is reinforced because of the negative coefficient of the future output expectations in the investment equation. (The parameter estimates are consistent although the standard errors are somewhat incorrect (see Appendix B)).

V. CONCLUSION

The purpose of this paper is two fold: a) give a simple and yet comprehensive account of the main issues related to the solutions of rational expectations models concerning the applied econometrician, b) derive a simple formula based on Lagrange principles to test the future expectations model and use it to check the ability of future output expectations, to stabilize the investment function in Spain.

Unfortunately, the behaviour of investment in the last two or three years remains unexplained, since output expectations are insignificant in the estimated equations. This leaves open the way for alternative explanations.

APPENDIX

A. HINTS ON THE FORWARD SOLUTION

This appendix shows by means of an example that the stability condition derived from the purely forward solution is not, in general, a necessary condition for the existence of a solution to the rational expectations model. Let us start by considering some results on conditional expectations. If (x, y, z) are three random variables and f(-) stands for a density function we can write,

$$f(y,x,z) = f(y/x,z)\ f(x/z)\ f(z) = f(y,x/z)\ f(z) \qquad (A.1)$$

from where we get

$$f(y/x,z)\ f(x/z) = f(y,x/z) \qquad (A.2)$$

so that

$$E\ y^{t}_{t+1}/\ I_{t-1} = \int [\int y_{t+1}\ f(y_{t+1}/\ \Delta\ I_t\ I_{t-1})\ d\ y_{t+1}]\ .$$

$$f\ (\Delta\ I_t/\ I_{t-1})\ d\ \Delta\ I_t$$

$$= \int y_{t+1}\ f\ (y_{t+1}/\ I_{t-1})\ d\ y_{t+1}$$

$$= y^{t-1}_{t+1} \qquad (A.3)$$

Let us suppose now that we have a simple model like,

$$y_t = a\ y^{t-1}_{t+1} + \psi + \varepsilon_t \qquad (A.4)$$

Taking expectations of this expression one gets easily

$$y_{t+s}^{t-1} = a\ y_{t+s+1}^{t-1} + \psi \tag{A.5}$$

so that by recursive substitution we have,

$$y_{t}^{t-1} = a^{s}\ y_{t+s}^{t-1} + \left(\sum_{r=o}^{s-1} a^{r}\right) \psi \tag{A.6}$$

Since we require a bounded solution, it is tempting to conclude that the stability condition is $|a| < 1$ (This is the condition given by Wallis (1980)). We are going to see below that this does not have necessarily to be so. Using the methods of section II we can write

$$y_{t+1} - y_{t+1}^{t-1} = \varepsilon_{t+1} + \delta_{1}\ \varepsilon_{t} \tag{A.7}$$

and substitution into (A.4) yields

$$y_{t} = \alpha\ y_{t-1} + \emptyset + \varepsilon_{t} \tag{A.8}$$

with

$$\emptyset = -\alpha\ \psi$$

$$\alpha = 1/a$$

$$a\ \delta_{1} = 1 \tag{A.9}$$

The value of 'δ_1' is chosen arbitrarily among the infinite solutions of the model. The stability condition in this case is then $|\alpha| < 1$, or $|a| > 1$.

Now, we show how this fact can be made compatible with (A.6). First from (A.8) one gets

$$y_{t}^{t-1} = \alpha\ y_{t-1} + \emptyset \tag{A.10}$$

$$y_{t+s}^{t-1} = \alpha^{s+1}\, y_{t-1} + \left(\sum_{r=o}^{s} \alpha^{r}\right) \emptyset \qquad \text{(A.11)}$$

and plugging this last expression in (A.6)

$$y_{t}^{t-1} = \alpha\, y_{t-1} + \left(\sum_{r=o}^{s} \alpha^{r}\right) \emptyset\, a^{s} + \left(\sum_{r=o}^{s-1} a^{r}\right) \psi$$

$$= \alpha\, y_{t-1} + \emptyset \qquad \text{(A.12)}$$

(using (A.9)). That is, although y_{t+s}^{t-1} is bounded as $s \rightarrow \infty$, $(a^{s}\ y_{t+s}^{t-1})$ is not, and one of it components makes up for the term in braces in (A.6). If $|a| < 1$, then $|\alpha| > 1$, and following the methodology of section II one can show easily that the only stable solution implies $\delta_{1} = 0$, so that

$$y_{t} = \varepsilon_{t} + \psi/(1-a)$$

$$y_{t+s}^{t-1} = \psi/(1-a),\ s \geqslant 0 \qquad \text{(A.13)}$$

Therefore, we get uniqueness imposing the forward condition, but this is just one possible solution in an infinite set. As another example, consider the model

$$y_{t} = a\, y_{t+1}^{t-1} + x_{t};\ x_{t} = \varepsilon_{t}/(1-\rho L);\ \varepsilon_{t} \sim \text{i.i.d}\ (0,\sigma) \qquad \text{(A.14)}$$

Then,

$$y_{t}^{t-1} = a^{s}\, y_{t+s}^{t-1} + \left(\sum_{r=o}^{s-1} (a\,\rho)^{r}\right) \rho\, x_{t-1} \qquad \text{(A.15)}$$

and it looks as if a necessary condition for stability is $|a\rho| < 1$. But substituting (A.7) and rearranging one gets

$$y_t = \alpha\, y_{t-1} + w_t; \qquad w_t = D\,(L)\,\varepsilon_t \tag{A.16}$$

where

$$D\,(L) = 1 + \delta_1 L - \frac{L}{1-\rho L} \tag{A.17}$$

and the stability condition is in general $|\alpha| < 1$, that is, $|a| > 1$. If $|a| < 1$, and $|\alpha| > 1$, this root has to disappear to get stability so that then we would require $D\,(a) = 0$.

B. ALTERNATIVE ESTIMATION PROCEDURES FOR RATIONAL EXPECTATIONS MODELS

Let us suppose that we have the two equation expectational model,

$$x_t = a\, y^t_{t+1} + z'_t\, \gamma + \varepsilon_t \qquad (B.1)$$

$$y_t = \emptyset_1\, y_{t-1} + \emptyset_2\, y_{t-2} + v_t \qquad (B.2)$$

Since in general we will not know the model governing 'y_t', equation (B.2) can be thought of as an approximation to the AR representation of 'y', that has been cut off at some arbitrarily specified lag, but otherwise sufficiently long.

We can rewrite the first equation as

$$x_t = \pi_1 y_t + \pi_2 y_{t-1} + z'_t \gamma + \varepsilon_t \qquad (B.3)$$

but now we have two parameters (π_1, π_2), instead of one (a) to be estimated. That is, in general the efficient procedure will require estimation of (B.1).

Let us consider now the first equation written as follows,

$$x_t = a\ (\emptyset_1\, y_t + \emptyset_2\, y_{t-1}) + z'\ \gamma + \varepsilon_t \qquad (B.4)$$

On intuitive grounds, one can guess that joint estimation of (B.1), (B.2) may be more efficient because there are some cross equation restrictions. Assuming that $\sigma_{\varepsilon v}=0$, the maximun likelihood approach leads to minimization of

$$s\ (\rho,\ \emptyset) + \Sigma\ (\emptyset) \qquad (B.5)$$

where $\rho = (a, \gamma)$, $\emptyset = (\emptyset_1, \emptyset_2)$, and s, Σ are, respectively, the sums of squared residuals in the first and second equation. The covariance matrix will be the inverse of

$$\begin{bmatrix} s_{\emptyset\emptyset} + \Sigma_{\emptyset\emptyset} & s_{\rho\emptyset} \\ s_{\emptyset\rho} & s_{\rho\rho} \end{bmatrix} \qquad (B.6)$$

conveniently rescaled. It is easily checked that the off diagonal elements will not vanish assymptotically so that the joint procedure is effectively more efficient for $\emptyset$ and for ρ.

This is the basic reason, from the estimation point of view, to seek a solution for the rational expectations model. The estimation procedure that considers the parameters of the structural model, after having substituted the solution for the expectations, is more efficient than reduced form type of approaches.

If we had future expectations further into the future in equation (B.1), the problem of setting up the criterion (B.5) becomes cumbersome. A short-cut, widely used in the literature consist then of two steps: first (B.2) is estimated by OLS, and second, expectations are generated and plugged in (B.1), which is finally estimated by OLS (Nickell (1980), Sargent (1978)). This procedure yields consistent parameter estimates while the standard errors are somewhat incorrect (overvalued in general). To see this, let us look at a simplified case, as follows,

$$y = x B + \varepsilon$$
$$= \hat{x} B + \varepsilon + (x - \hat{x}) B$$
$$= \hat{x} B + w \qquad (B.7)$$

where $x = \hat{x} + \hat{v}$ and $\hat{x}'\hat{v} = 0$ by construction. Then, it is easy to check that

$$(\tilde{B} - B) \sqrt{T} \underset{A}{\sim} N \left(0, \text{plim} \left(\frac{\hat{x}' \hat{x}}{T}\right)^{-1} \sigma_{\varepsilon}^{2}\right) \qquad (B.8)$$

but the standard OLS procedure will pick $(\hat{w}'\hat{w}/T)$ rather than $\hat{\sigma}_{\varepsilon}^{2}$, and this last quantity is effectively smaller. The same problem arises in the literature of expected and unexpected components. That is, we must estimate jointly, the reduced form model for the expectational variable and the structural equation, if we want to get correct standard errors.

There are other approaches to estimate equation (B.1). First, we could substitute y_{t+1}^{t} by its actual value and get a model of the type

$$x_t = a\, y_{t+1} + z'_t \gamma + w_t \qquad (B.9)$$

where now 'w_t' will follow a moving average, in general, and $E\, y_{t+1} w_t \neq 0$. This equation can be estimated by the method of instrumental variables, and the instruments can be selected, along the lines of section III (McCallum (1976), Muellbauer and Winter (1983)). If the non diagonality of $E(w\, w')$, is not taken into account, the standard errors will be incorrect, although the parameter estimates will be consistent. (This point has also been made by Sargan (1983)). This procedure, will not be very efficient, but has the advantage of not imposing an arbitrary reduced form for the expectational variables. Also, the arbitrary parameters in the rational expectations solution, can be estimated, and this can be a first step to implement a more efficient procedure. As an example, let us assume that instead of (B.2) we have,

$$y_t = \emptyset_1 y_{t-1} + \emptyset_2 x_t + v_t \tag{B.10}$$

and substituting for 'x_t' we get

$$y_t = \emptyset_1 y_{t-1} + (\emptyset_2 a) y^t_{t+1} + z_t' (\gamma \emptyset_2) + u_t \tag{B.11}$$

The solution of this equation can be obtained by the methods of section II. If for example, the z's are non-stochastic, and we know after estimation of (B.9) that the polynomial on 'y' has two real solutions, one stable 'α', and the other unstable 'ρ', the solution for 'y_t' is,

$$y_t = \bar{y}_t + w_t$$

$$w_t = u_t / (1 - \alpha L)$$

$$\bar{y}_t = K_1 z_{t-1}' \gamma / (1-\alpha L) + K_2 z_t' \gamma / (1 - \theta F) \tag{B.12}$$

where $\theta\rho = 1$, and $LF = 1$. (Note that $L\, y^{t-1}_{t+1} = y^{t-2}_t$, and

$F\, y^{t-1}_{t+1} = y^{t-1}_{t+2}$, so that $LF \neq 1$ when the operators are

applied to conditional expectations). The solution for $\bar{y}_t$ is obtained noting that

$$(1 - \alpha L)(1 - \rho L) = (-\rho L)(1 - \alpha L)(1 - \theta F)$$

$$\frac{1}{(1-\alpha L)\ (1-\rho L)} = \frac{-\theta\ F}{(1-\alpha L)\ (1-\theta F)}$$

$$= \frac{n}{1\ -\ \alpha L} + \frac{mF}{1\ -\ \theta F} \qquad \text{(B.13)}$$

where

$$n = \alpha\ \theta\ /\ (\alpha\ \theta\ -\ 1)$$
$$m = \theta\ /\ (\alpha\ \theta\ -\ 1) \qquad \text{(B.14)}$$

Then,

$$y^{t}_{t+1} = \tilde{y}_{t+1} + w^{t}_{t+1}$$

$$= \bar{y}_{t+1} + \alpha\ w_t$$

$$= \bar{y}_{t+1} - \alpha\ \bar{y}_t + \alpha\ y_t \qquad \text{(B.15)}$$

Since 'α' and 'ρ' are functions of $\emptyset_1$ and ($\emptyset_2$ a), this last expression can be substituted back in (B.1) and we obtain the system of two equations (B.1) and (B.10), in terms of observables. If both roots are stable, the solution will depend on a third parameter 'δ', so that,

$$w_t = [(1\ -\ \delta L)\ /\ ((1\ -\ \alpha L)\ (1\ -\ \rho L))]\ u_t$$

$$\tilde{y}_t = z'_{t-1}\ \gamma\ /\ [(1-\alpha L)\ (1\ -\ \rho L)] \qquad \text{(B.16)}$$

Then y^{t}_{t+1} can be defined correspondingly and substituted in (B.1) to get the estimable form of the model.

C. SELECTING THE INSTRUMENTS FOR THE INVESTMENT EQUATION

When we introduce y_{+1}, we want to use as an instrument for it, $\bar{y}_{t+1}$ (in the notation of section III). Then, since the dummies D1, D2, are nonstochastic they are obvious candidates for instrumental variables. That means that we should select an independent set of instruments among the set ($D1$, $D2$, $D1_{+1}$, $D2_{+1}$). Let us consider this set in a simple case,

D1	D2	$D1_{+1}$	$D2_{+1}$
1	0	1	0
1	0	1	0
1	0	1	0
1	0	1	0
0	1	1	0
0	1	0	1

Since D1+D2 is the vector unity, $D2_{+1}$, can be obtained as an exact combination of the remaining vectors as follows,

$$D2_{+1} = D1 + D2 - D1_{+1}$$

and more generally

$$D2_{+s} = D1 + D2 - D1_{+s}$$

Then, if we add the expectation y_{+s}, we only have one more instrument, that is, $D1_{+s}$.

D. THE CASE OF CURRENT EXPECTATIONS

When there are only expectations for time 't' made at time (t-1) in the system, the solution is particularly simple. (This is one of the cases considered by K. Wallis (1980). Let us consider then the model

$$B\, y_t + D\, y_t^{t-1} + c\, z_t = \varepsilon_t \qquad (D.1)$$

If we assume $z_t^{t-1} = z_t$, and substituting y_t^{t-1} by y_t we get finally

$$(B + D)\, y_t + c\, z_t = u_t \qquad (D.2)$$

where 'u_t' is an error with the usual properties, and in particular, is serially uncorrelated if 'ε_t' is. We can now get the reduced form

$$y_t = -\,(B + D)^{-1}\, c\, z_t + v_t$$

$$= \pi\, z_t + v_t \qquad (D.3)$$

and consider the estimation of π subject to the restrictions

$$(B + D)\, \pi + C = 0 \qquad (D.4)$$

The identification condition can be dealt with in the usual way. The only difference is that we have now the augmented matrix (B + D) to consider. Single equation estimation by an instrumental variable or a limited information method can also be dealt with in the usual way.

The conventional way of solving the problem of identifying and estimating model (D.1) consists of getting first the solution for y_t^{t-1}. The second step is to plug it back in (D.1) and get the model expressed as a function of observables. This procedure, is more complicated and leads to the same model as that given in (D.3,4). In order to see that, we get from (D.3)

$$y_t^{t-1} = - (B + D)^{-1} C z_t \tag{D.5}$$

and substituting in (D.1) yields

$$B y_t + (C - D (B + D)^{-1} C) z_t = \varepsilon_t \tag{D.6}$$

which in turn has the reduced form,

$$y_t = -B^{-1} (C - D (B + D)^{-1} C) z_t + v_t \tag{D.7}$$

We now prove that this coefficient matrix, is precisely that of (D.3). Then,

$$\begin{aligned} & B^{-1} (I - D (B + D)^{-1}) C - (B + D)^{-1} C \\ & = B^{-1} C - B^{-1} D (B + D)^{-1} C - (B + D)^{-1} C \\ & = B^{-1} C - (B^{-1} D + 1) (B + D)^{-1} C \\ & = B^{-1} (I - (D + B) (D + B)^{-1}) C \\ & = 0 \end{aligned} \tag{D.8}$$

E. A NOTE ON THE POWER OF THE TEST GIVEN IN SECTION III

In order to understand the properties of the test proposed in this paper, it may be useful to analyze an alike case. Consider then the following model,

$$y = x_1\lambda + x_2\ \theta\ \lambda + \varepsilon$$

$$= x_1\ b_1 + x_2\ B_2 + \varepsilon \qquad \text{(E.1)}$$

where 'λ' and 'θ' are scalars.

This is similar to the rational expectations situation; 'x_1' takes the place of the expectational variable and 'x_2' that of the moving average component in the error. Now, under standard assumptions, the maximum likelihood estimator of λ is B_1 OLS. We can set up a Wald test to check the null $H_o : B_1 = 0$. Alternatively, we could think of testing that hypothesis by means of a Lagrange test. Consider then,

$$x\ \left(\frac{\delta\ B}{\delta\ P}\right)' = (x_1,\ x_2)\ \begin{bmatrix} 1 & 0 \\ \theta & \lambda \end{bmatrix}$$

$$= (x_1 + \theta\ x_2\ ,\ x_2\ \lambda) \qquad \text{(E.2)}$$

where $p = (\lambda, \theta)$. Under H_o, $\lambda = 0$, so that this last expression does not have full column rank and the Lagrange test therefore is not well defined.

Since under H_o, $B_1 = B_2 = 0$, it may be more powerful to test jointly this last hypothesis, and this is the type of test proposed in this paper (The power will now depend on the value of θ). The power of the Wald test against the null $B_1 = 0$, will be given by the following non centrality parameter, when $\lambda \neq 0$

$$\lambda^2 (x_1' M_2 x_1)^{-1}; \quad M_2 = I - x_2 (x_2' x_2)^{-1} x_2' \qquad (E.3)$$

and in the case of the joint test, by

$$\lambda^2 (x_1' M_2 x_1)^{-1} + \theta^2 \lambda^2 (x_2' M_1 x_2)^{-1} -$$

$$- 2 \theta \lambda^2 (x_1' M_2 x_1)^{-1} x_1' x_2 (x_2' x_2)^{-1} \qquad (E.4)$$

If $\theta = 0$ the power is the same in both cases. If $x'_1 x_2 = 0$ and $\theta \neq 0$, the second procedure is more powerful, and if $x'_1 x_2 \neq 0$ there is no general solution.

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