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Agustín Maravall

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Agustín Maravall
Bank of Spain (*)

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1. INTRODUCTION

The seasonal adjustment method presently used at the Board for adjusting M_1 is a blend of X-11 and judgmental corrections applied to the monthly series of demand deposits and currency. Since the monthly (unadjusted) series are averages of daily values, an obvious alternative procedure for obtaining the monthly adjusted figure would be to adjust the daily series and then aggregate them into months. In other words, temporal aggregation can precede or succeed seasonal adjustment. The use of a daily method was recommended by the Bach Committee on Monetary Statistics (1). In this paper we compare the official Board method with the daily one developed at the Board, which is an extension of the one actually used by the Bach Committee.

It is argued that there are reasons for preferring a daily approach and reasons for preferring the monthly one. It seems safe to conclude that, to the extent that neither of the two approaches is unquestionably superior, they provide two alternative methods for adjusting the series. Insofar as the two methods do not yield identical results, the difference between them reflects and ambiguity in the computation of the series of interest (monthly seasonally adjusted M_1). Logically this ambiguity sets a lower limit to the precision with which the series can be measured and therefore has the character of a measurement error (2). Naturally, if in most practical situations the difference between the two methods happens to be very small, the ambiguity could be easily ignored. On the other hand it may be that on occasion the judgment as to whether growth of M_1 has been adequate or not and therefore the direction of monetary policy depends crucially on the method used for seasonal adjustment.

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(1) See Advisory Committee on Monetary Statistics (1976, pp. 4 and 39-40).

(2) It is an example of the errors that arise from "imperfection and incompleteness in the description of the phenomenon to be measured," as categorized by Morgenstern (1963, Chap. II).

Of course, economic time series are used by decision makers when setting policy. These series, we know, have errors, yet this knowledge is usually ignored. One can wonder what part of economic policy is simply the result of measurement errors . This paper can be viewed as a crude attempt at providing some insights into the problem for a specific error of measurement in the context of monetary policy (1).

In the next section we apply the two methods of seasonal adjustment to the M_1 series for the period 1971-1977. This yields a series of differences between the two adjusted values which we proceed to analyze. Then we provide an estimate of how likely it is that the direction of monetary policy depends on the method used for seasonal adjustment. Next, the ambiguity in measuring seasonally adjusted M_1 is translated into an equivalent imprecision in terms of interest rate movements. It is also seen how relatively minor changes in the way in which intermediate targets are expressed may decrease the effect of the ambiguity. Finally we consider how the two seasonal adjustment methods can be combined to yield a seasonally adjusted value subject to smaller revision errors.

2. DAILY VERSUS MONTHLY SEASONAL ADJUSTMENT OF M_1

The first series we consider is the seasonally adjusted monthly money supply (M_1) series published by the Federal Reserve. This series is obtained by applying X-11, and then judgmentally modifying the factors obtained, to the two components of M_1: currency and private demand deposits. The procedure is discussed in Fry (1976) and in Pierce *et al.* (1978).

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(1) Attempts at quantifying measurement errors are infrequent in economics. According to Morgenstern, "there is much less occupation with errors than in other fields. This is undoubtedly one of the reasons why the social sciences have had a rather uncertain development " (1963, p. 7).

The second series we consider is the monthly average of seasonally adjusted daily values of M_1, obtained also as the sum of adjusted currency and demand deposits components. The daily seasonal adjustment procedure we consider was developed at the Federal Reserve as an extension of the one used by the Bach Committee. The procedure is described in Pierce and Fries (1978)(1).

Perhaps some general comments on the two methods are in order:

Aside from the stochastic characteristics of the series, optimality of a seasonal adjustment method depends on the use to be made of the series (2). In our application, this use is the setting of monetary policy by the Federal Open Market Committee (FOMC). Since the growth of M_1 (seasonally adjusted) has been for some time a crucial variable in monetary control, optimal seasonal adjustment of M_1 would require separate estimation of the seasonal which is the result of past policy action and the one which is due to "exogenous" causes. Otherwise, monetary policy would be contaminated by undesirable variables, such as for example, past errors in the estimation of seasonal factors (3).

Thus, strictly speaking, optimality of the seasonal adjustment of the monetary aggregates in the context we consider possibly requires the use of an appropiate econometric model. Although some progress in that direction has been made by Plosser (1978,79), practical enforcement of such methods in on-going monetary policy is still distant (4). Meanwhile, what can be said of the relative merits of the two methods we consider?

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(1) The method is an extension of the daily method described in Pierce et al. (1978), along the lines developed in Pierce (1978). A brief description is given in appendix B.

(2) See Grether and Nerlove (1970).

(3) This point is discussed in the report of the Bach Committee (see Advisory Committee on Monetary Statistics (1978), pp. 37-39).

(4) See Lombra (1978). Possibly this distance is reinforced by the additional (optimality) requirement of specifying the correct aggregate loss function of the 12 FOMC members.

A theoretical reason for preferring a daily approach is provided in Geweke (1978), where it is shown that "for virtually every conceivable time series ... and a reasonably inclusive class of potential adjustment procedures, minimum mean-square-error adjustment implies that seasonal adjustment should always precede temporal or sectorial aggregation." Yet his result cannot be applied blindly. First, it is hard to assess whether the combination of X-11 and judgment falls into his "reasonably inclusive class" of adjustment procedures (1). Second, as shown in Taylor (1978), "it is not the preaggregation adjustment that is crucial for optimality, but rather the utilization of all observable components simultaneously." Insofar as the judgmental corrections may use daily information, Geweke's result would not apply.

Other relative advantages of the method based on the daily procedure are that it allows for a more complete treatment of special-day events, and that it makes possible the construction of perfectly consistent seasonally adjusted figures expressed in different time intervals.

On the other hand, there are advantages associated with the aggregate monthly method. First, the computational complications of the daily approach are not trivial. Second, monthly data tends to have smaller measurement errors (2). Finally, misspecification may be more of a problem in the daily approach (3).

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(1) See Lovell (1978).

(2) See Porter et al (1978b.)

(3) The preference for the daily approach expressed by several of this paper's referees is in contrast with those of the members of the Federal Reserve Advisory Committee on Seasonal Adjustment and with that in Shiskin (1978).

Thus it seems safe to conclude that neither of the two approaches can be seen as unquestionably superior. Since presently one of the methods is being used, and the other is being considered for use by the Federal Reserve, they provide a meaningful comparison for answering the question of how sensitive monetary policy is to alternative seasonal adjustment methods (1).

Let m_t stand for the seasonally adjusted monthly figure of M_1. Since monetary policy utilizes annualized rates of growth over two-month periods, we shall operate with the variable:

$$r_t = 6\left[(m_{t+2} - m_t)/m_t\right] .$$

When m is the official Board estimate, the variable r_t shall be denoted r_t^0. When m is computed through the daily approach, we shall write r_t^a. The series $[r_t^0]$ and $[r_t^a]$ are displayed in Table 1. Table 2 displays their autocorrelation function (ACF). For both series the functions are extremely close. No seasonality seems to remain, and only low order autocorrelation is significantly different from zero. Also both series are seen to be stationary. Possibly the most noticeable difference between the two series is the difference in variances, since the variance of r_t^0 is larger than the variance of r_t^a. Thus more seasonality is removed from the series when the daily procedure is used.

Let δ denote the difference between the two series:

$$\delta_t = r_t^0 - r_t^a .$$

(1) Naturally, comparison among other seasonal adjustment methods in terms of their effect on economic policies would also be of interest.

Tables 1, 2 and Fig.1 display the series δ_t and its ACF. Despite the fact that no seasonality can be detected in the r-series, their difference exhibits strong 6-months and annual patterns.

3. THE EFFECT OF ALTERNATIVE MEASUREMENTS ON MONETARY CONTROL: TOLERABLE VERSUS NONTOLERABLE GROWTH OF M_1

3.1. Alternative measurements of M_1 and monetary control.

At each monthly meeting, the Federal Open Market Committee sets a range for the rate of growth of the monetary aggregates (seasonally adjusted) which is considered tolerable over a two-month period. Policy action will depend - among other things -on whether or not the growth of the aggregates falls within this range (1).

In the case we are analysing, there are two alternative ways of measuring the rates of growth. Consider two monetary authorities, A and B, both of wich follow identical behavior. They set a range of tolerance for the rate of growth of M_1 seasonally adjusted, and rates outside the interval trigger some kind of intervention. Assume A and B are identical in every respect, with the only difference being the way they adjust for seasonality. A adjusts the monthly aggregates, while B adjusts the daily data, then aggregates them into months. Specifically, A operates with r^0, while B operates with r^a.

Let A and B each hold a meeting at time t in which they set a range of tolerance for the growth of M_1 over a two-month period. Assume that for the previous period, both measures of r yielded the same figure (thus $\delta_{t-2}=0$) so that, everything being equal, the range of tolerance will be the same for both. For a

(1) For a more complete description, see Wallich and Keir (1978) and Lombra and Torto (1975).

	JAN.	FEB.	MARCH	APRIL	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
	220797 (*)	222746	224362	225828	228302	229436	230723	231854	232270	232714	233099	234012
1971	221358 (**)	223177	224437	225685	227711	228753	229806	231011	232306	233073	233370	234069
	(***)		1,3426	1,5568	1.7824	1.4312	0.8439	0.3988	-2.5047	-3.1281	-0.6082	0.7840
	235732	237721	240025	241407	242205	242860	245099	247265	249237	251050	252179	255276
1972	235867	238041	240040	241288	241783	242448	244654	246646	249367	251079	252404	254675
	0.3584	-0.6723	0.3127	1.1172	1.0909	0.7280	0.0463	0.4949	-1.4306	-1.601	-0.2249	1.5087
	257737	258162	258179	259012	261896	264043	264677	265186	265075	266168	268629	270474
1973	257839	258304	258069	259023	261236	263076	263852	264532	265065	266174	268471	270885
	0.3051	-1.7674	0.4931	0.3046	1.2745	2.2659	0.3642	-0.7233	-1.8573	-1.5028	0.3366	-0.9145
	271761	273107	274839	275389	276228	277832	278316	278967	279615	280704	282311	282822
1974	271841	273371	274768	275435	275847	277109	278026	278746	279599	280864	282328	283528
	-0.5376	0.3342	0.3354	0.4838	0.6765	1.6786	-0.2050	-1,0946	-0.5948	-0.8212	-0.0715	-1.167
	282637	282787	284975	284756	287635	291205	291068	292405	293442	292923	295430	294509
1975	283017	283133	284941	285042	287185	290261	290609	292418	293217	292813	295418	295518
	-0.7697	0.7626	0.8821	0.1330	0.8766	2.6025	0.0088	-1.9856	-0.4918	0.2525	-0.4403	-2.2941
	295918	297936	299328	301462	303347	303537	304134	305840	306756	310133	310563	312576
1976	296281	297853	299047	301692	303029	303006	304469	306002	307224	309746	310630	313119
	-0.7604	2.2393	1.313	-0.6327	0,0664	1.5166	-1.2944	-1.3797	-0.2563	1.0815	0.7944	-1.8077
	314919	316336	318313	322008	322382	324306	327463	329231	331649	334730	334941	337225
1977	314407	316309	317986	321865	322221	323797	327611	329279	331798	333811	334657	336610
	.1,1202	1.105	-0.3627	0.2183	-0.3220	0.6823	-0.5812	-1.0488	0,0032	1.765	0.7854	-0.5581

(*) Seasonally Adjusted Monthly Series of M_1: Published (M_t^0)

(**) Seasonally Adjusted Monthly Series of M_1: Obtained from the daily method (M_t^a)

(***) Monthly Series of $\delta_t (=r_t^0 - r_t^a)$.

AUTOCORRELATION FUNCTIONS

a) Variable r_t^o

Lags	1	2	3	4	5	6	7	8	9	10	11	12	Error
1-12	.50	.02	-.06	-.12	.10	.17	-.01	.04	.12	.14	.16	-.08	.11
13-24	-.23	-.07	.07	.22	.15	.06	-.05	-.15	-.14	.04	.01	-.08	.15

b) Variable r_t^a

Lags	1	2	3	4	5	6	7	8	9	10	11	12	Error
1-12	.49	-.02	-.05	-.01	.18	.17	.00	.04	.11	.15	.15	-.09	.11
13-24	-.21	-.08	.09	.24	.21	-.01	-.11	-.09	-.06	.02	-.05	-.14	.15

c) Variable δ_t

Lags	1	2	3	4	5	6	7	8	9	10	11	12	Error
1-12	.34	-.24	-.12	.01	-.18	-.49	-.15	.09	-.09	-.12	.31	.59	.11
13-24	.13	-.20	-.07	.03	-.14	-.29	-.07	-.02	-.11	.01	.28	.33	.18

	Mean	S D
r_t^o	.061	5.196
r_t^a	.061	4.704
t	.054	1.165

specific growth of M_1 over the period, what are the chances that A and B would disagree as to whether or not that growth is tolerable?

The probability of disagreement is the sum of three components:

1) The probability that r_t^0 falls outside the interval of tolerance and r_t^a falls inside. Denote this probability by P_I.

2) The probability that r_t^0 falls inside the interval of tolerance and r_t^a does not. Denote this probability by P_{II}.

3) The probability that both r_t^0 and r_t^a fall outside the interval but on opposite sides. Denote this probability by P_{III}.

Assume that the forecast of r_t^0 at the time the meeting is held is the midpoint of the interval of tolerance (0 in figure 2) (1). The forecast error e_t is then the distance between r_t^0 and 0. Let α denote the midrange of the interval of tolerance. It is easily seen that (deleting the subscript t for notational simplicity):

1) When r^0 is outside the tolerance interval, then r^a will be inside if and only if

$$e+\alpha > \delta > e-\alpha$$

2) When r^0 is inside the interval, then r^a will be outside whenever

$$\delta < e-\alpha \qquad \text{or} \qquad \delta > e+\alpha$$

(1) A typical "Record of Policy Actions" of FOMC (published in the Federal Reserve Bulletin) contains the following statement: "The Committe agreed that if growth rates of the aggregates over the 2-month period appeared to be deviating significantly from the midpoints of the indicated ranges, the operational objective in the weekly-average Federal funds rate should be modified..." Also, Lombra and Torto (1975, p. 11) state that "from the viewpoint of the staff the ranges .... represent a standard error around a point estimate at the midpoint of the range." Thus the assumption that the midpoint of the range can be seen as an approximation to a staff forecast seems reasonable. Of course, this will not always be the case.

3) When r^0 is outside, for r^a to be also outside but on the opposite side it has to be that

$$|\delta| > e+\alpha$$

The first case is illustrated in figure2 .

Therefore the probabilities P_I, P_{II} and P_{III} can be expressed as:

$$P_I = 2\int_{e=\alpha}^{\infty}\int_{\delta=e-\alpha}^{e+\alpha} P(\delta/e)\ P(e)\ d\delta\ de,$$

$$P_{II} = 2\int_{e=0}^{\alpha} \left|1-\int_{\delta=e-\alpha}^{e+\alpha} P(\delta/e)\ d\delta\right| P(e)\ de,$$

$$P_{III} = 2\int_{e=\alpha}^{\infty}\int_{\delta=e+\alpha}^{\infty} P(\delta/e)\ P(e)\ d\delta\ de,$$

and the total probability of disagreement is equal to

$$P = P_I + P_{II} + P_{III}$$

In order to compute the probabilities P_I, P_{II} and P_{III}, we need the joint distribution of δ_t and e_t. In appendix A we conclude that this distribution can be assumed to be bivariate normal, with zero mean vector and covariance matrix Ω, as given in the appendix.

One implicit assumption in our formulation is that the p.d.f. of e is independent of α. This can only be seen as a simplification, since it could be that a wider tolerance range

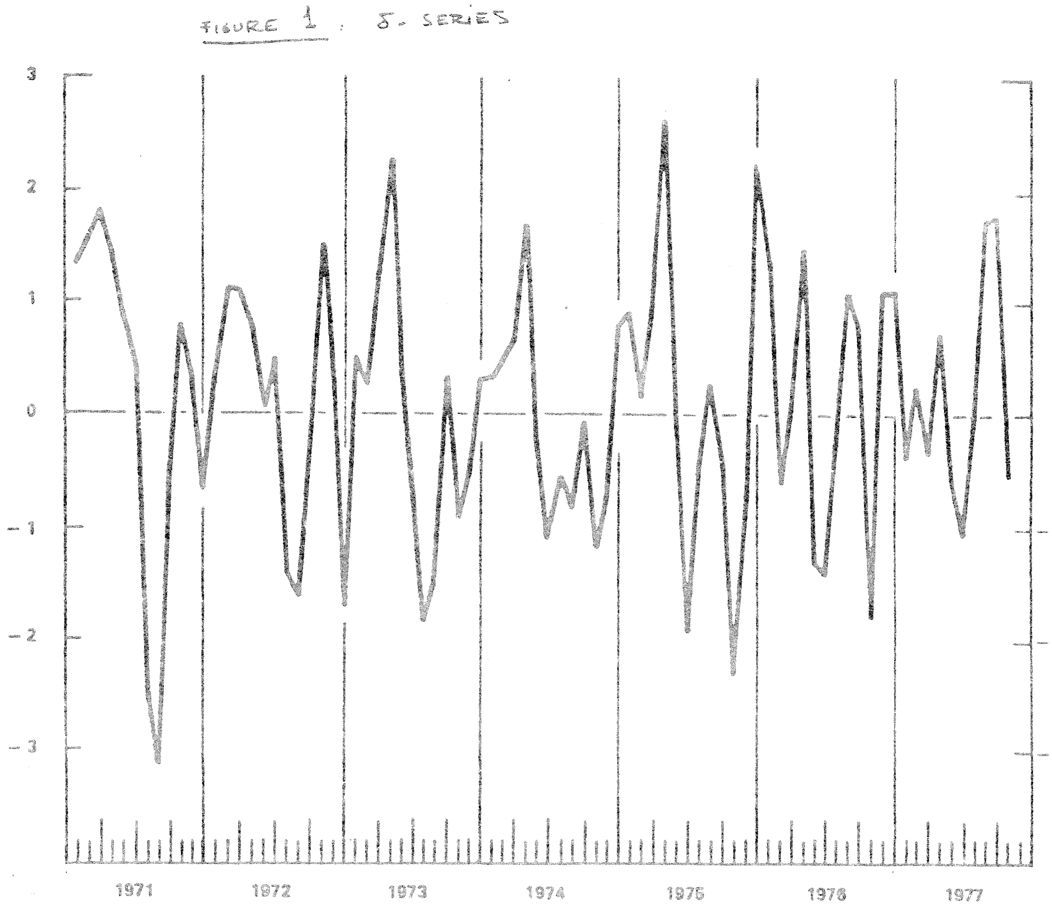
FIGURE 1 : δ - SERIES
3
2
1
0
-1
-2
-3
1971
1972
1973
1974
1975
1976
1977

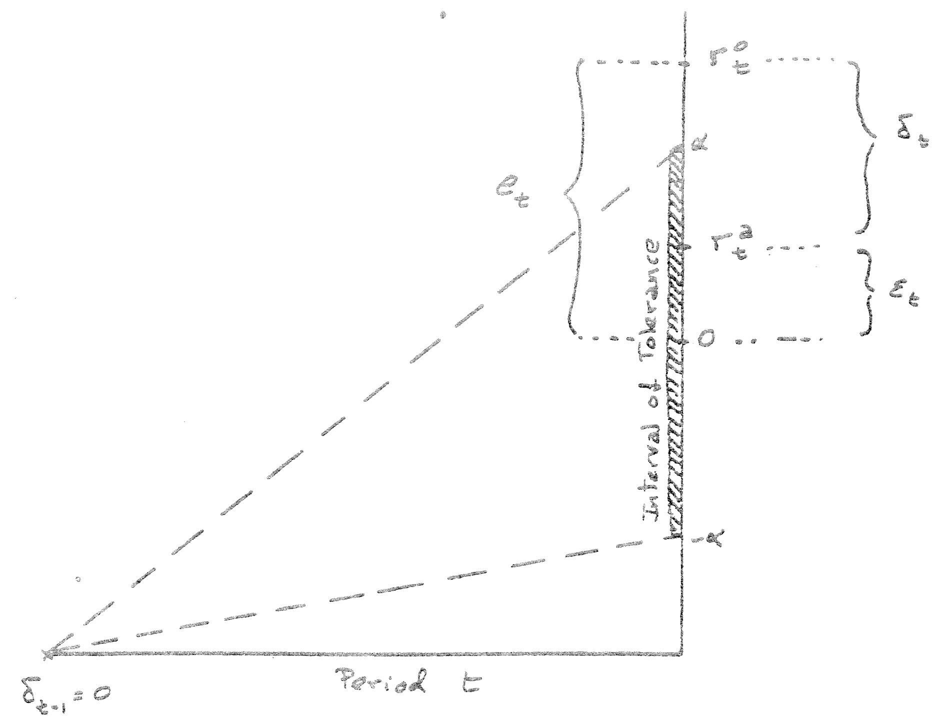
r_t^o
δ_t
α
e_t
r_t^a
ε_t
0
Interval of Tolerance
$-\alpha$
$\delta_{t-1} = 0$
Period t

Figure 2

indicates a greater willingness by the FOMC to tolerate wider departure of M_1 from the range midpoint (1).

3.2. The effect of alternative measurement on monetary intervention

Since the joint distribution $P(\delta_t, e_t)$ is given by:

$$P(\delta_t, e_t) = \frac{1}{2\pi\ |\Omega|^{1/2}} \exp \{-1/2(\delta_t \quad e_t)\Omega^{-1}\begin{pmatrix}\delta_t \\ e_t\end{pmatrix}\}$$

the probabilities P_I, P_{II} and P_{III} can be estimated through the expressions in (4), once the parameters are replaced by their estimates. The value of α, the midrange of the interval of tolerance is published in the Federal Reserve Bulletin. For the past years it has oscillated between 2 and 3. We set α=2.5. Before performing the integrations, a minor modification has to be made. In order to simplify the comparison between the two monetary authorities we set before $\delta_{t-2}=0$. Thus in our computation, the variance σ^2_δ should be replaced by $\sigma^2_{\delta_t/\delta_{t-2}=0}$. That is, instead of the marginal distributions of δ_t, we should employ the distribution of δ_t conditional on $\delta_{t-2}=0$ (2). From the expression derived in appendix A, it is obtained that $\sigma^2_{\delta_t/\delta_{t-2}=0} = 1.22$.

Numerical computation yields the following results:

$$P_I = .076 \qquad P_{II} = .053 \qquad P_{III} = .028\ (10^{-5})$$

$$P = .13$$

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(1) I am grateful to an anonymous referee for pointing out this qualification.

(2) Naturally, using $\sigma^2_{\delta_t/\delta_{t-2}=0}$ in the computation of P biases downwards the estimated probability of disagreement.

P_I is the probability that, for the same growth of M_1 in period t, B considers the growth tolerable and A does not; P_{II} is the probability that A considers the growth tolerable and B does not. Obviously P_{III} can be neglected (1).

Thus, for a given growth of M_1 in period t, there would be a 7.6 % probability that A would think monetary intervention was granted and B would think the opposite, a 5.3 % probability that A would think monetary intervention was not granted and B would think the opposite, and in general, a 13 % probability that the two monetary authorities would disagree. Approximately, one out of eight times disagreement would occur.

The estimated probabilities of disagreement are, so to speak, unconditional probabilities in that they do not take account of economic or policy conditions which may affect the joint p.d.f. For example, it could happen that deviations of M_1 from range midpoints are positively correlated with the rate of expansion of economic activity. In general, it would not be correct to assume that the probability of disagreement is invariant to different economic conditions and monetary policy directives (2).

3.3. Sensitivity of the result to parameter changes. The effect of forecasting improvement

The numerical result of the previous paragraph was computed for values of α and σ_e typical of recent years. Naturally, the smaller the value of α, the larger would be the probability of disagreement. Since over the last years, α has oscillated between 2 and 3, the probability of disagreement would have ranged between 13.8 and 11.5 per cent.

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(1) A necessary (though by no means sufficient) condition for r^0 and r^a to fall outside the tolerance range and on opposite sides, is $|\delta|>2\alpha$. Since $P(|\delta|>2\alpha)=2P(z>4.3)$, where $z \sim N(0,1)$, it follows that P_{III} is smaller than (.04) 10^{-3}. Therefore, for the rest of the analysis, the P_{III} type probability of disagreement can be safely ignored.

(2) I am grateful to an anonymous referee for pointing out this qualification.

More interesting is the relationship between the probability of disagreement and σ_e, the standard error of the forecast of the rate of growth of M_1 over the two-month period. We shall assume that α remains constant, although it would seem plausible that improvements in the ability to forecast M_1 could lead to a reduction of the interval of tolerance. However, the oscillations of α over the past years seem to have been motivated by other considerations. Also, we shall assume that σ_δ remains constant.

In the same way that e_t denotes the forecast error of r_t^0, let ε_t denote the forecast error of r_t^a. It follows that

$$\varepsilon_t = e_t - \delta_t$$

and since e and δ are uncorrelated,

$$\sigma_\varepsilon^2 = \sigma_e^2 + \sigma_\delta^2 \quad .$$

When $\sigma_e \to \infty$, the probability of r^0 and r^a falling both in the tolerance interval (α constant) approaches zero. Since the probability that they fall on opposite sides can be neglected, the probability of disagreement would tend to zero as $\sigma_e \to \infty$.

On the other hand, when $\sigma_e \to 0$, the probability that r^0 falls inside the interval approaches one. Thus, disagreement can only occur when r^0 falls outside. Since σ_ε approaches its minimum value as $\sigma_e \to 0$, the probability of r^a falling outside the interval becomes smaller and smaller. Thus, the probability of disagreement decreases as σ_e approaches zero.

Hence, the function that relates P to σ_e tends to a minimum for $\sigma_e \to 0$ and $\sigma_e \to \infty$. This means that, for a positive value of σ_e, a maximum for P has to exist (since, according to (4), the function is bounded). Thus in some region, improvements in forecasting accuracy increase the probability of disagreement.

Figure 3 plots the probability of disagreement as a function of σ_e, the standard deviation of the forecast error. Two things can be noticed:

a) The result obtained (disagreement would occur one out of eight times) is robust with respect to present (and at least short-run expectations of) forecasting accuracy. For the range $2.5<\sigma_e<4$, P oscillates between 11 % (for $\sigma_e=4$) and 13.3 % (for $\sigma_e=2.4$).

b) A maximum for P is reached when $\sigma_e=2.4$. Since our present accuracy implies larger values of σ_e, in the short-run, it is likely that improvements in forecasting accuracy increase the probability of disagreement.

Figure 4 decomposes P into the two probabilities P_I and P_{II}. Again both curves have a well defined maximum in the interval considered, although the maximum of P_I corresponds to a bigger value of σ_e (3 as opposed to 1.5).

The probabilities P_I and P_{II} can be given an interesting interpretation. Assume that r^a is the correct variable, but the monetary authority operates with r^0. Our probability of disagreement becomes then the probability that the monetary authority makes an error. In particular, P_I represents the probability that the rate of growth is judged non-tolerable when it is tolerable, and P_{II} the probability that the rate of growth is judged tolerable when it is not.

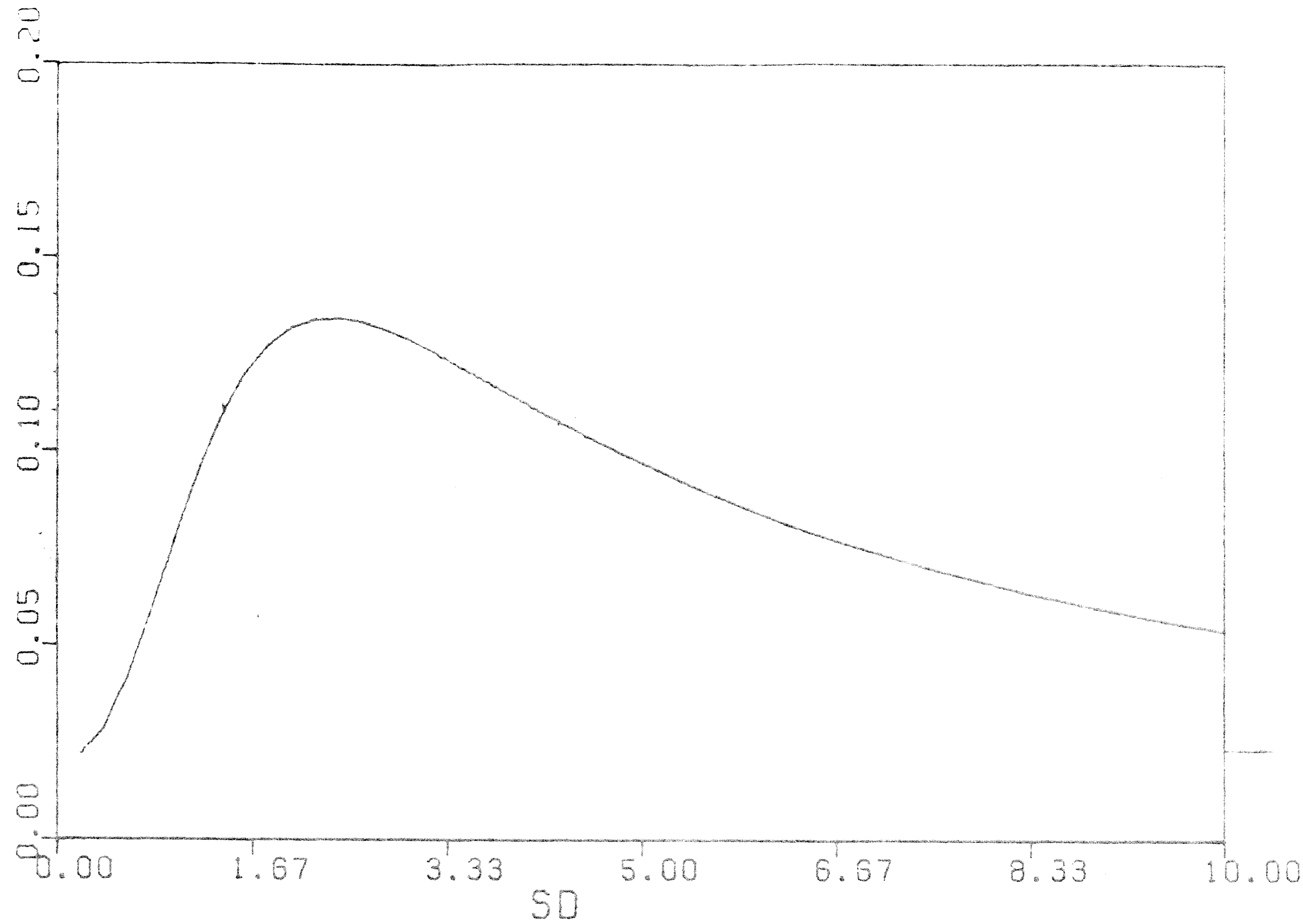

Figure 3 : TOTAL PROBABILITY OF DISAGREEMENT

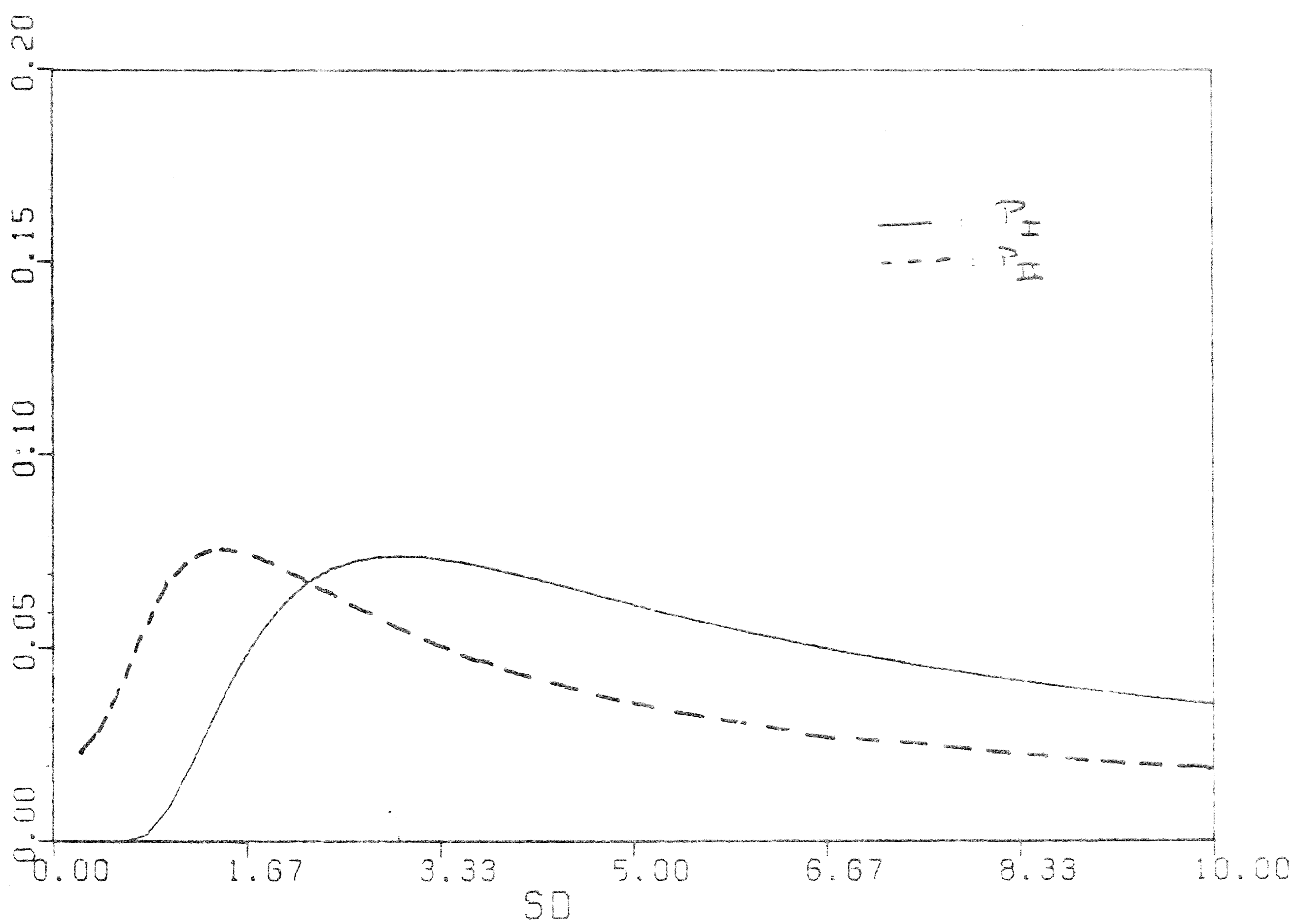

Figure 4 : DECOMPOSITION OF THE PROBABILITY OF DISAGREEMENT

Thus P_I and P_{II} can be seen as the probabilities of type I and type II errors, respectively. If nontolerable growth implies intervention and vice versa, then P_I represents the probability of intervention when none should have been granted, and P_{II} represents the probability of no intervention when there should have been some. From the values computed, it is seen that, over the relevant range of σ_e, the probability of a type I error is slightly larger than the probability of a type II error. This may reflect an implicit FOMC belief that it is more dangerous not to intervene when intervention is granted than to intervene when it is not needed.

4. THE EFFECT OF ALTERNATIVE MEASUREMENTS: AN INTEREST RATE APPROACH

In the previous section we provided a quantitative measure to a qualitative question. We estimated what would be the probability that two alternative seasonal adjustment methods would lead to disagreement in terms of whether or not the monetary authority should act to modify the funds rate. In this section we provide an answer to a different question: what would be the difference in interest rates when the two alternative seasonal adjustment methods are used? Since the difference between the two methods can be interpreted as an imprecision in the measurement of the seasonally adjusted series of M_1, the difference in funds rate changes (according to which method is used) can be therefore interpreted as an imprecision in detecting federal funds movements. In that sense, it sets a lower limit to the "finesse" with which monetary policy can be instrumented given present operating procedures.

In providing an answer to the question, we consider a monetary authority with a reaction function which specifies that changes in the funds rate are related to a set of variables, some of which are related to the seasonally adjusted rate of growth of M_1, while others are independent of M_1 measurements.

Assume that monthly changes in the Federal funds rate, y_t, are expressed as a function of forecast errors of the two-month rate of growth of M_1 (seasonally adjusted), e_t, plus a set of variables such as changes in the unemployment rate, in the rate of inflation, etc, which we assume unaffected by errors of measurement in M_1 (1). Let $\underline{z}$ denote the vector of these variables, so that

$$y_t = \xi(B)\ e_t + F(\underline{z})$$

where $\xi(B)$ denotesa polynomial lag.

If r_t^a were to be used, the forecast error would become ε_t, where $\varepsilon_t = e_t - \delta_t$. The difference in y_t from using one rate or the other is, therefore, (2),

$$\Delta y_t = \xi(B)\ \delta_t\ .$$

In Farr (1978) several reaction functions are estimated. The filter:

$$\xi(B) = (1-\xi_0 B)^{-1}\ (\xi_1+\xi_2 B)$$

with parameters equal to $\xi_0=.4$, $\xi_1=.03$, $\xi_2=.01$, seems to provide a formulation in agreement with all the results obtained. Thus

$$\xi(B) = .03 + .022 \sum_{i=1}^{\infty} (.4)^{i-1}\ B^i.$$

Applying $\xi(B)$ (truncated after the first twelve elements of the summation) to the series δ_t, a series Δy_t is

(1) This last assumption is, again, a simplification. Since,for example, measurements of M_1 influence policy and policy influences inflation, indirect relationships could exist.

(2) We are assuming that the lag structure $\xi(B)$ is the same wether we use officialadjustment or adjustment based on the daily method. Since the ACF of r_t^0 and r_t^a are extremely close, possibly the differences in $\xi(B)$ would be small, and the results presented below would not be much affected.

obtained. Expressed in terms of basis points (1), its first two moments are given by:

$$E(\Delta y_t) = .22$$

$$\sigma_{\Delta y} = 6.2$$

Since the standard deviation of month-to-month changes in the funds rate (over the sample period) is, approximately, 50 basis points, these results imply that 13 % of it could be accounted for simply by the difference in the two seasonal adjustment methods we considered (when applied to the series). Also, since $2\sigma_{\Delta y}=12.4$, a change in the funds rate of 1/8 of a percent point could be attributed to that difference (2).

To obtain an additional estimate, we used the reaction function estimated in De Rosa and Stern (1977) for the period 1970-74. In their formulation, month-to-month percentage change in the funds rate is related to a three-month moving average percentage change in M_1 through the filter:

$$\xi^*(B) = c_0 \; (1 - c_1 B - c_2 B^2 - c_3 B^3)^{-1}$$

where c_0, c_1, c_2 and c_3 are estimated as 14.64, .304, -.282 and .016, respectively. Since the measure of M_1 is different from the rate of change used previously, we computed the difference:

$$\delta_t^* = \bar{m}_t^0 - \bar{m}_t^a$$

where $\bar{m}_t^0$ is the three-month moving average percentage change in M_1 when seasonally adjusted published figures are used, and $\bar{m}_t^a$ is the same variable that uses seasonally adjusted values of

(1) A basis point represents a change of 1 %.

(2) Since the mean of δ can be accepted as zero, one could conclude that, in the long run, the effects would cancel out. But this would only be true if the reaction function is symmetric.

M_1 through the daily procedure (1). The first two moments of the series:

$$\Delta y_t^* = \xi^*(B)\ \delta_t^*$$

are estimated by

$$E(\Delta y^*) = .32$$

$$\sigma_{\Delta y^*} = 7.43$$

where the unit again is basis points. Therefore, the imprecision in detecting month-to-month funds changes, derived from the difference between the two adjustment methods we consider, is of similar magnitude for the two formulations of the reaction function.

5. SOME IMPLICATIONS

5.1. Ambiguities in Seasonal Adjustment

In the previous two sections we have analysed the effects of using the seasonally adjusted monthly average of daily values of M_1 versus the monthly average of seasonally adjusted daily values of M_1. We found that, depending on which one is used, the monetary authority would disagree one out of eight times as to whether or not intervention is warranted; and that a difference of 1/8 of a percent point in the Federal funds rate could separate policies using the two alternative methods. So despite the fact that no criterion seems to be available that would establish which of the two methods is the right one to use, the implications of choosing one versus the other do not appear to be negligible.

(1) The similarities in the AC function of δ^* and δ are striking. For example, the largest AC are as follows:

Lags	1	6	12
δ	.34	-.49	.59
δ^*	.45	-.47	.58

From a general point of view, the problem we have analysed reinforces the idea that seasonal adjustment may possibly affect the data considerably more than is intended. First, there is the well known problem of revision errors. Second, there are the shortcomings of the seasonal adjustment methods presently available. But even if (for example) by divine revelation the "true" filters to apply and all future observations were known, there would still be problems left, due to conceptual ambiguities. ambiguities that simply arise from the fact that we do not know exactly what we want to measure (1).

Despite all its limitations, there is little doubt that seasonal adjustment along the lines we have analysed will continue to be used in the setting of monetary policy, at least for the time being. Then, the undesirable effects of seasonal adjustment errors can be smoothed out with simple modifications in operating procedures.

The first modification that comes to mind is extracting the irregular component from the seasonally adjusted figures. Thus, the series of monetary aggregates would simply consist of the trend-cycle component. Since the official adjustment does not provide final estimates of the irregular component, we cannot compute how the δ-series of errors would change if such a replacement were to be enforced. In the next subsection we briefly discuss the effects of other types of modifications in present operating procedures.

5.2. Changes in Operating Procedures

The analysis of sections 3 and 4 was based on a simplified version of present operating procedures. Under these procedures, the key variable is the monthly rate of growth of M_1

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(1) Another example of this type of ambiguity is provided in Maravall (1978), where the effect of using seasonally adjusted rates versus rates of seasonally adjusted levels of M_1 are discussed and found to be non-trivial.

seasonally adjusted over a two-month period. The measurement of this variable, we concluded, cannot be very precise, and many of the problems associated with its measurement arise from the need to use seasonally adjusted values. This raises the question of whether some other time series related to the growth of M_1 could be measured with more precision. Two possibilities suggest themselves as likely improvements. One is to modify the measurement of the rate of growth in such a way as to smooth out monthly variability. The other is to reduce the frequency at which policy directives are set, so that fewer seasonal effects would be included in the series.

A rigorous analysis of the two possibilities is beyond the scope of this paper, since many more elements would be involved. We simply illustrate some improvements that could be expected

Let v denote the variable used to measure the rate of growth (seasonally adjusted) of M_1. Let v^0 represents the one computed by adjusting monthly values and v^a the one based on daily seasonally adjusted values. As a rough measure of the imprecision associated with v, we use the first two sample moments of the difference:

$$d = v^0 - v^a$$

When v represents, for example, the monthly annualized rate of growth of consecutive nonoverlapping three-month moving averages, i.e.,

$$v_t = \left(\frac{m_{t+1} + m_t + m_{t-1}}{m_{t-2} + m_{t-3} + m_{t-4}} - 1\right) \times 4 \quad ,$$

we obtain:

	Var.	Mean
δ_t	1.35	.054
d_t	.45	.025

so that var $(d_t)=1/3$ var(δ_t). Thus the additional smoothing introduced by the three-month moving average does indeed reduce the imprecision of the measurement.

To see the effect of varying the frecuency of meetings, we consider meetings every 2, 3, 4 and 6 months, where the rate of growth is measured as the annualized change over periods of 2, 3, 4 and 6 months, respectively. Let d^k represent the corresponding difference in the two seasonally adjusted rates of growth when the meeting is held every k months. The decrease in the importance of the difference can be seen in the following table:

(in percent units)

	Var.	Mean
δ	1.35	.054
d^2	.81	.051
d^3	.55	.048
d^4	.38	.033
d^6	.10	.034

5.3. Improving seasonal factor revisions

As we already mentioned, of the two alternative seasonal adjustment methods considered in this paper, neither can be seen as unquestionably superior; more reasonably, the two methods could complement each other. Therefore, one can think of combining them to improve upon the results that each one provides when taken individually.

It is well known that forecasts obtained with different models can be combined into a weighted average, where the weights are chosen so as to minimize the MSE of the forecast. Similarly, one can think of combining seasonally adjusted estimates of a variable obtained through alternative methods. However, an important difference is the fact that the true seasonal factor, s_t, is unobservable, no matter how large the time series becomes, and the error of the estimate s_t is therefore never known. As an alternative criterion for choosing the weights, some function of the revision errors can be employed. In particular, we could compute the weights of the linear combination of seasonal factors so as to minimize the variance of the revision error, for some chosen period of time.

In our application, let

$$r_t^u = r_t^0 + s_t^0$$

$$r_t^u = r_t^a + s_t^a ,$$

where s^0 and s^a denote the seasonal estimates provided by the two methods, and r^u the seasonally unadjusted rate of growth of M_1. Denote by r_t^β the seasonally adjusted rate given by

$$r_t^\beta = \beta r_t^0 + (1-\beta) r_t^a .$$

Since $r_t = r_t^{\beta} + s_t^{\beta}$, it follows that,

$$s_t^{\beta} = \beta\, s_t^0 + (1-\beta)\, s_t^a$$

which implies that the revision errors (ω) of the three seasonal components are related by the equation:

$$\omega_t^{\beta} = \beta \omega_t^0 + (1-\beta)\, \omega_t^a \,.$$

Let β_0 be the value of β that minimizes the variance of ω_t^{β}. It is easily seen that:

$$\beta_0 = \frac{\sigma_{\omega a}^2 - \rho\, \sigma_{\omega 0}\, \sigma_{\omega a}}{\sigma_{\omega a}^2 + \sigma_{\omega 0}^2 - 2\rho\, \sigma_{\omega 0}\, \sigma_{\omega a}}$$

where ρ is the (contemporaneous) crosscorrelation between ω_t^0 and ω_t^a (1).

As a consequence, if r_t^{β} is used instead of either r_t^0 or r_t^a, a decrease in the magnitude of the revision errors can be expected.

To get an idea of the possible magnitude of this expected improvement, we consider the revision errors of the Demand Deposit component of M_1. First, we compute the seasonal factors for the years 1974, 75, 76 and 77 when the time series available ends on December 31,1976, and obtain their revisions after a new year of data (1977) becomes available. The comparison between the revision errors corresponding to the published seasonal factors and the ones obtained with the daily method is given in Pierce and Fries (1978), and the daily method is seen to produce smaller

(1) As suggested by one referee, an alternative computationally efficient way of computing β_0 would be by writing

$$\omega_t^a = \beta_0 (\omega_t^a - \omega_t^0) + \omega_t^{\beta} \,,$$

then estimating β_0 by OLS.

revision errors. We have computed the revision errors corresponding to s_t^β, for $\beta = \beta_0$. The comparison of these errors with the ones obtained through the daily method, in terms of the mean square error, absolute average error and average error, is presented in the following table:

	MSE		AE		AAE	
	ω^β	ω^a	ω^β	ω^a	ω^β	ω^a
1974	.92	.91	-2.45	-2.58	.77	.76
1975	1.25	1.41	2.82	3.00	.92	.96
1976	2.11	2.30	-.58	.88	1.29	1.27
1977	3.66	3.89	-4.81	-4.97	1.73	1.72

(MSE = x 10^6; AE = x 10^{-4}; AAE = x 10^{-3})

It is seen that there is an overall decrease in the magnitude of the revision errors when the linear combination of the two methods is used. However, since $\beta_0 = .095$, the weight given to the method based on the daily procedure is very large, and the improvement is therefore relatively small. In fact, only values inside the interval $0 < \beta < .2$ yield a MSE smaller than that of ω_t^a.

Although a method with smaller revision errors is not necessarily better, it is nevertheless an important advantage, particularly when the series are used in policy making. One is tempted to conclude that, when this empirical superiority of the daily method is put together with the theoretical result provided by Geweke, more attention should be placed on the daily seasonal adjustment of the monetary aggregate series.

6.- SUMMARY

Monthly seasonally adjusted rates of growth of the monetary aggregates have become the key elements in the design of US monetary policy. Since the monthly series are aggregates of daily values, two possibilities are open, depending on the order in which the two operations - aggregation and seasonal adjustment - are performed. Although the currently published figures are based on a direct monthly adjustment method, an alternative method based on daily adjustment has been developed at the Board for possible consideration. This paper compares the difference between the results obtained with the two procedures from the point of view of monetary policy.

The first part of the paper estimates the difference between the two methods for the period 1971-77. In order to assess the relevance of this difference in terms of monetary policy, we provide two measures. First, section 3 deals with the following question:

Our simplified policy decision making process is based on a range of tolerance for the rate of growth of M_1, so that growth outside this range triggers intervention. Thus the judgment of whether growth of M_1 is tolerable or not becomes the basic element that induces changes in the instrument.

Within this framework, we consider two monetary authorities, identical in all respects except in the fact that, when computing the rates of growth, one uses the monthly adjustment method, while the other uses the method based on the daily data. What is then the probability that, starting with the same initial conditions, a given growth of M_1 over one period would be judged tolerable by one authority and nontolerable by the other?.

Under some simplifying assumptions it is found that, under present operating procedures, disagreement could be expected to happen one out of eight times, approximately.

Several additional results are obtained. First, the estimated probability of disagreement is quite robust with respect to reasonable assumptions for the width of the range of tolerance and for the accuracy of short-run forecasts. Second, somewhat surprisingly, it is found that short-run improvements in this accuracy may lead to increases in the probability of disagreement. Third, under the assumption that the "appropiate" method is the one where adjustment precedes aggregation, probabilities of type I and type II errors for present operating procedures are computed, reflecting the probability of intervention when none is warranted and the probability of nonintervention when some is deserved. Probabilities of the first type are seen to be somewhat larger.

Section 4 tries to assess the relevance of the difference between the two methods by looking at its effect in terms of interest rates. It is seen that a change in the federal funds rate of approximately one-eighth of a percent point could be accounted for simply by the difference between the two methods.

Summarizing, insofar as the present state of the art does not tell us which method is to be preferred, there seems to be a non-negligible lower bound to the accuracy with which monetary policy can be implemented, stemming partly from an imprecision in the definition of the variable of interest.

Naturally, the analysis is conditional on present operating procedures. The last part of the paper discusses how (minor) changes in the procedures may attenuate undesirable effects of present seasonal adjustment methods. First, series that smooth the rate of growth over longer periods would certainly be more stable. Second, combinations of the alternative methods of computing the series could decrease the magnitude of the revision errors implied by seasonal adjustment. Incidentally, analysis of the revision errors indicates a marked superiority (in that respect) of the method based on daily adjustment.

APPENDIX A

1. THE DISTRIBUTION OF δ

Prewhitening of the δ-series is achieved through the filter:

(A.1) $$a_t = \psi(B)^{-1}\, \delta_t \quad ,$$

where

(A.2) $$\psi(B) = \frac{1-\phi B}{1-\theta B}\,(1-\gamma_1 B^6-\gamma_2 B^{12}).$$

Maximum likelihood estimates of the parameters are given by:

$$\phi = -.27 \; ; \quad \theta = -.88 \; ; \quad \gamma_1 = -.27 \; ; \quad \gamma_2 = .47$$

The ACF of the residuals a_t is displayed in table 3, and no lagged autocorrelation is significantly different from zero. The variance of a_t is 42 % of the variance of δ and its mean can be assumed zero (t=.106) Let $\tilde{a}_t = a_t/\sigma_a$. We wish to test the hypothesis that the series $\tilde{a}_t$ is normally distributed.

Let μ_3 and μ_4 denote the coefficients of skewness and kurtosis, respectively. In samples from a normal process, the distribution of μ_3 is symmetric about zero, with variance approximately given by $var(\mu_3) \simeq 6/T$, where T is the sample size. Similarly, as T becomes larger, the distribution of μ_4 approaches normality with a mean of 3 and a variance given by $var(\mu_4) \simeq 24/T$ (1).

Since our sample yields the values:

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(1) See Durand (1971, pp. 346-349). For samples of more than 50 observations the approximation to the distribution of μ_3 works relatively well; the sample coefficient μ_4 approaches normality more slowly.

$$\hat{\mu}_3 = .123 \quad ; \quad \sigma(\mu_3) = .27$$

$$\hat{\mu}_4 = 2.83 \quad ; \quad \sigma(\mu_4) = .54 ,$$

both coefficients, μ_3 and μ_4, can be assumed to be generated by a sample from a normal distribution.

This conclusion is further reinforced by performing Kolmogorov-Smirnov ahd Chi-Square tests. In the first case, the sample quatity:

$$D_T = \max |F_T(x) - F(x)|$$

where $F(x)$ is $N(0,1)$ and $F_T(x)$ the empirical c.d.f. of the sample, yields the value:

$$D_T = .048$$

Since at test levels of $\alpha=.10$, .05, .01 and for T=84, the hypothesis is rejected when D>.09, .099, .113, respectively, we can accept that $F_T(x)$ is $N(0,1)$; the empirical distribution $F_T(x)$ comfortably lies everywhere within the acceptance bands (1).

When the Chi-Square goodness of fit test is performed, with the observations grouped in 10 classes, the quantity

$$c^2 = T \sum_i (\hat{P}_i - P_i)^2 / P_i ,$$

where $\hat{P}_i$ and P_i are the empirical and theoretical frequencies in the ith class, yields the value $c^2=3.22$. Since $\chi^2_{.05}(9)=16.9$, again the normality hypothesis for a_t is not rejected.

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(1) We use critical values of D corrected for the case in which the mean and variance of the distribution have been estimated from the sample (see Lilliefors (1967)).

2. THE EFFECT OF REVISION ERRORS

Since the series we analyse are the ones available as of July 1978 and the sample spans the period 1971-77, the seasonal factors for the last years of this period will be affected by future revisions. This raises the possibility that the distribution of δ could be different for recent data, still subject to revision errors.

To test for this possibility we divide the sample period into two subperiods, one covering the first five years, the other the last two years. The statistic

$$f = \frac{n-1}{m-1} \sum_{t=1}^{58} (a_{1t}-\bar{a}_1)^2 / \sum_{t=59}^{82} (a_{2t}-\bar{a}_2)^2 = 1.32 ,$$

where m=58, n=24, and $\bar{a}_1$ and $\bar{a}_2$ denote the sample means of the two superiods, is smaller than $F_{59,23}(.975)=2.20$. Hence we do not reject the assumption of equal variances. Since the statistic

$$t = \frac{\left[\bar{a}_1-\bar{a}_2-(\mu_1-\mu_2)\right] \sqrt{m+n-2}}{\sqrt{1/m + 1/n}\left(\sum_{t=1}^{58} (a_{1t}-\bar{a}_1)^2 + \sum_{t=59}^{82} (a_{2t}-\bar{a}_2)^2\right)^{1/2}} ,$$

under the assumption $\mu_1=\mu_2$, takes the value t=1.15, we do not reject the assumption that the means are equal (at the 5 % level).

3. THE VARIANCE OF $\delta_t/\delta_{t-2}=0$

In the computation of the probability of disagreement of section 3 we need an estimate of $\text{var}(\delta_t/\delta_{t-2}=0)$.

From (A.1) and (A.2),

$$(1-\phi B)\delta_t = (1-\theta B)\, v_t ,$$

where $\gamma(B)\ v_t = a_t$, and $\gamma(B) = 1-\gamma_1 B^6-\gamma_2 B^{12}$.

Thus:

$$\delta_t = \phi^2\delta_{t-2} + v_t + (\phi-\theta)\ v_{t-1} - \phi\theta v_{t-2}\ .$$

Since v_t, v_{t-1} and v_{t-2} are uncorrelated,

$$\text{var}(\delta_t/\delta_{t-2}=0) = \sigma_v^2\ [\ 1+(\phi-\theta)^2 + \phi^2\theta^2\ ]\ ,$$

and

$$\sigma_v^2 = \frac{1-\gamma_2}{1+\gamma_2}\ \frac{1}{[(1-\gamma_2)^2-\gamma_1^2]}\ \sigma_a^2\ .$$

4. THE DISTRIBUTION OF THE e-SERIES

In section 3 we argued that, to a first approximation, the deviation of r_t^0 from the midpoint of the tolerance interval could be interpreted as a staff forecast error corresponding to the forecast done at the time of the FOMC meeting. The series of such forecast errors has been analysed in Porter et. al. (1978 a, pp. 5-19) for the period 1972-1977, where it is shown that the error distribution can be assumed to be Normal.

Since the forecasts of r_t^0 are month-to-month forecasts of a rate of growth over a two-month period, e_t cannot be considered a one-step ahead forecast error. Instead, two consecutive forecast errors will span two two-month periods with a month of overlap. Hence we should expect e_t to follow a first order moving average process. This is in complete agreement with the sample autocorrelation of e_t, displayed in table 3. Estimation of the MA(1) model:

AUTOCORRELATION FUNCTIONS

a) Variable a_t (innovations of δ_t)

Lags	1	2	3	4	5	6	7	8	9	10	11	12	Error
1-12	-.07	-.03	-.14	-.13	.06	-.05	.00	.05	-.11	-.04	.09	.05	.11
13-24	-.06	-.12	-.11	.09	.07	.10	.02	-.10	-.10	-.01	.14	.04	.15

b) Variable e_t

Lags	1	2	3	4	5	6	7	8	9	10	11	12	Error
1-12	.37	-.18	-.18	-.05	.08	-.02	-.17	-.06	.01	.02	.10	.02	.12
13-24	-.10	.10	.04	.21	.17	-.10	-.09	-.08	-.16	.00	.03	.07	.15

c) Variable b_t (innovations of e_t)

Lags	1	2	3	4	5	6	7	8	9	10	11	12	Error
1-12	-.05	-.11	.09	-.05	.08	.01	-.16	-.13	.08	-.13	.15	-.02	.12
13-24	-.06	-.08	.02	.14	.05	-.10	-.12	-.05	.08	.09	-.03	.04	.18

CROSSCORRELATION FUNCTION BETWEEN a_t AND b_t

Lag-zero crosscorrelation = .21

Correlation of b_t with future values of a_t:

Lags	1	2	3	4	5	6	7	8	9	10
1-10	.13	-.09	.19	-.11	-.10	.05	-.05	-.09	-.06	.01
11-20	.14	.16	-.21	.09	.09	-.07	.01	-.08	.20	.07

Correlation of b_t with past values of a_t:

Lags	1	2	3	4	5	6	7	8	9	10
1-10	-.02	.02	-.12	-.12	-.00	.04	-.02	-.08	-.05	.15
11-20	.15	.03	-.01	.07	-.02	-.08	.00	-.13	.00	-.15

$$e_t = (1-\lambda B)\ b_t$$

yields $\lambda = -.60$, $\sigma_b = 2.53$. Table 3 also exhibits the ACF of the residuals b_t, where again no lagged autocorrelation is significantly different from zero.

5. THE JOINT DISTRIBUTION OF δ_t AND e_t

Although the marginal distributions of δ_t and e_t are Normal, this does not imply that the joint distribution has to be Normal. Yet two uncorrelated random variables with marginal Normal distributions will be jointly normally distributed if and only if the variables are mutually independent (1). Thus if we find that the difference between the two seasonally adjusted estimates and the forecast errors are independent, we can conclude that their joint distribution is Normal. The tests for independence will be carried out on the innovations of the δ and e processes, that is, on the a- and b-series.

On the bivariate sample $[a_t, b_t]$ we perform first a contingency table test for independence. The statistic Tl^2, where l^2 is given by:

$$l^2 = \sum_{i=1}^{r} \sum_{j=1}^{s} \frac{T_{ij}^{\ 2}}{T_i\ T_j} - 1,$$

when a and b are independent, converges in probability to χ^2 with (r-1)(s-1) d. of f. Setting r=3, s=3, we obtain $Tl^2 = 1.92$. Since $\chi^2_{.05}(4) = 9.5$, we do not reject the hypothesis of independence.

Second, we perform a Spearman's rank correlation test for independence. Let a_k be the A_kth largest value of a in the sample, and b_k be the B_kth largest value of b (k=1,...,T). If a and b are statistically independent, then the test statistics

(1) See Anderson (1958, p. 38).

$$R = 1 - \frac{6}{T(T^2-1)} \sum_{k=1}^{T} (A_k - B_k)^2$$

is asymptotically Normal with zero mean and variance equal to 1/T-1. Since, for a two-tailed test,

$$\frac{\mu_{1-.05/2}}{\sqrt{T-1}} = .24$$

and, for our sample, R=.21, we conclude that the hypothesis of independence cannot be rejected (at the .05 level of significance) Finally, table 3 presents the crosscorrelation function between the innovations a_t and b_t (1).

Summarizing, the joint distribution of (δ_t, e_t) can be assumed to be bivariate Normal, with zero mean vector and covariance matrix given by:

$$\Omega = \begin{pmatrix} \sigma_\delta^2 & 0 \\ 0 & \sigma_e^2 \end{pmatrix} ,$$

where σ_δ^2=1.34 and σ_e^2=3.25.

(1) After all , on a priori grounds it seems plausible that the forecast error of the rate of growth r_t^0 independent of the difference between two alternative seasonal adjustment methods.

APPENDIX B

DAILY SEASONAL ADJUSTMENT

The Advisory Committee on Monetary Statistics recommended a statistical method of seasonal adjustment based on daily data (1). The method, first suggested by professor Milton Friedman, was developed in Pierce, Vanpeski and Fry 1978 , and further extended and modified in Pierce and Fries 1978. With some minor changes, this is the method we have followed. In this appendix we simply sketch a brief outline; a more general description can be found in the last reference mentioned.

Let Y_t denote the series under consideration, for which we assume the multiplicative model:

$$Y_t = P_t \, S_t \, U_t \qquad \text{(B.1)}$$

where P, S and U denote the trend, seasonal and irregular factors. If small letters denote logs, equation (B.1) can be rewritten:

$$y_t = p_t + s_t + u_t \; .$$

Our aim is to estimate

$$y_t^{SA} = y_t - s_t = p_t + u_t$$

from a time series on y_t.

First, the trend p_t is estimated with the symmetric moving average filter:

$$p_t = \mu(B) \, y_t$$

(1) See Advisory Committee on Monetary Statistics [1976, pp. 39-40].

where $\mu(B)=(1/365) \sum_{j=-182}^{182} B^j$ and b is the familiar lag operator. Defining $\nu(B)=1-\mu(B)$, the detrended series are given by:

$$z_t = \nu(B)\ y_t$$

The filter $\nu(B)$ eliminates a linear or quadratic time-varying trend; a more general discussion can be found in Fuller (1976, Chap. 9) and Durbin (1963). Since, by definition

$$\nu(B)\ s_t = s_t$$

the filter preserves the seasonal component. However, some low-order autocorrelation is induced in the irregular, though the effect can be neglected (see Durbin (1963)).

Once the series have been detrended, dummy variables are used to remove some deterministic seasonal effects. Seventeen dummy variables were eventually selected, corresponding to day-of-week effects, first of month effect, tax date effect, and holiday effects. (In general, the day-of-week effect of the day before the holiday carries over the holiday).

If d_{it} denotes a dummy variable, then

$$s^d_{it} = \sum_{i=1}^{17} \beta_i\ d_{it}$$

represents a first set of deterministic seasonal effects. Let α_t denote the residual:

$$\alpha_t = z_t - s^d_{it}$$

Other deterministic seasonal effects are estimated by fitting a Fourier series to these residuals:

$$\alpha_t = \sum_{j=1}^{182} (a_j \cos 2\pi f_j t + b_j \sin 2\pi f_j t) + \varepsilon_t$$

where f_j, $j=1,\ldots,182$, are the harmonics of the fundamental annual frequency. To allow for a more parsimonious representation, the frequencies are ranked according to their contribution to the sum of squares of α. The ones that jointly represent 99% of this contribution are then selected and the regression is rerun. If J denotes the set of j's corresponding to the selected f_j's, the component

$$s^d_{2t} = \sum_{j \in J} (a_j \cos 2\pi f_j t + b_j \sin 2\pi f_j t)$$

represents a second set of deterministic seasonal effects. The total deterministic seasonal component is therefore:

$$s^d_t = s^d_{1t} + s^d_{2t}$$

Inspection of the autocorrelation function of the residual:

$$e_t = y_t - p_t - s^d_t$$

indicates the presence of seasonality that has not been removed from the series (1). This seasonality will be the subject of a stochastic extraction, but before doing so we need to recover the first and last six months, lost in the detrending of the series. To recover the first six months is trivial,

(1) Since the autocorrelation function of e_t indicates that the process is stationary, OLS estimation of dummy variables and sine-cosine regression is asymptotically efficient (see Fuller (1976 , p. 393)).

since previous observations on y_t are available. Recovery of the last six months is achieved through the following steps:

Let T denote the last observation of the series y_t. Through the detrending, the series p_t and s_t^d will end at $t = T - 182$. Since s_t^d is deterministic, we can extend s_t^d one year (i.e., up to T + 182). Then we compute:

$$h_t = y_t - s_t^d \quad , \text{ for } t = T-182,\ldots,T$$

and fit the function:

$$h_t = \gamma_o + \gamma_1 t + n_t \, , \quad (t = T-182,\ldots,T)$$

with an ARIMA model for n_t. Once this is performed, we forecast h_t for $t = T + 1, \ldots, T + 182$, and compute:

$$y_t = h_t + s_t^d \quad \text{for} \quad t = T+1, \ldots , T+182.$$

Hence the series y_t have been extended up to T+182. On this extended series we detrend as before and obtain p_t up to T. Similarly, s_t^d and e_t will end now at period T. (Notice that with this detrending procedure, only one forecast of y_t is used to compute p_t, s_t^d and e_t fot t=T+1. Similarly, two forecasts of y_t are used to estimate the components for t=T+2, and so on). On the series e_t some modifications for outliers are performed.

Let x_t be the series e_t extended to time T, with the outliers corrected. The autocorrelation of x_t indicates the presence of seasonality which has not been removed with the deterministic approach (1).

The autocorrelation function of x_t also exhibits some low order autocorrelation. Using a low order nonseasonal filter, $\delta(B)$, the series x_t is transformed into a series v_t:

(B.2) $$v_t = \delta(B) \; x_t$$

such that autocorrelation is only present at seasonal lags. Let

(B.3) $$v_t = s_t^* + \eta_t$$

where s_t^* is purely seasonal and η_t is white noise. Assume v_t has the moving average representation:

$$v_t = \Psi(B) \; c_t$$

where $\Psi(B)$ is the ratio of two finite polinomials. Hence the spectrum of v_t is given by:

$$\emptyset_v(\omega) = \frac{1}{2\pi} \; \Psi(B) \; \Psi(B^{-1}) \, \sigma_c^2 \, , \quad B = e^{-i\omega}.$$

In order to be able to identify the two unobservable components in (B.3) we assume that the variance of η_t, the noise, has to be maximized (2). Also, from (B.3),

$$\emptyset_v(\omega) = \emptyset_{s*}(\omega) + \emptyset_\eta(\omega)$$

and it can be shown that maximizing the variance of η is equi-

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(1) See Pierce [1978]

(2) This is a standard assumption. See for example Box, Hillmer, Tiao (1978) and Pierce (1978).

valent to setting (1)

$$\phi_{\eta}(\omega) = \min_{\omega} \phi_V(\omega).$$

Therefore, the spectrum of s* can be estimated as:

$$\phi_{s*}(\omega) = \phi_V(\omega) - \min_{\omega} \phi_V(\omega)$$

and the τ-lag autocovariance of s_t^* can be computed as the inverse Fourier transform:

$$\lambda_{s*}(\tau) = \int_{-\pi}^{\pi} e^{i\tau\omega} \phi_{s*}(\omega)\, d\omega .$$

From this, we can form the filter:

$$\lambda_{s*}(B) = \sum_{\tau} \lambda_{s*}(\tau)\, B^{\tau}$$

and, finally, s_t^* can be computed through:

$$s_t^* = [\lambda_V(B)]^{-1} \lambda_{s*}(B)\ v_t \quad ,$$

where $\lambda_V(B)=\psi(B)\psi(B^{-1})\sigma_c^2$.

The stochastic seasonal component, s_t^s, can be recovered through (B.2),

$$s_t^s = \delta(B)^{-1}\ s_t^* .$$

This finishes the decomposition of the series y_t. The total seasonal component is given by:

$$s_t = s_t^d + s_t^s$$

and the seasonally adjusted series is given by:

$$y_t^{SA} = y_t - s_t$$

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(1) See Wecker (1978).

or, in terms of the levels:

$$S_t = e^{s_t^d} \; e^{s_t^s}$$

$$Y_t^{SA} = e^{y_t^{SA}}$$

The seasonally adjusted M_1 monthly series implied by the daily procedure used in the paper were computed by applying the previous method to the daily series of Demand Deposits and Currency for the period January 1/71 to December 31/78. The two adjusted components were then added and the monthly average of daily values computed.

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