CLIMATE-CONSCIOUS MONETARY POLICY

2023

BANCO DE ESPAÑA
Eurosistema

Documentos de Trabajo
N.º 2334

Anton Nakov and Carlos Thomas

CLIMATE-CONSCIOUS MONETARY POLICY (*)

Anton Nakov

ECB AND CEPR

Carlos Thomas

BANCO DE ESPAÑA

(*) The views expressed here are the responsibility of the authors only, and do not necessarily coincide with those of the ECB or the Banco de España. We are very grateful to Martin Bodenstein, Diego Comin, Natasha Hinterlang, Òscar Jordà, Michel Juillard, Conny Olovsson, Melina Papoutsi, Thomas Stoerk, Mathias Trabandt, and seminar participants at the ECB and Banco de España for their very helpful comments and suggestions; and to Francisco Alonso and Marta Suárez-Varela for their excellent research assistance. Any remaining errors are ours.

Documentos de Trabajo. N.º 2334

November 2023

https://doi.org/10.53479/34755

The Working Paper Series seeks to disseminate original research in economics and finance. All papers have been anonymously refereed. By publishing these papers, the Banco de España aims to contribute to economic analysis and, in particular, to knowledge of the Spanish economy and its international environment.

The opinions and analyses in the Working Paper Series are the responsibility of the authors and, therefore, do not necessarily coincide with those of the Banco de España or the Eurosystem.

The Banco de España disseminates its main reports and most of its publications via the Internet at the following website: http://www.bde.es.

Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

© BANCO DE ESPAÑA, Madrid, 2023

ISSN: 1579-8666 (on line)

Abstract

We study the implications of climate change and the associated mitigation measures for optimal monetary policy in a canonical New Keynesian model with climate externalities. Provided they are set at their socially optimal level, carbon taxes pose no trade-offs for monetary policy: it is both feasible and optimal to fully stabilize inflation and the welfare-relevant output gap. More realistically, if carbon taxes are initially suboptimal, trade-offs arise between core and climate goals. These trade-offs however are resolved overwhelmingly in favor of price stability, even in scenarios of decades-long transitions to optimal carbon taxation. This reflects the untargeted, inefficient nature of (conventional) monetary policy as a climate instrument. In a model extension with financial frictions and central bank purchases of corporate bonds, we show that green tilting of purchases is optimal and accelerates the green transition. However, its effect on CO2 emissions and global temperatures is limited by the small size of eligible bonds' spreads.

Keywords: Ramsey optimal monetary policy, climate change externalities, Pigouvian carbon taxes, green QE.

JEL classification: E31, E32, Q54, Q58.

Resumen

Estudiamos las implicaciones del cambio climático y las medidas de mitigación asociadas para la política monetaria óptima, utilizando un modelo neo-Keynesiano canónico con externalidades climáticas. Si están en su nivel socialmente óptimo, los impuestos al carbono no plantean ningún conflicto para la política monetaria: es posible y óptimo estabilizar completamente la inflación y la brecha de producción relevante para el bienestar. Si, de modo más realista, los impuestos al carbono son inicialmente subóptimos, surgen conflictos entre los objetivos primarios y los climáticos. Sin embargo, estos conflictos se resuelven de forma abrumadora a favor de la estabilidad de precios, incluso en escenarios en que la transición a una imposición óptima al carbono tarda décadas. Esto refleja la naturaleza no focalizada e ineficiente de la política monetaria (convencional) como instrumento climático. En una extensión del modelo con fricciones financieras y compras de bonos corporativos por el banco central, mostramos que un sesgo de dichas compras hacia bonos verdes es óptimo y acelera la transición ecológica. Sin embargo, su efecto sobre las emisiones de CO_2 y las temperaturas globales se ve limitado por el tamaño reducido de los diferenciales de los bonos elegibles.

Palabras clave: política monetaria Ramsey-óptima, externalidades climáticas, impuestos al carbono pigouvianos, expansión cuantitativa verde.

Códigos JEL: E31, E32, Q54, Q58

Non-Technical Summary

The scientific community has reached a consensus on the need for decarbonization to address climate change. While carbon taxation and emissions trading are seen as key policy tools, there is less agreement on the role of other policy areas, including monetary policy. Policymakers have differing views on whether central banks should consider climate change in their monetary policy frameworks. This paper explores the questions surrounding the integration of climate goals into monetary policy.

The paper uses a New Keynesian model with climate externalities to analyze the impact of climate-conscious monetary policy. The model incorporates the use of green and fossil energy in production, with fossil energy contributing to carbon emissions and global warming. The government can impose a carbon tax to address these externalities. The paper focuses on the Ramsey optimal monetary policy, where the central bank considers climate externalities as part of its decision-making process.

The paper establishes a benchmark result: if carbon taxes follow the socially optimal path, monetary policy faces no trade-offs and can fully stabilize inflation and the welfare-relevant output gap. Strict inflation targeting allows the central bank to replicate the socially efficient equilibrium, including the optimal path of CO2 emissions. However, this assumes that carbon taxes are set optimally from the start, which is unrealistic given the slow progress in carbon taxation observed in practice. Therefore, the paper examines a scenario of a "slow green transition" where the carbon tax gradually converges to its optimal level.

In the slow green transition scenario, a tension arises between price stability and climate goals. Suboptimal carbon taxes during the transition lead to excessive fossil energy consumption. The central bank may have an incentive to depress output to reduce energy consumption, but this comes at the cost of lowering output below its natural level and accepting temporarily lower inflation. The trade-off between price stability and climate goals is quantified using a calibrated model. The results show that the trade-off is resolved in favor of price stability, with only a minimal and short-lived departure from strict inflation targeting.

The paper also considers the use of "green QE" as a targeted instrument for addressing climate change. Green QE refers to central banks tilting their corporate bond portfolios toward green bonds and away from brown bonds. The analysis shows that under optimal policy, full QE is implemented alongside optimal carbon taxation. In the slow green transition scenario, the central bank initially refrains from purchasing any brown bonds due to the large gap between actual and optimal carbon taxes. Once the carbon tax gap narrows, the central bank buys brown bonds to implement the optimal fossil energy price. Green tilting allows the central bank to compensate for the shortfall in carbon taxation.

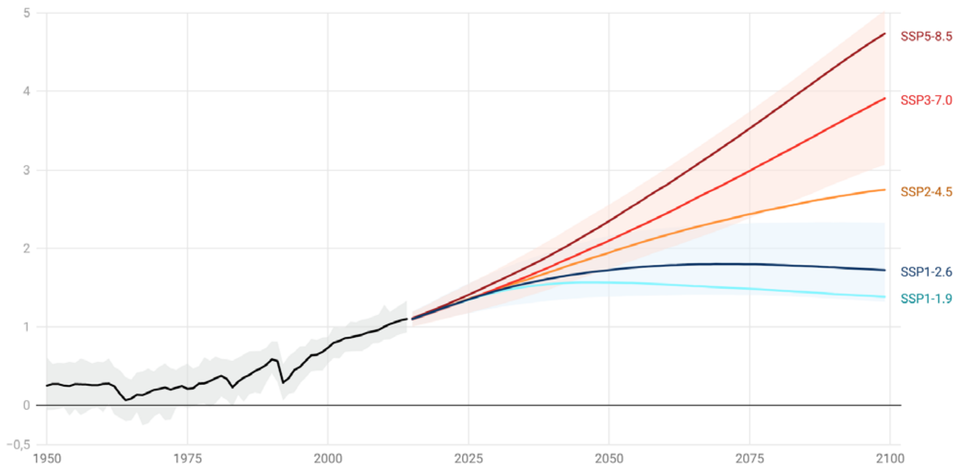
The trade-offs under the slow green transition are still resolved in favor of price stability even with green QE. Green tilting accelerates the transition, but its impact on atmospheric carbon concentration and global temperatures remains small. This is because the effectiveness of green tilting depends on the extent to which it can tighten financing conditions for brown firms and ease them for green firms, which is limited by the eligibility criteria of central bank purchase programs.

In conclusion, while monetary policy can play a role in addressing climate change, its ability to do so is limited due to the untargeted nature of conventional instruments and design restrictions on unconventional tools. The paper emphasizes the importance of optimal carbon taxation and the challenges of integrating climate goals into monetary policy.

1 Introduction

The World scientific community has come to a consensus on the need for decarbonization of the global economy in order to combat climate change, in view of the rise of global temperatures in recent decades and projections of what could happen if decisive action is not taken (see Figure 1).

Figure 1: Global temperatures since 1950 and projections. Source: IPCC, 2021



While genuinely targeted policies, such as carbon taxation and emissions trading schemes, are widely seen as the key policy levers to mitigate climate change, there is much less agreement on what role other policy areas should play. This is especially the case for monetary policy. Different policy-makers have expressed rather opposing views on whether central banks should adopt climate change considerations in their monetary policy frameworks, given their current legal mandates (see e.g. Lagarde, 2021, and Powell, 2023).

Even if one takes the view that monetary policy should adopt climate goals, this still raises a number of crucial normative questions. How should monetary policy respond to climate change, given its obligation to pursue its core statutory goals, notably price stability? Is there a trade-off between core goals and climate goals? If so, how do these trade-offs depend on what (genuine) climate authorities are doing? And how should those trade-offs be resolved, given the monetary policy instruments at central banks' disposal?[1]

In this paper, we address the above questions using a canonical New Keynesian model extended with climate externalities. Our modelling of the latter follows Golosov et al (2014) closely: production requires the use of green and fossil energy, whereby the latter produce CO2 emissions that add to carbon concentration in the atmosphere and hence to global warming, which causes damages to the economy's productive capacity. As in Golosov et al (2014), the government

[1] As argued by Hansen (2022), monetary policy tools are much less potent than fiscal ones when it comes to confronting climate change.

can impose a tax on fossil energy production, henceforth "carbon tax". In this context, we analyze the Ramsey optimal monetary policy by a benevolent (i.e. social welfare-maximizing) central bank. Therefore, the central bank internalizes the climate externalities from fossil energy consumption. It is in this sense that we refer to monetary policy as being "climate-conscious".

We first establish analytically a benchmark result. Provided carbon taxes follow their socially optimal path at all times,[2] monetary policy does not face any trade-offs: it is both feasible and optimal to fully stabilize inflation and the *welfare-relevant* output gap (i.e. the gap between the actual and the socially efficient level of output). Thus, strict inflation targeting allows the central bank to replicate the social planner equilibrium, including the socially optimal path of CO2 emissions. The intuition for this results is simple: if carbon taxes are set at their optimal (Pigouvian) level, then all agents internalize perfectly the climate externalities from fossil energy use. This leaves nominal rigidities as the only distortion left, which the central bank can offset through a policy of strict price stability.[3]

While useful as a normative benchmark, the assumption that carbon taxes are set optimally since the very first period is unrealistic, given the rather slow pace of progress in carbon taxation and similar policies observed in practice, even in advanced economies. For this reason, we focus the remainder of our analysis on the case of a "slow green transition", which we define as a scenario in which, starting from zero, the carbon tax converges slowly towards its socially optimal path. In this case, a tension arises between price stability and climate goals. Because carbon taxes are suboptimal during the transition, the economy consumes too much fossil energy. Aware of this, the central bank has an incentive for depressing output –bringing it closer to its socially efficient path, i.e. narrowing the welfare-relevant output gap– in order to reduce overall energy consumption, including consumption of fossil energy. However, this comes at the expense of lowering output below its *natural* (i.e. flexible-price) level and thus accepting a transitory fall in inflation below target.

In order to quantify this trade-off, we use a calibrated version of our model economy. We find that the trade-off is resolved overwhelmingly in favor of price stability. In particular, under a very slow green transition in which optimal carbon taxation is reached after 30 years, the optimal departure from strict inflation targeting is very small, of barely 10 basis points in the first quarter, and short-lived. This is mirrored by a small, short-lived fall in output below its natural level, which barely helps reduce output towards its (much lower) socially efficient level. As a result, optimal monetary policy barely affects the path of fossil energy consumption and CO2 emissions, compared to a scenario in which monetary policy ignores climate considerations and sticks strictly to its price stability mandate.

The intuition for this result is the following. Due to its untargeted nature, conventional (interest-rate) monetary policy is a rather blunt, inefficient tool for climate-related purposes: reducing CO2 emissions requires the central bank to reduce overall (i.e. fossil, but also green)

[2]The optimal carbon tax has the same shape as Golosov et al's (2014) well-known formula: as a proportion of output, optimal carbon taxes depend only on households' subjective discount factor and on the parameters governing the accumulation of atmospheric carbon concentration and the economic damages from global warming.

[3]As in much of the literature on optimal monetary policy in New Keynesian models, we assume that monopolistic distortions are offset by means of an appropriately chosen revenue subsidy. As a result, strict inflation targeting replicates the socially efficient equilibrium, *provided* the carbon tax equals its optimal level. This assumption allows us to isolate the effect of climate change and carbon taxes on monetary policy trade-offs from the effect of monopolistic distortions.

energy consumption, which in turn requires depressing economic activity and lowering inflation below target, all of which is rather costly in social welfare terms. Faced with this unfavorable trade-off, the central bank optimally decides to deviate minimally from strict inflation targeting.

In practice, however, central banks have other, more targeted instruments for addressing climate change. The most prominent one is the so called "green QE", i.e. the possibility of tilting their portfolio of corporate bond holdings in favor of "green bonds" –understood as bonds satisfying certain climate-related eligibility criteria– and in detriment of "brown bonds".[4] To analyze optimal green tilting of QE, we extend our baseline model with a simple specification of financial frictions that allow central bank purchases of corporate bonds to affect the cost of bond financing for green and fossil energy producers and thus, through a standard cost channel, the relative price of both energy sources.

We first show that our benchmark normative result generalizes to the model with corporate QE: as long as carbon taxation is optimal, it is again feasible and optimal to fully stabilize inflation and the welfare-relevant output gap. The difference is that now optimal policy also entails "full QE", whereby the central bank absorbs as many bonds (both green and brown) as needed to offset the financial friction in both energy sectors. We then show that, in the slow green transition scenario, full *green* QE continues to be optimal, but *brown* QE has two distinct phases. Initially, the gap between actual and optimal carbon taxes is large enough that green tilting cannot raise brown bond spreads sufficiently to implement the socially optimal fossil energy price: the best the central bank can do is not to purchase any brown bonds at all. Subsequently, once the carbon tax gap becomes sufficiently small, the central bank buys as many brown bonds as needed for brown bond spreads to reach the level necessary to implement the optimal fossil energy price. In this second phase, green tilting allows the central bank to exactly compensate for the shortfall in carbon taxation as the latter catches up with its optimal level.

Finally, we quantify how the trade-offs under slow green transition change in the presence of QE. As in the baseline model, the trade-offs continue to be resolved clearly in favor of price stability. The main difference is that optimal green tilting of QE accelerates somewhat the green transition: fossil energy use reaches its socially optimal level a year and a half earlier, compared to a climate-oblivious scenario of strict inflation targeting and no QE. However, the impact on atmospheric carbon concentration and global temperatures is very small. This reflects the fact that the effectiveness of green tilting at reducing carbon emissions depends on how much it can tighten financing conditions for brown firms and ease them for green firms. Since central banks' purchase programs typically restrict the set of eligible bonds to those with high credit quality (i.e. with investment-grade rating), their average spreads are relatively small even in the absence of central bank purchases –a fact that we incorporate in our calibration–, thus limiting the scope of green tilting for altering the relative spreads of brown vs green bonds.

In sum, our analysis suggests that, while monetary policy can play a role in confronting climate change –and *should* play it, under a welfare-maximizing criterion for optimal policy–, the extent to which it can do so is rather limited, reflecting either the untargeted, inefficient nature of its conventional instruments, or the design restrictions on its unconventional tools.

[4] The ECB and the Bank of England are two prominent examples of major central banks that have explicitly incorporated green criteria in its corporate bond purchase programs; see ECB (2022) and Bank of England (2021).

1.1 Related Literature

Building on the seminal work by William Nordhaus on integrated climate-economy models,[5] the literature on climate change and the macroeconomy has grown considerably over the last decade. Standard environmental policies such as taxes, subsidies, and caps were studied in RBC models by Fischer and Springborn (2011), Heutel (2012) and Angelopoulos et al (2013). Optimal carbon taxation was analyzed in Golosov et al (2014), from whom we borrow our specification of climate externalities, and more recently in Barrage (2020). Annicchiarico and Di Dio (2015), Ferrari and Nispi Landi (2022), Airaudo, Pappa and Seoane (2023) and Olovsson and Vestin (2023) have all explored the macroeconomic effects of climate change mitigation policies in New Keynesian DSGE models, including the possibility of green policy-induced inflation, or "greenflation".[6]

A recent literature explores the role of monetary policy and other macroeconomic policies in New Keynesian DSGE models with climate change. Benmir and Roman (2020) assess different types of fiscal, monetary, and macroprudential policies aimed at reducing CO2 emissions. Ferrari and Pagliari (2021) explore the cross-country implications of climate-related mitigation policies in a two-country model with country-specific fiscal and monetary policies and the possibility of cooperation between them. Diluiso et al. (2020) use a model with financial frictions and climate policy to study the risks a low-carbon transition poses to financial stability and how central bank (monetary and financial) policies can be used to manage these risks. Ferrari and Nispi Landi (2021, 2023) study the effectiveness of temporary and permanent green QE, respectively, at mitigating CO2 emissions in models with environmental externalities.

In a real model with climate externalities and financial frictions, Papoutsi, Piazzesi and Schneider (2023) study how the sectoral composition of central bank asset purchases shapes their environmental impact. They also analyze the optimal asset purchase policy. They find that if an optimal carbon tax is in place, asset purchases should focus only on minimizing financial frictions and not take climate externalities explicitly into account;[7] whereas in the absence of an optimal carbon tax, green monetary policy can improve welfare.

Importantly, the above contributions do not study the Ramsey optimal monetary policy, both conventional and unconventional, in a New Keynesian environment. In this regard, our analysis clarifies the trade-offs that monetary policy faces between its core goals (such as price stability) and climate goals, and how such trade-offs depend on the path of other climate policies such as carbon taxes.

2 Baseline Model

Our baseline model is a standard New Keynesian framework extended with an energy block and climate change externalities *à la* Golosov et al. (2014). The economy consists of five types of agents: households, final goods producers, energy producers, the government (which acts as the climate authority), and a monetary authority. We next describe each in turn.

[5] See e.g. Nordhaus (2008).

[6] The macroeconomic impact of carbon taxes and other mitigation policies has also been analyzed in dedicated reports by international organizations. See e.g. IMF (2022).

[7] Our result on the absence of trade-offs under optimal carbon taxation in the model extention with QE can therefore be seen as a generalization of Papoutsi et al.'s (2023) above result to a nominal framework with inflation and conventional (interest-rate) monetary policy.

2.1 Households

There exists a representative household that maximizes lifetime welfare,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \frac{\chi}{1+\varphi} N_t^{1+\varphi} \right],$$

where $C_t = \left(\int_0^1 c_{z,t}^{(\epsilon-1)/\epsilon} dz \right)^{\epsilon/(\epsilon-1)}$ is a Dixit-Stiglitz basket of consumption varieties (with $\epsilon > 1$), N_t is labor supply, and $\beta \in (0,1)$ is a discount factor, subject to the following budget constraint in nominal terms,

$$\int_0^1 P_{z,t} c_{z,t} dz + B_t = R_{t-1} B_{t-1} + W_t N_t + \Pi_t + T_t,$$

where $P_{z,t}$ is the price of consumption variety $z \in [0,1]$, B_t are holdings of one-period nominal debt, R_t is the gross nominal interest rate, W_t is the nominal wage, and Π_t and T_t are nominal lump-sum profits from firms and government subsidies, respectively. Cost minimization implies $c_{z,t} = (P_{z,t}/P_t)^{-\epsilon} C_t$ and $\int_0^1 P_{z,t} c_{z,t} dz = P_t C_t$, where $P_t = \left(\int_0^1 P_{z,t}^{1-\epsilon} dz \right)^{1/(1-\epsilon)}$ is the aggregate price index. The first-order conditions of the intertemporal problem can then be expressed as

$$\chi N_t^{\varphi} C_t = \frac{W_t}{P_t} \equiv w_t, \tag{1}$$

$$\frac{1}{C_t} = \beta R_t \mathbb{E}_t \left\{ \frac{P_t}{P_{t+1} C_{t+1}} \right\}.$$

2.2 Final goods producers

Each consumption variety $z \in [0,1]$ is produced by a monopolistic producer. The production function of variety-z producer is

$$y_{z,t} = \left[1 - D\left(S_t\right) \right] A_t F\left(N_{z,t}, E_{z,t}\right). \tag{2}$$

where A_t is exogenous total factor productivity (TFP), $N_{z,t}$ is the firm's labor demand and $E_{z,t}$ is its energy consumption. Each firm uses a combination of green and fossil energy,

$$E_{z,t} = \mathbf{E}(E_{z,t}^g, E_{z,t}^f),$$

where $E_{z,t}^g$ is green energy and $E_{z,t}^f$ is fossil energy (also known as "brown" energy). Both F and $\mathbf{E}$ have constant returns to scale. The term $D\left(S_t\right)$ is the so-called *damage function*, which depends on the stock of carbon concentration in the atmosphere, S_t.[8] The damage function represents the key externality through which climate change affects economic activity in the model.[9]

Producer z's cost minimization problem is as follows,

[8]As explained by Golosov et al. (2014), the function $D\left(S_t\right)$ can be seen as compounding a mapping from atmospheric carbon concentration into global temperature, T_t, and a mapping from global warming into economic damages. Denoting the latter two mappings by T and D_T respectively, we therefore have $D\left(S_t\right) \equiv D_T\left(T(S_t)\right)$

[9]A more general model specification could also include direct effects of climate change on household utility. See e.g. van der Ploeg and Withagen (2012) and Barrage (2020) for contributions adopting this approach.

$$\min_{N_{z,t},\{E^g_{z,t}\}_{i=g,f}} W_t N_{z,t} + \sum_{i=g,f} p^i_t P_t E^i_{z,t} - MC_{z,t}\left[1 - D\left(S_t\right)\right] A_t F\left(N_{z,t}, \mathbf{E}(E^g_{z,t}, E^f_{z,t})\right),$$

where p^g_t and p^f_t are the real price of green and fossil energy, respectively, and $MC_{z,t}$ is the Lagrange multiplier on (2), i.e. firm z's nominal marginal cost. The first-order conditions are given by

$$MC_t\left[1 - D\left(\cdot\right)\right] A_t F_N\left(N_{z,t}, E_{z,t}\right) = W_t, \tag{3}$$

$$MC_t\left[1 - D\left(\cdot\right)\right] A_t F_E\left(N_{z,t}, E_{z,t}\right) \mathbf{E}_{E^i}(E^g_{z,t}, E^f_{z,t}) = p^i_t P_t, \tag{4}$$

$i = g, f$, where we use the fact that, under constant returns to scale, marginal costs are equalized across firms: $MC_{z,t} = MC_t$ for all z.[10] The firm's total cost can then be expressed as $W_t N_{z,t} + \sum_{i=g,f} p^i_t P_t E^i_{z,t} = MC_t y_{z,t}$.

Producers' pricing problem is standard. Each producer faces a demand curve

$$y_{z,t} = \left(P_{z,t}/P_t\right)^{-\epsilon} C_t. \tag{5}$$

Firms receive a subsidy τ^y per unit of revenue.[11] We assume Calvo (1983) pricing, with θ denoting the fraction of randomly-selected producers not adjusting their price in a given period. A producer that has the opportunity of changing its price in period t chooses $P_{z,t}$ to maximize the expected future discounted stream of nominal profits over the (expected) life of the new price,

$$\sum_{t=0}^{\infty} \mathbb{E}_t \left\{ \Lambda_{t,t+s}\theta^s \left[\left(1 + \tau^y\right) P_{z,t} - MC_{t+s}\right] \left(\frac{P_{z,t}}{P_{t+s}}\right)^{-\epsilon} C_{t+s} \right\},$$

where

$$\Lambda_{t,t+s} \equiv \beta^s \frac{P_t C_t}{P_{t+s} C_{t+s}} \tag{6}$$

is the stochastic discount factor. The first order condition is

$$\sum_{t=0}^{\infty} \mathbb{E}_t \left\{ \Lambda_{t,t+s}\theta^s \left(\left(1 + \tau^y\right) P^*_t - \frac{\epsilon}{\epsilon - 1} MC_{t+s} \right) \left(\frac{P^*_t}{P_{t+s}}\right)^{-\epsilon} C_{t+s} \right\} = 0, \tag{7}$$

where P^*_t is the common optimal price chosen by all time-t price-setters. The overall price level follows

$$P_t = \left[\left(1 - \theta\right)\left(P^*_t\right)^{1-\epsilon} + \theta P^{1-\epsilon}_{t-1}\right]^{1/(1-\epsilon)}. \tag{8}$$

2.3 Energy sectors

Each type of energy is produced by a representative producer. Both types of energy are measured in Gigatons of oil equivalents (Gtoe).[12] As in Golosov et al's (2014), production in both energy sectors is linear in labor,[13]

[10] Under the assumption of constant returns to scale in F and $\mathbf{E}$, the marginal products F_N and F_E depend only on the labor-energy ratio $N_{z,t}/E_{z,t}$, whereas $\mathbf{E}_{E^i}, i = f, g$, depend only on the green-fossil energy ratio $E^g_{z,t}/E^f_{z,t}$. Since factor prices are common to all firms, those ratios are equalized across producers, and therefore so are marginal products and the nominal marginal cost.

[11] As will become clear shortly, we introduce a revenue subsidy in order to offset the monopolistic distortion and focus the analysis on the trade-offs created by nominal rigidities and climate change externalities.

[12] In measuring energy in Gtoe we depart from Golosov et al (2014), who measure it in Gigatons of carbon (GtC) emissions. One difficulty with measuring energy of both types in GtC is that one needs to assign a carbon content to green energy sources –which by definition produce no, or almost no, emissions– when calibrating the model. This difficulty is avoided by measuring energy in Gtoe.

[13] In particular, they assume linearity in labor in the complete characterization of their model. Their key theoretical result, the closed-form solution for the optimal carbon tax, is obtained under a more general specification both for energy and final goods production technologies.

$$E_t^i = A_t^i N_t^i,$$

for $i = g, f$, where A_t^i is sector-specific exogenous productivity. Each type of energy is sold in a perfectly competitive market at real price p_t^i. Fossil-fuel energy production is subject to a per-unit tax τ_t^f. The representative firm in energy sector i chooses N_t^i to maximize real profits,

$$\frac{\Pi^i}{P_t} = \left(p_t^i - \tau_t^i\right) A_t^i N_t^i - w_t N_t^i,$$

for $i = g, f$, with $\tau_t^g = 0$. The first-order conditions are

$$p_t^g A_t^g = w_t, \tag{9}$$

$$\left(p_t^f - \tau_t^f\right) A_t^f = w_t. \tag{10}$$

2.4 Climate externalities

As in Golosov et al (2014), the damage function $D(S_t)$ is such that

$$1 - D(S_t) = e^{-\gamma_t (S_t - \bar{S})},$$

where γ_t is an exogenously time-varying elasticity and $\bar{S}$ is pre-industrial atmospheric carbon concentration. Following also Golosov et al (2014), the law of motion for atmospheric carbon concentration is linear in past carbon emissions from fossil energy consumption,

$$S_t - \bar{S} = \sum_{s=0}^{t+T} (1 - d_s)\, \xi E_{t-s}^f, \tag{11}$$

where ξ is the carbon content of fossil energy, defined as tons of carbon per ton of oil equivalent (tC/toe), such that ξE_t^f measures fossil energy consumption in gigatons of carbon (GtC).

2.5 Government and the monetary authority

The government's nominal budget constraint is

$$\tau^y \int_0^1 P_{z,t} y_{z,t} dz + T_t + R_{t-1} B_{t-1} = P_t \tau_t^f E_t^f + B_t.$$

Without loss of generality, we assume a balanced-budget rule. Finally, the monetary authority (the "central bank") sets the nominal interest rate R_t. In section 3, we will analyze optimal monetary policy, such that the central bank chooses R_t so as to maximize social welfare.

2.6 Market clearing

Final goods market clearing requires $y_{z,t} = c_{z,t}$ for each variety z. We define aggregate output as $Y_t = \left(\int_0^1 y_{z,t}^{\epsilon/(\epsilon-1)} dz\right)^{(\epsilon-1)/\epsilon}$. It follows that $Y_t = C_t$. Labor market clearing requires

$$N_t = \sum_{i=g,f} N_t^i + N_t^y,$$

where $N_t^y \equiv \int_0^1 N_{z,t} dz$ is labor demand by final goods producers. Equation (2) can be expressed as $y_{z,t} = [1 - D(S_t)] A_t F(1, E_t/N_t^y) N_{z,t}$, where we use the fact that energy-labor ratios are equalized across firms at the level E_t/N_t^y (where E_t is total energy demand). Aggregating across firms, and using equation (5), we obtain

$$[1 - D(S_t)] A_t F(N_t^y, E_t) = \Delta_t Y_t, \tag{12}$$

where $\Delta_t \equiv \int_0^1 (P_{z,t}/P_t)^{-\epsilon} dz$ is an index of relative price dispersion with law of motion

$$\Delta_t = \theta \left(\frac{P_t}{P_{t-1}}\right)^{\epsilon} \Delta_{t-1} + (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon}. \tag{13}$$

Equation (12) implies that relative price dispersion increases the amount of labor and energy inputs needed to satisfy a certain level of aggregate consumption demand. Equations (13) and (8) imply that nonzero inflation ($P_t/P_{t-1} \neq 1$) gives rise to relative price distortions, just as in the standard New Keynesian framework.

3 Optimal conventional monetary policy

We start our analysis of optimal monetary policy by establishing analytically a simple benchmark result, related to the special case in which the carbon tax is set at all times at its socially optimal level. For this purpose, we first characterize the social planner equilibrium, which also allows us to derive optimal carbon taxation.

3.1 Social planner equilibrium

The social planner maximizes household welfare,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \frac{\chi}{1+\varphi}\left(N_t^y + \sum_{i=g,f} N_t^i\right)^{1+\varphi}\right],$$

subject to the aggregate resource constraints,

$$[1 - D(S_t)] A_t F\left(N_t^y, \mathbf{E}(E_t^g, E_t^f)\right) = C_t, \tag{14}$$

$$A_t^i N_t^i = E_t^i \quad i = g, f,$$

and the law of motion of atmospheric carbon concentration, equation (11). As shown in Appendix A, the first-order conditions of this problem can be combined into the following three conditions for social efficiency,

$$[1 - D(S_t)] A_t F_N(\cdot) = \chi N_t^{\varphi} C_t, \tag{15}$$

$$[1 - D(S_t)] A_t F_E(\cdot) \mathbf{E}_{E^g}(\cdot) = \frac{\chi N_t^{\varphi} C_t}{A_t^g}, \tag{16}$$

$$
\begin{aligned}
\left[1-D\left(S_{t}\right)\right] A_{t} F_{E}(\cdot) \mathbf{E}_{E^{f}}(\cdot) &= \frac{\chi N_{t}^{\varphi} C_{t}}{A_{t}^{f}}+C_{t} \mathbb{E}_{t}\left\{\sum_{s=0}^{\infty} \beta^{s} \xi\left(1-d_{s}\right) \gamma_{t+s} \frac{Y_{t+s}}{C_{t+s}}\right\} \\
&= \frac{\chi N_{t}^{\varphi} C_{t}}{A_{t}^{f}}+Y_{t} \mathbb{E}_{t}\left\{\sum_{s=0}^{\infty} \beta^{s} \xi\left(1-d_{s}\right) \gamma_{t+s}\right\}, \qquad (17)
\end{aligned}
$$

where the second equality in (17) uses the fact that $C_t = Y_t$ for all t.

Equation (15) is the standard efficiency condition in the basic New Keynesian model (except for the presence of the climate externality factor, $1-D(S_t)$). It requires that the marginal product of labor equals its marginal utility cost (χN_t^{φ}) expressed in consumption units (i.e. rescaled by marginal consumption utility, $1/C_t$). Equation (16) is the analogous efficiency condition for green energy: it requires that its marginal contribution to the production of final goods equals the marginal utility cost of producing it (again, expressed in consumption units).

Equation (17) is the corresponding efficiency condition for fossil energy. It differs from its green energy counterpart in the presence of the second term in the right hand side, which is Golosov et al's (2014) well-known formula for the marginal externality damage of carbon emissions.[14] The latter term captures the expected present-discounted value of the future economic damages produced by an additional unit of fossil energy consumption. Fossil energy consumption produces carbon emissions today in the amount ξ. These emissions add to atmospheric carbon concentration at each future date $t+s$, $s \geq 0$, in an amount determined by the carbon depreciation structure, $1-d_s$. This in turn reduces future output in the amount $\gamma_{t+s}Y_{t+s}$, or $\gamma_{t+s}Y_{t+s}C_{t+s}^{-1}$ once expressed in utils, which simplifies to γ_{t+s} given that in this model all output is consumed. Finally, these externality damages are expressed in units of today's consumption by dividing them by the time-t marginal utility of consumption, $C_t^{-1} = Y_t^{-1}$, such that they are proportional to current output.

3.2 Flexible-price equilibrium

We now return to the decentralized economy. As customary in analyses of optimal monetary policy in New Keynesian models, it is useful to investigate the properties of equilibrium under flexible prices, because it represents the equilibrium achieved under sticky prices when the monetary authority follows a policy of strict inflation targeting. Under flexible prices ($\theta = 0$), equation (7) becomes

$$P_{z,t} = (1+\tau^y)^{-1} \frac{\epsilon}{\epsilon-1} MC_t.$$

Therefore, all firms choose the same price, $P_{z,t}/P_t = 1$ for all z, and relative price distortions are eliminated, $\Delta_t = 1$. The real marginal cost is then given by $MC_t/P_t = (1+\tau^y)\frac{\epsilon-1}{\epsilon}$. Combining this with (1), (3), (4), (9) and (10), we obtain

$$(1+\tau^y)\frac{\epsilon-1}{\epsilon}\left[1-D(S_t)\right] A_t F_N(\cdot) = \chi N_t^{\varphi} C_t, \qquad (18)$$

$$(1+\tau^y)\frac{\epsilon-1}{\epsilon}\left[1-D(S_t)\right] A_t F_E(\cdot)\mathbf{E}_{E^g}(\cdot) = \frac{\chi N_t^{\varphi} C_t}{A_t^g}, \qquad (19)$$

[14] The only difference between our formula and theirs is the presence of the parameter ξ, i.e. the carbon content of fossil energy. This stems from the fact that –as explained before– we measure energy in Gtoe instead of in GtC (the measure used in Golosov et al 2014). Thus, transforming fossil energy consumption into carbon emissions requires multiplying the former by its carbon content.

$$(1+\tau^{y})\frac{\epsilon-1}{\epsilon}\left[1-D\left(S_{t}\right)\right]A_{t}F_{E}\left(\cdot\right)\mathbf{E}_{E^{f}}\left(\cdot\right)=\frac{\chi N_{t}^{\varphi}C_{t}}{A_{t}^{f}}+\tau_{t}^{f}. \tag{20}$$

Comparing the above three equations with the social efficiency conditions (15) to (17), it follows that setting

$$\tau^{y}=\frac{\epsilon}{\epsilon-1}-1, \tag{21}$$

$$\tau_{t}^{f}=Y_{t}\mathbb{E}_{t}\left\{\sum_{s=0}^{\infty}\beta^{s}\left(1-d_{s}\right)\xi\gamma_{t+s}\right\}\equiv\tau_{t}^{f*}, \tag{22}$$

allows the flexible-price equilibrium to replicate the social planner equilibrium.[15] Equation (21) shows a well-known result in normative New Keynesian theory: setting the revenue subsidy rate τ^{y} equal to the net price markup under flexible prices, $\frac{\epsilon}{\epsilon-1}-1$, allows policy-makers to offset the monopolistic distortion. Equation (22) reflects Golosov et al's (2014) key theoretical result: if the carbon tax is set equal to the marginal externality damage of carbon emissions, then the decentralized economy replicates the social planner equilibrium. Intuitively, if the carbon tax equals τ_{t}^{f*} (also known as the optimal Pigouvian tax), then all agents perfectly internalize the climate externality from fossil energy use. In our model, the same result is true for the flexible-price equilibrium, provided equation (21) is also satisfied.

3.3 Optimal monetary policy: the case of optimal carbon taxation

We are now ready to obtain our main analytical result. As in the standard New Keynesian framework, in our model a policy of strict inflation targeting, $\pi_{t}\equiv P_{t}/P_{t-1}=1$ for all $t\geq 0$, allows the decentralized economy to replicate the flexible-price equilibrium. To see this, notice from equation (8) that such a policy ensures that newly-set optimal prices always equal the overall price level, $P_{t}^{*}=P_{t}$, as in the flexible-price equilibrium. Intuitively, if the price level is held constant by monetary policy, those firms that have the chance of resetting their price in a given period have no reason for actually changing it. Equation (13) then implies $\Delta_{t}=\theta\Delta_{t-1}+1-\theta$, such that relative price dispersion at any given time depends only on past dispersion.

In what follows, we make two maintained assumptions. First, we assume that there is no initial price dispersion ($\Delta_{-1}=1$), such that from $t=0$ onwards price dispersion arises only to the extent that there is nonzero inflation. Under a zero (net) inflation policy, we thus have $\Delta_{t}=1$ for all $t\geq 0$, as in the flexible price equilibrium. Second, we assume that the revenue subsidy satisfies condition (21) in all scenarios. These assumptions allow us to isolate monetary policy trade-offs from the influence of initial price dispersion and monopolistic distortions, and thus focus our analysis on how climate externalities and carbon taxation affects those trade-offs, which is the key objective of our paper.[16]

From the above and the discussion in section 3.2, it follows that, provided the carbon tax equals the optimal Pigouvian tax in equation (22), a policy of strict inflation targeting allows the central bank to also replicate the social planner equilibrium. In particular, output replicates

[15] Notice that, given $\Delta_{t}=1$, equation (12) also replicates its social planner counterpart, equation (14).

[16] As shown by Benigno and Woodford (2005) and Woodford (2003), monopolistic distortions give the central bank a reason for transitorily deviating from a zero inflation policy under the time-0 optimal monetary policy commitment, with inflation converging asymptotically to its optimal long-run value of zero. For an analysis of optimal monetary policy when there exists initial price dispersion, see Yun (2005).

its socially efficient level, which we may denote by Y_t^*, such that the *welfare-relevant output gap* is fully stabilized: $Y_t/Y_t^* = 1$ for all $t \geq 0$. Thus, the optimal monetary policy is strict inflation targeting.

We summarize the above discussion in the following proposition. A formal proof can be found in Appendix B, which also lays out the general optimal monetary policy commitment problem.

Proposition 1 *Provided the carbon tax is set at its socially optimal level (equation 22) at all times, the optimal monetary policy is strict inflation targeting:* $\pi_t = 1$. *This policy allows the decentralized economy to replicate the social planner equilibrium.*

The intuition for this key result is simple: if the carbon tax is set at its socially optimal level, then all agents internalize perfectly the negative externality from carbon emissions. This leaves nominal price rigidity as the only distortions left, which can be offset by the central bank through a strict inflation targeting policy. In sum, as long as climate authorities set them at the optimal level, *carbon taxes pose no trade-off for monetary policy*: it is both feasible and optimal to fully stabilize inflation and the welfare-relevant output gap.

4 Quantitative analysis

While Proposition 1 provides a useful normative benchmark, the assumption of optimal carbon taxation from the very first period may be seen as unrealistic, given the sluggish pace of progress in carbon taxation and other mitigation policies worldwide. Therefore, we next turn our attention to the more realistic case in which carbon taxation is suboptimal, which creates trade-offs between core and climate goals. We start by calibrating the model.

4.1 Calibration

Preferences. As is standard in the New Keynesian literature, we set the (inverse) labor supply elasticity φ to 1. We normalize initial labor supply to 1, which requires setting the scale parameter of labor disutility χ to 1.[17] The elasticity of substitution across final goods varieties, ϵ, is also set to a standard value of 7. As in Nordhaus (2008), we assume a household net discount rate of 1.5% a year, which in our quarterly model implies $\beta = 0.985^{1/4}$.

Technology. Following Golosov et al (2014), we assume a Cobb-Douglas functional form for the final-goods production technology,

$$F(N, E) = N^{1-\alpha} E^{\alpha},$$

and a CES function for the energy basket,

$$E = \mathbf{E}(E^g, E^f) = [\omega (E^g)^\rho + (1-\omega)(E^f)^\rho]^{1/\rho}. \tag{23}$$

As in Golosov et al (2014), we set $\alpha = 0.04$, which corresponds approximately with the energy share of World GDP. Based on the empirical evidence in Papageorgiou et al (2017), we set ρ to

[17] See Appendix C for further details.

0.65, which implies an elasticity of substitution between green and fossil energy, $1/(1-\rho)$, close to 3. The remaining parameters are calibrated as follows. Under the above functional forms, the relative price of green energy is given by[18]

$$\frac{p_0^g}{p_0^f} = \frac{\partial F/\partial E^g}{\partial F/\partial E^f} = \frac{\omega}{1-\omega}\left(\frac{E_0^f}{E_0^g}\right)^{(1-\rho)}.$$

Using data from BP, we obtain levels of World annual consumption of fossil and green energy in 2019 of 11.70 and 3.28 Gtoe, respectively,[19] such that $E_0^f = 11.70/4$ and $E_0^g = 3.28/4$. Using data from the International Renewable Energy Agency, we estimate a relative price of green over fossil energy of $p_0^g/p_0^f = 0.54$.[20] Given these targets, we solve for ω as

$$\omega = \frac{1}{1 + (p_0^f/p_0^g)(E_0^f/E_0^g)^{1-\rho}}.$$

The productivity factors of both energy sectors, A_0^g and A_0^f, are then calibrated to match E_0^g and E_0^f (see Appendix C for further details).[21]

The carbon process and the damage function. In order to calibrate the carbon content of fossil energy (ξ), we take the carbon content of each of the three sources of fossil energy (coal, gas and oil) and average them using as weights the share of each source in fossil energy consumption.[22] This yields $\xi = 0.879$ tC/toe.

We assume the following depreciation structure for atmospheric carbon concentration,

$$1 - d_s = \phi_0 (1-\phi)^s,$$

for $t \geq 0$, where $1-\phi_0$ is the share of carbon emissions into the atmosphere that exits it within the same quarter (into the biosphere and the surface oceans), and ϕ is the rate at which the remaining share disappears from the atmosphere. Our two-parameter specification is a special case of Golosov et al's (2014), and is motivated by our need to have a well-defined terminal

[18]Notice that $\frac{\partial F}{\partial E^i} = \frac{\partial F}{\partial E}\frac{\partial \mathbf{E}}{\partial E^i} = \alpha \frac{F}{E}\omega_i \left(\frac{E}{E^i}\right)^{(1-\rho)}, i = f, g$, with $\omega_g = \omega = 1 - \omega_f$.

[19]We take data on World consumption of different energy sources in 2019 in Terawatt-hour (TWh) from BP's Statistical Review of World Energy, and transform them into Gtoe using conversion factors from the International Energy Agency's (IEA) unit converter. We then calculate green energy as the sum of energy from wind, nuclear, hydropower, traditional biomass, biofuels and other renewable sources; and fossil energy as the sum of energy from coal, gas and oil.

[20]We use the levelized cost of energy (LCOE) as a proxy for the price of energy of different sources. Using global LCOE estimates for 2019 from IRENA (2020), we calculate the price for green energy as the weighted average of LCOEs for biomass, hydroelectric, solar and wind energy, using as weights their share in World consumption of all these energy sources in the BP data. This gives a price of 0.061 USD per kWh. According to the same source, LCOE estimates for fossil fuel-generated power range from 0.05 to 0.177 USD/kWh; we take the simple average of both numbers, yielding 0.113 USD/kWh. Taking the ratio of green and fossil LCOEs gives a relative price of 0.537.

[21]For simplicity, we assume that productivity is constant in both energy sectors: $A_t^i = A_0^i$ for all $t \geq 0$, $i = f, g$. We make the same assumption for TFP in final goods production, A_t.

[22]We take the carbon content of oil and coal from Golosov et al (2014): 0.846 tC/ton for oil and 0.716 tC/ton for coal (anthracite). As noted by these authors, one ton of oil equivalents (toe) is 1.58 tons of coal, so the carbon content of coal expressed in tC/toe is $0.716 \times 1.58 = 1.131$. The carbon content of natural gas is 0.0153 tC/GJ (IPCC 2006, table 1.3), or equivalently 0.641 tC/toe (after using the conversion factor GJ/toe, equal to 41.87 according to the IEA Unit Converter). We then weight these contents in tC/toe by the share of each source in total fossil energy consumption in 2019 in the BP data (32% coal, 29% gas, 39% oil).

steady state in the model for computational purposes.[23] We calibrate both parameters such that our carbon depreciation structure mimics Golosov et al's as closely as possible over a 300-year horizon. Figure 2 shows how our $1 - d_s$ function compares with theirs.

As in Golosov et al (2014), we set the pre-industrial stock of atmospheric carbon concentration $\bar{S}$ to 581 GtC and the stock in the present S_0 to 802 GtC.

As regards the damage function, we follow Golosov et al (2014) in setting the elasticity γ to 2.4×10^{-5}. Given the values of $\bar{S}$ and S_0, the economic damage from global warming in the present amounts to $\gamma(S_0 - \bar{S}) = 0.53\%$.

Price stickiness. Finally, we set the Calvo parameter θ to 0.75, such that prices change on average once a year, which roughly corresponds with empirical evidence for the euro area. The first two panels of Table 1 summarize the calibration of the baseline model.

4.2 Results

We turn to the quantitative results for the case of a "slow" green transition. "Slow" means that the carbon tax is assumed to be phased in gradually, starting from zero and reaching its optimal level after 30 years. This gradualism in climate policy in principle provides ample scope for activist monetary policy. We will measure the extent of monetary activism by the deviations from strict inflation targeting (zero inflation), which as proved in Section 3 is the optimal monetary policy when carbon taxes follow their optimal path from time zero.

Figure 3 shows the evolution of the variables of interest. The blue solid lines display transition dynamics under the optimal carbon tax; as explained before, in this scenario all real variables replicate their paths in the social planner equilibrium. The latter is characterized by constant levels for all variables,[24] with the exceptions of atmospheric carbon concentration, which increases as a result of (socially optimal) positive net carbon emissions, and of output, which declines as a result of rising damages from global warming.

The red solid lines show the dynamics under the slow green transition, assuming also that monetary policy ignores climate externalities and sticks to strict inflation targeting (zero inflation), such that output replicates its natural (flexible-price) level. In this case, output starts out above its socially efficient level (i.e. the welfare-relevant output gap is positive), reflecting the lower levels of carbon taxation. Low carbon taxation implies that fossil energy is too cheap, im-

[23] Golosov et al (2014) use the three-parameter specification $1 - d_s = \phi_L + (1 - \phi_L)\phi_0 (1 - \phi)^s$, where ϕ_L is the share of carbon emitted into the atmosphere that stays in it forever. Under this specification, the model does not have a well-defined terminal steady state, at least as long as carbon emissions do not converge to zero, because atmospheric carbon concentration S_t grows unboundedly with new emissions as time goes by. Because our model is forward-looking, we need to have a terminal steady state in order to be able to solve it. This is achieved by setting ϕ_L to zero.

[24] To see this, rescale by $1 - D(S_t)$ all social-planner equilibrium conditions featuring that term and use $C_t = Y_t$ to obtain

$$A_t F(N_t^y, E(E_t^g, E_t^f)) = Y_t/[1 - D(S_t)] \equiv Y_t^D,$$
$$A_t \partial F(\cdot)/\partial N_t^y = \chi N_t^\phi Y_t^D,$$
$$A_t \partial F(\cdot)/\partial E_t^i = \chi N_t^\phi Y_t^D / A_t^i + 1_{i=f} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s (1 - d_s) \xi \gamma_{t+s},$$

$i = f, g$, where Y_t^D is gross-of-damages output. These four equations, together with $A_t^i N_t^i = E_t^i, i = f, g$, jointly determine the path of $N_{t\ i=y,f,g}^i, E_{t\ i=f,g}^g, Y_t^D$. Absent any change in TFP factors $(A_t, A_t^i, i = f, g)$, the former six variables are constant. Since $Y_t = [1 - D(S_t)] Y_t^D$, it follows that Y_t evolves in proportion to $1 - D(S_t)$.

Figure 2: Carbon decay structure

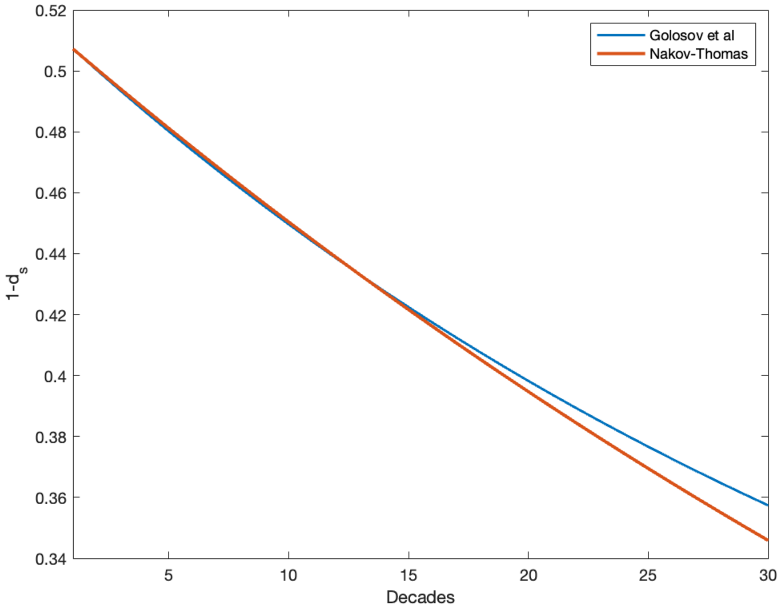
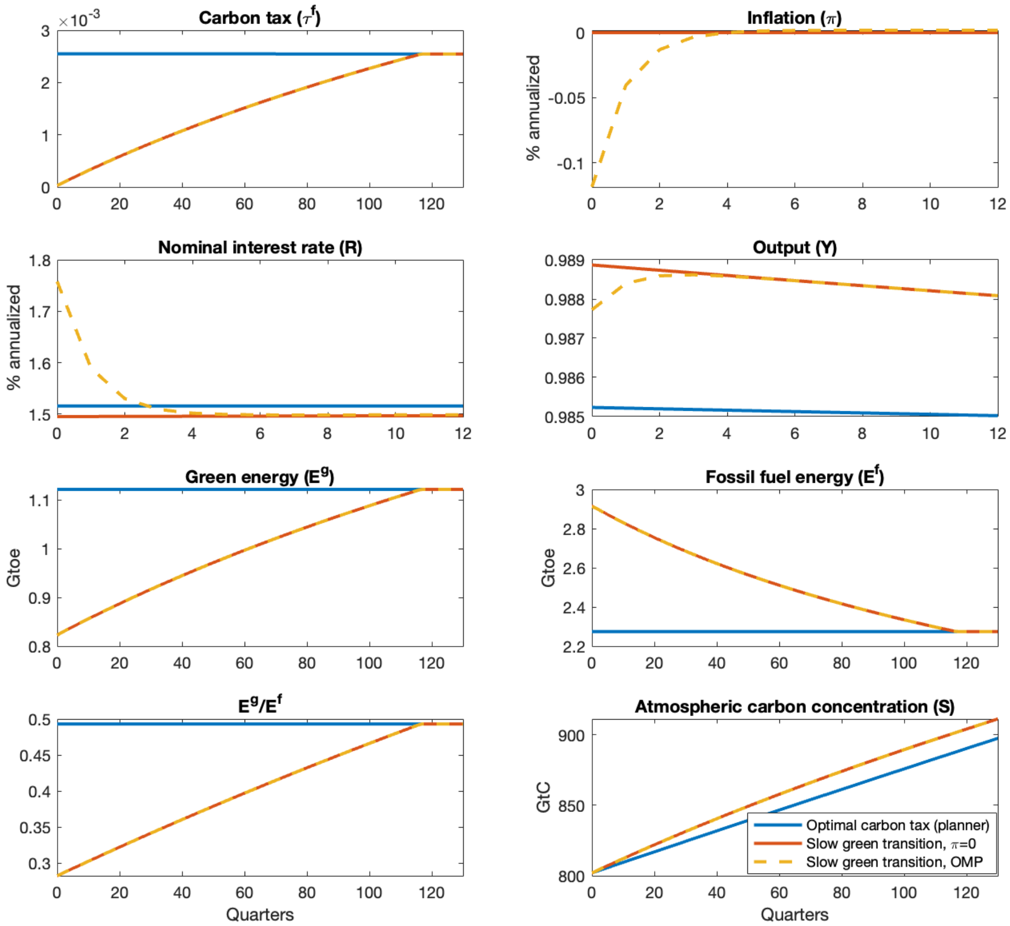


Table 1: Calibration

	Description	Value	Target/Source
New Keynesian block			
β	Household discount factor	$0.985^{1/4}$	Nordhaus (2008)
θ	Calvo parameter	0.75	Price adj. freq. 1 yr
ϵ	Elasticity of substitution	7	Standard
φ	(inv) elasticity labor supply	1	Standard
Energy & climate block			
α	Energy share of output	0.04	Golosov et al (2014)
ρ	(1-inv) elast subst g vs f	0.65	Papageorgiou et al (2017)
γ	Elasticity damage function	0.000024	Golosov et al (2014)
ϕ_0, ϕ	carbon depreciation structure	0.51 0.00033	Golosov et al deprec.
ω	weight of green energy	0.2571	$p^g/p^f = 0.54$
A^f	productivity fossil sector	84.2	$E^f = 11.7\ Gtoe$
A^g	productivity green sector	155.9	$E^g = 3.3\ Gtoe$
ξ	carbon content fossil energy (tC/toe)	0.879	IPCC (2006) tables
$\bar{S}, S_0$	Atmosph. carbon concentration (GtC)	$581, 802$	Golosov et al (2014)
QE extension			
κ_f	Spread sensitivity, brown bonds	0.024	Impact CSPP anncmnt
κ_g	Spread sensitivity, green bonds	0.160	Impact CSPP anncmnt
$\bar{B}^f$	Min. private absorption brown bonds	0.0172	Brown bond spreads
$\bar{B}^g$	Min. private absorption green bonds	0.0026	Green bond spreads
ψ	Bond leverage of energy firms	5	Lever. CSPP-eligible issuers

plying inefficiently high (low) levels of fossil (green) energy use. This in turn produces a faster accumulation of carbon in the atmosphere, and hence a faster increase in economic damages from global warming. Along the transition, output falls more quickly than (and hence converges towards) its efficient counterpart, reflecting both rising carbon taxation and a faster increase in climate-related damages.

Figure 3: Transitions in baseline model



The yellow dashed lines show the dynamics under the optimal monetary policy. The benevolent, climate-conscious central bank understands that, during the green transition, carbon taxes are suboptimally low and CO2 emissions excessively high. Aware of this, and compared to the zero inflation scenario, it implements a tighter interest rate path in order to reduce aggregate demand and thus overall energy consumption, including fossil energy consumption. This way, output comes closer to its socially efficient path, i.e. the welfare-relevant output gap becomes narrower. In return, the central bank accepts a fall in output below its *natural* level and hence a fall in inflation below its long-run target (zero). However, the deviation from strict inflation targeting is very small (around 10 basis points in the first period) and short-lived (barely a year). Therefore, the optimal policy can be characterized essentially as price stability. Fossil energy use does fall, although compared to the scale of its reduction along the transition the effect is indistinguishable from zero. Correspondingly, the path of atmospheric carbon concentration is essentially unaffected.

Why is optimal monetary policy not more climate-activist? The reason is that interest rate policy affects green and fossil energy use in basically the same way. Therefore, it is a rather blunt, untargeted, inefficient instrument for reducing carbon emissions. The central bank could always achieve a larger reduction in carbon emissions, but this would come at the expense of a deep recession –with the resulting fall in consumpion– and a severe fall in inflation below target, all of which would be excessively costly in social welfare terms.

5 Green QE

So far we have restricted our analysis to conventional (interest-rate) monetary policy, with the aim of showing in the simplest and most transparent way to what extent the existence of climate externalities create trade-offs for monetary policy. In practice, some central banks use other instruments in order to pursue climate-related goals. Among the latter, a prominent role is played by "green QE", whereby central banks tilt their bond portfolios in favor of "green bonds" that satisfy certain climate-related eligibility criteria, and against "brown bonds". In this section, we extend our baseline model by introducing a simple financial friction that allows central bank purchases of corporate bonds to play a role in affecting equilibrium allocations.

5.1 A simple model of green (and brown) QE

Assume that a fraction ψ of energy producers' operating costs must be pre-financed with working capital loans. In particular, these firms are assumed to issue short-term bonds at the start of the period. Bonds of sector-i energy firms are issued at a price $1/R_t^i$ –which is taken as given by firms– and have unit face value. Therefore, the number of bonds issued by each sector-i firm in period t equals $\frac{\psi w_t N_t^i}{1/R_t^i} = \psi R_t^i w_t N_t^i$, which is also the face value to be repaid.[25] We adopt the timing convention that bond repayments are due at the end of the period –i.e. bonds are intra-period. This assumption allows us to preserve the static nature of energy prices in our baseline model, which simplifies the algebra, but is otherwise essentially innocuous.[26] Thus, the net bond return $R_t^i - 1$ can also be interpreted as the bond spread relative to the frictionless intra-period net return (zero).

Under these assumptions, the maximization problem of firms in energy sector $i = f, g$ becomes

$$\max_{N_t^i} \left(p_t^i - \tau_t^i\right) A_t^i N_t^i - \left[1 + \psi\left(R_t^i - 1\right)\right] w_t N_t^i,$$

again with $\tau_t^g = 0$. The first-order conditions are now given by

$$p_t^g A_t^g = \left[1 + \psi\left(R_t^g - 1\right)\right] w_t, \tag{24}$$

$$(p_t^f - \tau_t^f) A_t^f = \left[1 + \psi\left(R_t^f - 1\right)\right] w_t. \tag{25}$$

The above expressions differ from their baseline model counterparts (equations 9 and 10) in the presence of the terms $\psi\left(R_t^i - 1\right)$, i.e. the product of the leverage factor ψ and the bond spread, $R_t^i - 1$. These terms create a wedge between the marginal revenue product of labor (net of carbon taxes, in the case of sector f) and its marginal (non-financial) cost, w_t. *Ceteris paribus*, these financial wedges raise the real price of both types of energy.

[25] Our baseline (no QE) model can be nested by setting $\psi = 0$.

[26] Alternatively, we could assume that bonds repayments are due at the beginning of the following period. This would make the firm's problem dynamic and, in particular, it would introduce time $(t+1)$-dated terms in equations (24) and (25) below. However, such modification would leave our numerical results essentially unchanged.

Corporate bonds are purchased by households and the central bank. Let B_t^i denote the market value of sector-i bond purchases by the household. Following Andrés, López-Salido and Nelson (2004), Chen, Curdia and Ferrero (2012) and Gertler and Karadi (2013), among others, we assume that households incur transaction costs when adjusting their corporate bond portfolio. In particular, the household budget constraint is now

$$P_t C_t + B_t + \sum_{i=g,f} B_t^i \left(1+\zeta_t^i\right) = R_{t-1} B_{t-1} + \sum_{i=g,f} R_t^i B_t^i + W_t N_t + \Pi_t + T_t,$$

where ζ_t^i is a transaction cost per sector-i bond. Notice that, since corporate bonds are intra-period, their repayments $\sum_{i=g,f} R_t^i B_t^i$ accrue within the period. As in Gertler and Karadi (2013), we assume quadratic transaction costs of the form

$$\zeta_t^i = \frac{\kappa_i}{2} \frac{\left(B_t^i - \bar{B}^i\right)^2}{B_t^i},$$

$i = g, f$, with $\kappa_i > 0$. The first-order conditions for $\{B_t^i\}_{i=g,f}$ are given by

$$R_t^i = 1 + \kappa_i \left(B_t^i - \bar{B}^i\right), \tag{26}$$

$i = g, f$. All other household first-order conditions are as in the baseline model. Market clearing for sector-i bonds requires that household demand equals firms' supply minus bonds absorbed by the central bank,

$$B_t^i = \psi w_t N_t^i - B_t^{i,cb}, \tag{27}$$

for $i = g, f$, where $B_t^{i,cb} \in [0, \psi w_t N_t^i]$ is the market value of the central bank's purchases of sector-i bonds. Therefore, central bank purchases of bonds of a given energy sector reduce the amount of such bonds to be absorbed by the private sector (households). From equation (26), this allows the central bank to reduce bond spreads for that sector and thus (from equations 24 and 25) lower the real price of that type of energy, p_t^i.

5.2 Optimal monetary policy with QE: analytical results

We now show that our key normative result from Section 3 carries over to an environment in which the central bank toolkit includes bond purchases.

Optimal monetary policy under optimal carbon taxation. Provided the carbon tax is set at its socially optimal level ($\tau_t^f = \tau_t^{f*}$, where the optimal Pigouvian carbon tax continues to be given by equation 22), it is trivial to show that optimal monetary policy combines strict inflation targeting ($\pi_t = 1$, as in our baseline model) with central bank purchases of corporate bonds in the amount necessary to fully eliminate bond spreads for *both* sectors: $R_t^g = R_t^f = 1$. Under this configuration of monetary policy, equations (24) and (25) collapse to their baseline model counterparts (equations 9 and 10) and the decentralized equilibrium replicates the social-planner allocation. From equations (26), eliminating corporate spreads requires reducing households' bond absorption to $B_t^i = \bar{B}^i$, which from equation (27) requires the central bank to absorb all bond supply over and above $\bar{B}^i$: $B_t^{i,cb} = \psi w_t N_t^i - \bar{B}^i$,$i = g, f$, for all $t \geq 0$. For brevity (and lack of a better name), we may refer to this policy as "full QE".

The above result can be summarized by the following proposition, which extends Proposition 1 to the model with QE.

Proposition 2 *Consider the model extension with corporate bond purchases. Provided the carbon tax is set at its socially optimal level at all times, optimal monetary policy combines strict inflation targeting ($\pi_t = 1$) and full QE, such that bond spreads of both energy sectors are eliminated: $R_t^g = R_t^f = 1$. This policy allows the decentralized economy to replicate the social planner equilibrium.*

The intuition for this results is again very simple. The extended model contains two additional frictions: the financial wedges affecting green and fossil energy prices ($\psi(R_t^i - 1), i = f, g$). Provided carbon taxes are socially optimal, all the central bank needs to do in order for energy prices to follow their optimal paths is to fully offset the financial frictions by eliminating green and brown bond spreads ($R_t^i = 1, i = f, g$). The climate externality is thus fully offset, leaving nominal rigidities as the only distortion, which the central bank addresses through strict inflation targeting.

Optimal monetary policy under suboptimal carbon taxation. We now consider the implications of the more realistic case of suboptimal taxation for optimal monetary policy.[27] We focus on the case of the "slow green transition" analyzed in the previous sections, such that $\tau_t^f < \tau_t^{f*}$ for an initial period, after which $\tau_t^f = \tau_t^{f*}$. It is again optimal for the central bank to do full *green* QE, i.e. to eliminate green bonds' spread ($R_t^g = 1$) at all times by absorbing their supply over and above $\bar{B}^g$: $B_t^{g,cb} = \psi w_t N_t^g - \bar{B}^g$ for all $t \geq 0$.

As regards fossil energy sector bonds –henceforth "brown bonds" –, optimal QE policy aims at tightening financing conditions for that sector as much as needed in order for the market price of fossil energy to replicate its socially optimal level. The latter is the level of p_t^f that solves equation (10) evaluated at the optimal Pigouvian carbon tax: $p_t^f = \tau_t^{f*} + \frac{w_t}{A_t^f}$. However, such an outcome is not always feasible. To see this, we solve for p_t^f in equation (25) and equate the resulting expression to the socially optimal price,

$$\tau_t^f + \left[1 + \psi\left(R_t^f - 1\right)\right]\frac{w_t}{A_t^f} = \tau_t^{f*} + \frac{w_t}{A_t^f}.$$

Solving the latter equation for the brown bond spread yields $R_t^f - 1 = \frac{1}{\psi}\frac{\tau_t^{f*} - \tau_t^f}{w_t/A_t^f}$. However, equation (26) for $i = f$ and the fact that households cannot hold more brown bonds than the amount supplied by firms ($B_t^f \leq \psi w_t N_t^f$) imply an upper bound on the brown bond spread,

$$R_t^f - 1 \leq \kappa_f(\psi w_t N_t^f - \bar{B}^f). \tag{28}$$

This allows us to obtain the following optimal rule for the brown bond spread,

$$R_t^f - 1 = \min\left\{\kappa_f\left(\psi w_t N_t^f - \bar{B}^f\right), \frac{1}{\psi}\frac{\tau_t^{f*} - \tau_t^f}{w_t/A_t^f}\right\}. \tag{29}$$

[27] Appendix D lays out the general optimal monetary policy problem in the model with QE.

Equation (29) can be translated into an optimal rule for brown bond purchases ("brown QE") by combining it with (26) and (27) for $i = f$, yielding

$$B_t^{f,cb} = \max\left\{0, \psi w_t N_t^f - \bar{B}^f - \frac{1}{\kappa_f \psi}\frac{\tau_t^{f*} - \tau_t^f}{w_t/A_t^f}\right\}.$$

We can now characterize optimal brown QE in a slow green transition. Provided the initial gap between actual and optimal carbon taxes $(\tau_t^{f*} - \tau_t^f)$ is large enough that $\frac{1}{\psi}\frac{\tau_t^{f*}-\tau_t^f}{w_t/A_t^f} > \kappa_f(\psi w_t N_t^f - \bar{B}^f)$, in the initial phase of the transition the central bank cannot make brown bond spreads high enough because it hits its short-selling constraint ($B_t^{f,cb} \geq 0$), and the best it can do is to maximize the spread on brown bonds by holding none of them. Once the carbon tax gap becomes small enough that $\frac{1}{\psi}\frac{\tau_t^{f*}-\tau_t^f}{w_t/A_t^f} \leq \kappa_f(\psi w_t N_t^f - \bar{B}^f)$, QE policy can fully replicate the first-best allocation by purchasing as many brown bonds as needed in order for their spread to reach its socially optimal level. In other words, the central bank allows the financial friction affecting fossil energy firms to do the job of making fossil energy sufficiently expensive, thus compensating for the shortfall in carbon taxation. Once the carbon tax gap is closed ($\tau_t^{f*} = \tau_t^f$), the central bank implements "full brown QE" by purchasing brown bonds in the amount $\psi w_t N_t^f - \bar{B}^f$, thus eliminating brown bond spreads ($R_t^f = 1$).

An implication of the preceding analysis is that, in the initial phase of the slow green transition, monetary policy cannot fully offset the climate externality and hence faces a trade-off between stabilizing inflation and the welfare-relevant output gap, similar to the one in the baseline model. Once the carbon tax gap becomes sufficiently narrow, the transition enters a second phase in which QE policy is able to achieve the socially optimal prices for both green and fossil energy, thus fully offsetting the climate externality. From that point onwards, there is no longer a trade-off between core and climate goals, allowing the central bank to replicate the first-best equilibrium.[28] The next subsection analyzes numerically the slow green transition scenario.

5.3 Numerical analysis

Calibration of new parameters. In order to calibrate the sensitivity of bond spreads to central bank purchases, $\kappa_i = \frac{d(R_t^i)}{d(B_t^{i,cb})}$, we first note that such sensitivity can be expressed as $\frac{d(R_t^i)}{d(B_t^{i,cb})/B_t^{i,s}}\frac{1}{B_t^{i,s}}$, where $B_t^{i,s} \equiv \psi w_t N_t^i$ is sector i's bond supply. Todorov (2020) estimates that the ECB's initial announcements on its corporate sector purchase program (CSPP) in March and April 2016 lowered yields of eligible bonds by a combined 52 basis points (bp). We thus target $4 \times d(R_i) = 0.5\%$, $i = f, g$, in our quarterly model.[29] Since its implementation, the CSPP has absorbed approximately 30% of eligible bonds.[30] We hence target $d(B_t^{i,cb})/B_t^{i,s} = 0.3$. Given

[28] Strictly speaking, the absence of trade-offs in the second phase of the green transition requires the absence of relative price distortions once that phase starts: $\Delta_{t^*-1} = 1$, where t^* denotes the time at which the central banks' short-selling constraint ceases to bind. The first phase ($t < t^*$) will typically involve some transitory nonzero inflation and hence some price dispersion. However, in our numerical simulations Δ_{t^*-1} is indistinguishable from 1.

[29] Todorov's (2020) study does not distinguish between green and brown issuers. However, since the original CSPP annnouncements did not make any explicit distinction between green and brown issuers, it is plausible to assume that the yield impact was similar for both types of issuers.

[30] ICMA estimates a universe of CSPP eligible bonds at end June 2022 with a nominal value of EUR 1,250 bn, which coupled with ECB holdings data implies that 28% of eligible bonds were held by the program.

the value of each sector's initial bond supply ($B_0^{i,s}$, $i = f, g$) implied by our calibration, we then obtain $\kappa_f = 0.024$ and $\kappa_g = 0.160$.

We calibrate the minimum level of bond absorption by private investors, $\{\bar{B}^i\}_{i=f,g}$, by targeting euro area energy sector bond spreads before the introduction of the CSPP. We use Bloomberg data as of 31 December 2015 on yields of all outstanding bonds issued by firms in the energy and utilities sectors,[31] and calculate each bond's spread relative to the same-maturitiy OIS rate. We define as "green" and "brown" bonds those issued by firms with emissions intensity (greenhouse gas emissions per sales) above and below the median, respectively. This results in average spreads somewhat below 1.5% for both bond types.[32] Based on this, we target $4 \times (R_i - 1) = 1.5\%$, $i = f, g$, in our quarterly model. Using equation (26), we then solve for $\bar{B}^i = B_t^i - (R_t^i - 1)/\kappa_i$, $i = f, g$.

Finally, using also Bloomberg data on wage costs and bond debt outstanding for the same euro area energy sector issuers, we set the leverage factor ψ to 5.[33]

The last panel of Table 1 displays the calibrated values of these new parameters.

Results. Figures 4 and 5 compare, for the extended model with QE, the dynamics under optimal carbon taxes (equivalently the social-planner allocation, in the case of the real variables; blue lines) and under the slow green transition, both in a climate-oblivious scenario with strict inflation targeting and no QE (red lines), and the optimal climate-conscious monetary policy (dashed yellow lines).

As shown in Figure 4, and in accordance with the analytical results in the previous subsection, the optimal monetary policy involves full green QE (such that green bond spreads are eliminated) and two different phases for brown QE. For most of the transition, the gap between actual and optimal carbon taxation is too big to be compensated for by green tilting of QE. The best the central bank can do is not to purchase any brown bonds and let their spread reach its maximum level (the one consistent with private investors absorbing the entire supply). In this first phase, the central bank implements 100% green tilting. Once the carbon tax gap becomes sufficiently small, the central bank starts purchasing brown bonds, in the amount necessary in order for fossil energy prices to exactly replicate their optimal level. This second phase, which lasts only a few quarters, is therefore characterized by partial green tilting. Once optimal carbon taxation is reached, green tilting stops and the central bank implements full brown QE too in order to eliminate brown bond spreads.

Figure 5 shows the corresponding macroeconomic dynamics. Compared to the baseline model (Figure 3), the main difference is that optimal QE allows the central bank to accelerate the green transition somewhat: compared to the zero inflation, no QE policy, fossil energy consumption reaches its socially efficient path six quarters earlier. Figure 4 reveals that this is not the result

[31] We include the following sectors (in Bloomberg's classification): Coal Operations, Exploration and Production or Integrated Oils, Oil and Gas Services and Equipment, Pipeline, Refining and Marketing, Renewable Energy, Power Generation and Utilities.

[32] In particular, 1.44% for green bonds, and 1.26% for brown bonds.

[33] In the model, ψ is the ratio between the value of outstanding bonds ($\psi w_t N_t^i$) and personnel costs ($w_t N_t^i$). We calculate the ratio of bond debt outstanding and personnel expenditures for the sample of bond-issuing energy firms as a whole (equivalently, the weighted average of the same ratio across firms, using as weights each firm's share of total personnel costs), which yields a ratio of 5.05. Notice that we have assumed a common leverage factor for green ($i = g$) and fossil energy firms ($i = f$). As it turns out, this is innocuous: using the same emissions intensity-based classification as before, we find almost identical leverage factors for green (5.01) and brown issuers (5.06).

of more expensive fossil energy. Instead, the elimination of green bond spreads implies cheaper green energy, leading final goods producers to substitute away from fossil energy and into clean one. However, the reduction in fossil energy use is again too small to make much of a difference for atmospheric carbon concentration. Finally, the optimal departure from strict inflation targeting is again minimal, as the central bank avoids a more forceful increase in nominal interest rate and a larger drop in output below its natural level

Why is green tilting of QE relatively ineffective at lowering carbon emissions? In our model, the effectiveness of green tilting depends on the size of energy firms' bond spreads in the absence of QE, which determines the extent to which the relative financing conditions of green vs brown energy firms can be affected by climate-oriented asset purchases. Our calibration is consistent with the fact that the ECB, and most other major central banks, restrict their purchases to bonds with high credit quality, as reflected e.g. in investment-grade rating. As a result, the spreads of bonds eligible under the CSPP and similar corporate bond purchase programs are relatively small to begin with, even in the absence of QE. This substantially limits the scope of green QE tilting for altering brown vs green firms' financing conditions and hence the relative price of dirty vs clean energy.

Figure 4: Transitions in green QE model: spreads

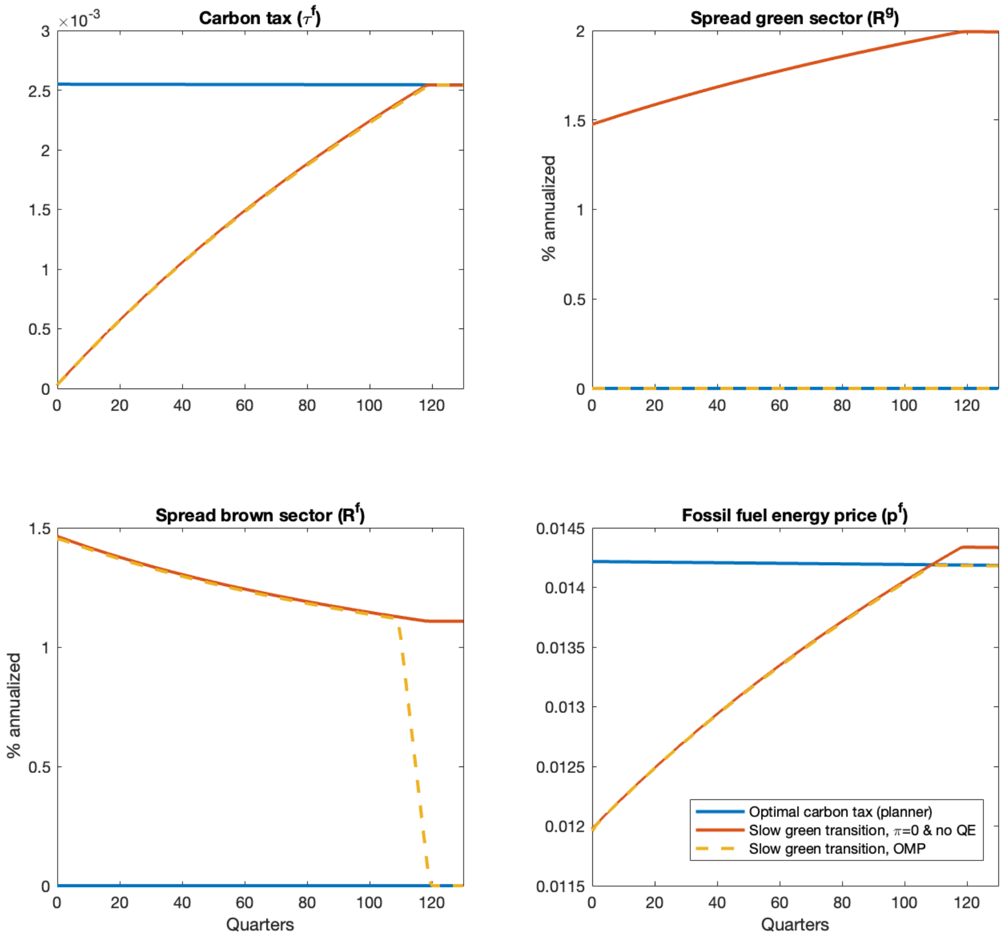
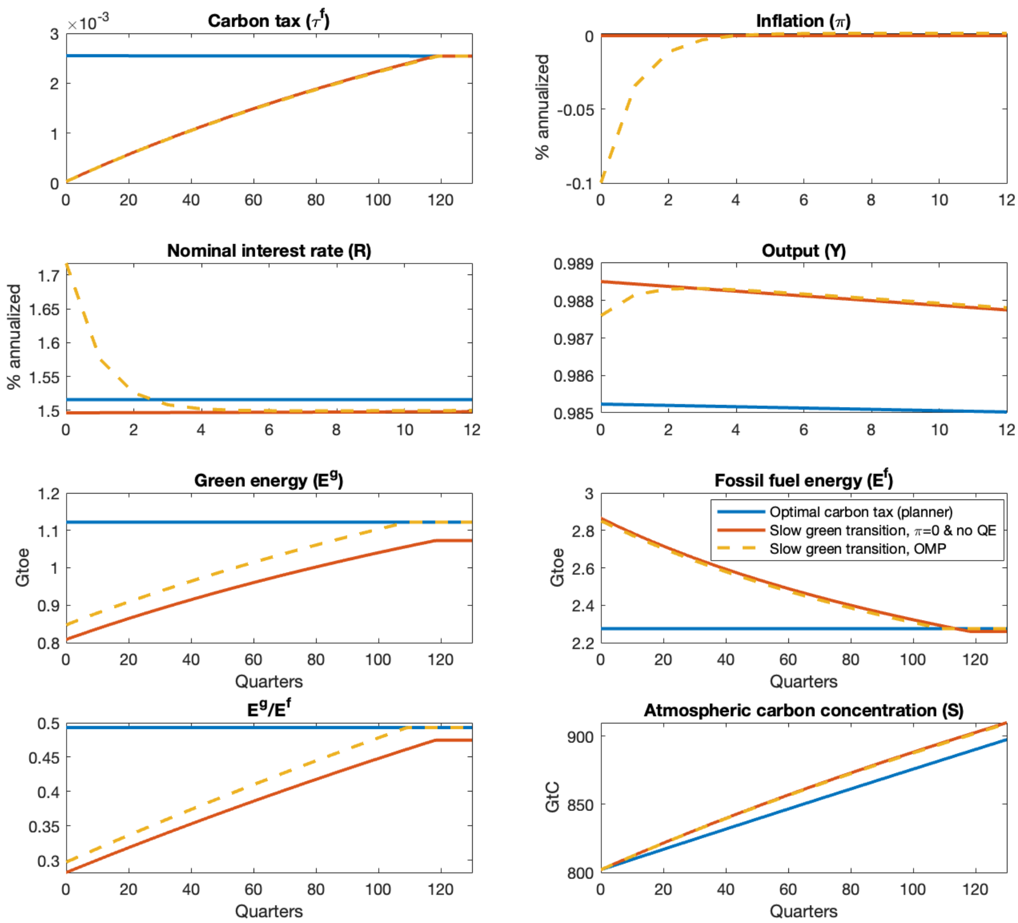


Figure 5: Transitions in green QE model: macro



6 Conclusions and directions for future research

This paper has provided a normative analysis of monetary policy in a canonical New Keynesian model with climate change externalities and carbon taxation. Our model is deliberately simple, with the aim of clarifying as transparently as possible the trade-offs that monetary policy faces between core and climate goals, given its tools and the path of other mitigating measures, such as carbon taxes. Thus, some caveats are in order, which also suggest directions for further research.

First, we have treated the World economy as a single jurisdiction in terms of both climate and monetary policies. While some international coordination of climate policies exists in practice (notably through the annual United Nations Conference of Parties), progress is far from being perfectly homogeneous. Therefore, it would be interesting to extend this analysis to a multi-region framework with asymmetric policies.

Second, we assume exogenous production technologies, implying fixed (and low, under reasonable calibrations) elasticities of substitution between different inputs. In models with directed technical change, elasticities of substitution are low in the short run but potentially high in the long run (see e.g. Hassler, Krusell and Olovsson, 2021). While our main results would largely hold in a model with this feature, given the short-term nature of the monetary policy trade-offs, allowing for endogenous technical change is likely to be important under scenarios of severe procrastination by climate authorities, in which the time needed for carbon taxes to reach their socially optimal level extends far beyond the horizon considered here.

Finally, in our green transition scenarios carbon taxes and CO2 emissions eventually converge to their socially optimal level, where social optimality is defined within the internal logic of our model, which balances the welfare benefits from lower carbon emissions against the costs from reduced economic activity and consumption. In this sense, the path of carbon emissions in our model need not coincide with the target paths set out in the Paris Agreement, which requires achieving net zero emissions by 2050. Therefore, it would be instructive to analyze how monetary policy would optimally respond in scenarios where carbon pricing evolves in a way that is consistent with the above targets.

References

Airaudo, Florencia, Evi Pappa and Hernan D. Seoane. (2023). "The green metamorphosis of a small open economy". CEPR Discussion Paper Series, 17863, Centre for Economic Policy Research. https://cepr.org/publications/dp17863

Andrés, Javier, J. David López-Salido and Edward Nelson. (2004). "Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium". *Journal of Money, Credit, and Banking*, 36(4), pp. 665-690. http://doi.org/10.1353/mcb.2004.0061

Angelopoulos, K., G. Economides and A. Philippopoulos. (2013). "First-and second-best allocations under economic and environmental uncertainty". *International Tax and Public Finance*, 20, pp. 360-380. http://doi.org/10.1007/s10797-012-9234-z

Annicchiarico, Barbara, and Fabio Di Dio. (2015). "Environmental policy and macroeconomic dynamics in a new Keynesian model". *Journal of Environmental Economics and Management*, 69, pp. 1-21. http://doi.org/10.1016/j.jeem.2014.10.002

Bank of England. (2021). *Bank of England publishes its approach to greening the Corporate Bond Purchase Scheme* [Press Release]. https://www.bankofengland.co.uk/news/2021/november/boe-publishes-its-approach-to-greening-the-corporate-bond-purchase-scheme

Barrage, Lint. (2020). "Optimal Dynamic Carbon Taxes in a Climate-Economy Model with Distortionary Fiscal Policy". *The Review of Economic Studies*, 87(1), pp. 1-39. http://doi.org/10.1093/restud/rdz055

Benigno, Pierpaolo, and Michael Woodford. (2005). "Inflation Stabilization and Welfare: The Case of a Distorted Steady State". *Journal of the European Economic Association*, 3(6), pp. 1185-1236. http://doi.org/10.1162/1542476057775012914

Benmir, Ghassane, and Josselin Roman. (2020). "Policy interactions and the transition to clean technology". Grantham Research Institute on Climate Change and the Environment Working Paper, 337, Grantham Research Institute on Climate Change and the Environment. https://www.lse.ac.uk/granthaminstitute/wp-content/uploads/2020/04/working-paper-337-Benmir-Roman-Jul21.pdf

Calvo, Guillermo A. (1983). "Staggered prices in a utility-maximizing framework". *Journal of Monetary Economics*, 12(3), pp. 383-398. http://doi.org/10.1016/0304-3932(83)90060-0

Chen, Han, Vasco Cúrdia and Andrea Ferrero. (2012). "The macroeconomic effects of large-scale asset purchase programmes". *The Economic Journal*, 122(564), pp. F289-F315. http://doi.org/10.1111/j.1468-0297.2012.02549.x

Diluiso, Francesca, Barbara Annicchiarico, Matthias Kalkuhl and Jan Christoph Minx. (2020). "Climate Actions and Stranded Assets: The Role of Financial Regulation and Monetary Policy". CESifo Working Paper, 8486, Munich Society for the Promotion of Economic Research - CESifo. http://doi.org/10.2139/ssrn.3676092

European Central Bank. (2022). *ECB takes further steps to incorporate climate change into its monetary policy operations* [Press release]. https://www.ecb.europa.eu/press/pr/date/2022/html/ecb.pr220704~4f48a72462.en.html

Ferrari, Alessandro, and Valerio Nispi Landi. (2022). "Will the green transition be inflationary? Expectations matter". ECB Working Paper, 2726, European Central Bank. http://doi.org/10.2139/ssrn.4226354

Ferrari, Alessandro, and Valerio Nispi Landi. (2023). "Whatever it takes to save the planet? Central banks and unconventional green policy". *Macroeconomic Dynamics*, pp. 1-26. http://doi.org/10.1017/S1365100523000032

Ferrari, Alessandro, and Valerio Nispi Landi. (2023). "Toward a green economy: the role of central bank's asset purchases". ECB Working Paper, 2779, European Central Bank. http://doi.org/10.2139/ssrn.4357535

Ferrari, Massimo Minesso, and Maria Sole Pagliari. (2021). "No Country is an Island. International Cooperation and Climate Change". Banque de France Working Paper, 815, Banque de France. http://doi.org/10.2139/ssrn.3868947

Fischer, Carolyn, and Michael Springborn. (2011). "Emissions targets and the real business cycle: Intensity targets versus caps or taxes". *Journal of Environmental Economics and Management*, 62(3), pp. 352-366. https://doi.org/10.1016/j.jeem.2011.04.005

Gertler, Mark, and Peter Karadi. (2013). "QE 1 vs. 2 vs. 3...: A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool". *International Journal of Central Banking*, January, pp. 5-53. https://www.ijcb.org/journal/ijcb13q0a1.pdf

Golosov, Mikhail, John Hassler, Per Krusell and Aleh Tsyvinski. (2014). "Optimal taxes on fossil fuel in general equilibrium". *Econometrica*, 82(1), pp. 41-88. http://hassler-j.iies.su.se/PAPERS/ecta.pdf

Hansen, Lars Peter. (2022). "Central banking challenges posed by uncertain climate change and natural disasters". *Journal of Monetary Economics*, 125, pp. 1-15. http://doi.org/10.1016/j.jmoneco.2021.09.010

Hassler, John, Per Krusell and Conny Olovsson. (2022). "Directed Technical Change as a Response to Natural Resource Scarcity". *Journal of Political Economy*, 129(11), pp. 3039-3072. http://doi.org/10.1086/715849

Heutel, G. (2012). "How should environmental policy respond to business cycles? Optimal policy under persistent productivity shocks". *Review of Economic Dynamics*, 15(2), pp. 244-264. http://doi.org/10.1016/j.red.2011.05.002

Intergovernmental Panel on Climate Change. (2006). *Guidelines for National Greenhouse Gas Inventories: Vol. 2: Energy.* Institute for Global Environmental Strategies. https://www.ipcc-nggip.iges.or.jp/public/2006gl/vol2.html

Intergovernmental Panel on Climate Change. (2021). *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change.* Cambridge University Press. https://doi.org/10.1017/9781009157896

International Monetary Fund. (2022). "Near-Term Macroeconomic Impact of Decarbonization Policies". En International Monetary Fund, *World Economic Outlook*, October. https://www.elibrary.imf.org/display/book/9798400218439/CH003.xml

International Renewable Energy Agency. (2020). *Renewable Power Generation Costs in 2019.* https://www.irena.org/publications/2020/Jun/Renewable-Power-Costs-in-2019

Lagarde, C. (2021). *Climate change and central banking: keynote speech by the President of the ECB, at the ILF conference on Green Banking and Green Central Banking* [Press release]. https://www.ecb.europa.eu/press/key/date/2021/html/ecb.sp210125~f87e826ca5.en.html

Nordhaus, William D. (2008). *A Question of Balance: Weighing the Options on Global Warming Policies.* Yale University Press. http://doi.org/10.12987/9780300165982

Olovsson, Conny, and David Vestin. (2023). "Greenflation?". Sveriges Riksbank Working Paper Series, 420, Sveriges Riksbank. https://www.riksbank.se/globalassets/media/rapporter/working-papers/2023/no.-420-greenflation.pdf

Papageorgiou, Chris, Marianne Saam and Patrick Schulte. (2017). "Substitution between Clean and Dirty Energy Inputs: A Macroeconomic Perspective". *The Review of Economics and Statistics,* 99(2), pp. 281-290. http://doi.org/10.1162/REST_a_00592

Papoutsi, Melina, Monika Piazzesi and Martin Schneider. (2023). *How unconventional is green monetary policy.* https://web.stanford.edu/~piazzesi/How_unconventional_is_green_monetary_policy.pdf

Powell, Jerome H. (2023). *Panel on "Central Bank Independence and the Mandate-Evolving Views": remarks by the chair Board of Governors of the Federal Reserve System at the Symposium on Central Bank Independence Sveriges Riksbank Stockholm, Sweden.* Bank for International Settlements. https://www.bis.org/review/r230111a.pdf

Todorov, Karamfil. (2020). "Quantify the quantitative easing: Impact on bonds and corporate debt issuance". *Journal of Financial Economics,* 135(2), pp. 340-358. http://doi.org/10.1016/j.jfineco.2019.08.003

Van der Ploeg, Frederick, and Cees Withagen. (2012). "Is there really a green paradox?". *Journal of Environmental Economics and Management,* 64(3), pp. 342-363. http://doi.org/10.1016/j.jeem.2012.08.002

Woodford, Michael. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press. https://www.jstor.org/stable/j.ctv30pnvmf

Yun, T. (2005). "Optimal Monetary Policy with Relative Price Distortions". *American Economic Review,* 95(1), pp. 89-109. http://doi.org/10.1257/0002828053828653

Appendix

A. Social planner equilibrium

The Lagrangian of the social planner's problem is given by

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \{ \log(C_t) - \frac{\chi}{1+\varphi} \left(N_t^y + \sum_{i=g,f} N_t^i \right)^{1+\varphi} \\
&+\lambda_t^y \left[\left[1 - D\left(S_t\right)\right] A_t F\left(N_t^y, \mathbf{E}(E_t^g, E_t^f)\right) - C_t \right] \\
&+ \sum_{i=g,f} \lambda_t^i [A_t^i N_t^i - E_t^i] + \zeta_t [S_t - \sum_{s=0}^{t+T} \left(1 - d_s\right) \xi E_{t-s}^f] \}.
\end{aligned}$$

The first-order conditions with respect to $C_t, N_t^y, \{N_t^i\}_{i=f,g}, E_t^g, E_t^f$ and S_t are given respectively by

$$\lambda_t^y = 1/C_t, \tag{30}$$

$$\lambda_t^y \left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial N_t^y} = \chi N_t^{\varphi}, \tag{31}$$

$$\lambda_t^i A_t^i = \chi N_t^{\varphi}, \quad i = g, f, \tag{32}$$

$$\lambda_t^y \left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^g} = \lambda_t^g. \tag{33}$$

$$\lambda_t^y \left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^f} = \lambda_t^f + \mathbb{E}_t \{ \sum_{s=0}^{\infty} \beta^s \left(1 - d_s\right) \xi \zeta_{t+s} \}. \tag{34}$$

$$\zeta_t = \lambda_t^y D'\left(S_t\right) A_t F\left(N_t^y, \mathbf{E}(E_t^g, E_t^f)\right). \tag{35}$$

Combining all the above conditions, and using $D'\left(S_t\right) = \gamma_t \left[1 - D\left(S_t\right)\right]$, we obtain

$$\left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial N_t^y} = \chi N_t^{\varphi} C_t,$$

$$\left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^g} = \frac{\chi N_t^{\varphi} C_t}{A_t^g},$$

$$\begin{aligned}
\left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^f} &= \frac{\chi N_t^{\varphi} C_t}{A_t^f} + C_t \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(1 - d_s\right) \xi \frac{Y_{t+s}}{C_{t+s}} \gamma_{t+s} \\
&= \frac{\chi N_t^{\varphi} C_t}{A_t^f} + Y_t \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(1 - d_s\right) \xi \gamma_{t+s},
\end{aligned}$$

where the second equality uses the fact that $C_t = Y_t$ for all t.

B. General optimal monetary policy problem and proof of Proposition 1

Let $p_t^* \equiv P_t^*/P_t, \pi_t \equiv P_t/P_{t-1}$. Using (6), we can then write firms' optimal price decision (equation 7) as

$$p_t^* = \frac{V_t^{num}}{V_t^{den}},$$

where

$$\begin{aligned} V_t^{num} &\equiv \sum_{t=0}^{\infty} (\beta\theta)^s \mathbb{E}_t \left\{ \left(\frac{P_{t+s}}{P_t} \right)^{\epsilon} \frac{\epsilon / (\epsilon - 1)}{1 + \tau^y} mc_{t+s} \right\} \\ &= \frac{\epsilon / (\epsilon - 1)}{1 + \tau^y} mc_t + \beta\theta \mathbb{E}_t \left\{ \pi_{t+1}^{\epsilon} V_{t+1}^{num} \right\}. \end{aligned}$$

$$V_t^{den} \equiv \sum_{t=0}^{\infty} (\beta\theta)^s \mathbb{E}_t \left\{ \left(\frac{P_{t+s}}{P_t} \right)^{\epsilon - 1} \right\} = 1 + \beta\theta \mathbb{E}_t \left\{ \pi_{t+1}^{\epsilon-1} V_{t+1}^{den} \right\}.$$

The Lagrangian of the Ramsey optimal monetary policy problem is

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \{ \log(Y_t) - \frac{\chi}{1+\varphi} \left(\sum_{i=y,g,f} N_t^i \right)^{1+\varphi} \\ &+ \lambda_t^y \left[[1 - D(S_t)] A_t F \left(N_t^y, \mathbf{E}(E_t^g, E_t^f) \right) - \Delta_t Y_t \right] \\ &+ \sum_{i=g,f} \lambda_t^i [A_t^i N_t^i - E_t^i] + \zeta_t [S_t - \sum_{s=0}^{t+T} (1 - d_s) \xi E_{t-s}^f] \\ &+ \lambda_t^w \left[\chi \left(\sum_{i=y,g,f} N_t^i \right)^{\varphi} Y_t - w_t \right] + \lambda_t^{p^f} \left[w_t / A_t^f + \tau_t^f - p_t^f \right] \\ &+ \lambda_t^{p^g} [w_t / A_t^g - p_t^g] + \lambda_t^N \left[mc_t [1 - D(S_t)] A_t \frac{\partial F(\cdot)}{\partial N_{z,t}} - w_t \right] \\ &+ \sum_{i=g,f} \lambda_t^{E^i} \left[mc_t [1 - D(S_t)] A_t \frac{\partial F(\cdot)}{\partial E_{z,t}^i} - p_t^i \right] \\ &+ \lambda_t^{\Delta} \left[\theta \pi_t^{\epsilon} \Delta_{t-1} + (1-\theta)(p_t^*)^{-\epsilon} - \Delta_t \right] \\ &+ \lambda_t^{\pi} \left[(1-\theta)(p_t^*)^{1-\epsilon} + \theta \pi_t^{\epsilon-1} - 1 \right] + \lambda_t^{p^*} \left[V_t^{num} / V_t^{den} - p_t^* \right] \\ &+ \lambda_t^{num} \left[\frac{\epsilon / (\epsilon - 1)}{1 + \tau^y} mc_t + \beta\theta E_t \pi_{t+1}^{\epsilon} V_{t+1}^{num} - V_t^{num} \right] \\ &+ \lambda_t^{den} \left[1 + \beta\theta E_t \pi_{t+1}^{\epsilon-1} V_{t+1}^{den} - V_t^{den} \right] \}. \end{aligned}$$

The first-order conditions are

$$\frac{1}{Y_t} + \lambda_t^w \chi \left(\sum_{i=y,g,f} N_t^i \right)^{\varphi} = \lambda_t^y \Delta_t, \tag{Y_t}$$

$$\begin{aligned} \chi N_t^{\varphi} &= \lambda_t^y [1 - D(S_t)] A_t \frac{\partial F(\cdot)}{\partial N_t^y} + \lambda_t^w \chi \varphi N_t^{\varphi - 1} Y_t \\ &+ \left[\lambda_t^N \frac{\partial^2 F(\cdot)}{\partial N_{z,t}^2} + \sum_{i=g,f} \lambda_t^{E^i} \frac{\partial^2 F(\cdot)}{\partial E_{z,t}^i \partial N_t^y} \right] [1 - D(S_t)] A_t, \end{aligned} \tag{36}$$

$$\chi N_t^{\varphi} = \lambda_t^i A_t^i + \lambda_t^w \chi \varphi N_t^{\varphi - 1} Y_t, \tag{$N_t^i, i = f, g$}$$

$$0 = \lambda_t^y [1 - D(S_t)] A_t \frac{\partial F(\cdot)}{\partial E_t^i} - \lambda_t^i - \mathbf{1}_{i=f} \mathbb{E}_t \{ \sum_{s=0}^{\infty} \beta^s (1 - d_s) \xi \zeta_{t+s} \} \tag{37}$$

$$+\left[\lambda_t^N \frac{\partial F^2(\cdot)}{\partial N_{z,t}\partial E_t^i} + \lambda_t^{E^i}\frac{\partial F^2(\cdot)}{\partial\left(E_{z,t}^i\right)^2} + \lambda_t^{E^{j\neq i}}\frac{\partial F^2(\cdot)}{\partial E_{z,t}^i \partial E_{z,t}^{j\neq i}}\right][1-D(S_t)]A_t,$$

$$\zeta_t = \lambda_t^y D'(S_t) A_t F(\cdot) + \left[\lambda_t^N \frac{\partial F(\cdot)}{\partial N_{z,t}} + \sum_{i=g,f}\lambda_t^{E^i}\frac{\partial F(\cdot)}{\partial E_{z,t}^i}\right] mc_t D'(S_t) A_t, \tag{S_t}$$

$$\lambda_t^w + \lambda_t^N = \sum_{i=g,f}\lambda_t^{p^i}/A_t^i, \tag{w_t}$$

$$0 = \lambda_t^{p^i} + \lambda_t^{E^i}, \tag{$p_t^i, i=f,g$}$$

$$0 = \left[\lambda_t^N \frac{\partial F(\cdot)}{\partial N_{z,t}} + \sum_{i=g,f}\lambda_t^{E^i}\frac{\partial F(\cdot)}{\partial E_{z,t}^i}\right][1-D(S_t)]A_t + \lambda_t^{num}\frac{\epsilon/(\epsilon-1)}{1+\tau^y}, \tag{mc_t}$$

$$\lambda_t^y Y_t = \beta\theta\mathbb{E}_t\left\{\lambda_{t+1}^{\Delta}\pi_{t+1}^{\epsilon}\right\} - \lambda_t^{\Delta}, \tag{Δ_t}$$

$$\lambda_t^{p^*} = \lambda_t^{\Delta}(1-\theta)(-\epsilon)(p_t^*)^{-\epsilon-1} + \lambda_t^{\pi}(1-\theta)(1-\epsilon)(p_t^*)^{-\epsilon}, \tag{p_t^*}$$

$$0 = \lambda_t^{\Delta}\theta\epsilon\pi_t^{\epsilon-1}\Delta_{t-1} + \lambda_t^{\pi}\theta(\epsilon-1)\pi_t^{\epsilon-2} + \lambda_{t-1}^{num}\theta\epsilon\pi_t^{\epsilon-1}V_t^{num} + \lambda_{t-1}^{den}\theta(\epsilon-1)\pi_t^{\epsilon-2}V_t^{den}, \tag{π_t}$$

$$\lambda_t^{num} = \lambda_t^{p^*}\frac{1}{V_t^{den}} + \lambda_{t-1}^{num}\theta\pi_t^{\epsilon}, \tag{V_t^{num}}$$

$$\lambda_t^{den} = -\lambda_t^{p^*}\frac{V_t^{num}}{\left(V_t^{den}\right)^2} + \lambda_{t-1}^{den}\theta\pi_t^{\epsilon-1}. \tag{V_t^{den}}$$

Proof of Proposition 1. We now show that, under the maintained assumption that equation (21) holds, and provided (22) is satisfied, the zero inflation equilibrium satisfies the above conditions. Let $\pi_t = p_t^* = \Delta_t = \frac{V_t^{num}}{V_t^{den}} = mc_t = 1$ for all t. Then the first-order conditions become

$$\frac{1}{Y_t} + \lambda_t^w\chi\left(\sum_{i=y,g,f}N_t^i\right)^{\varphi} = \lambda_t^y, \tag{38}$$

$$\chi N_t^{\varphi} = \lambda_t^y[1-D(S_t)]A_t\frac{\partial F(\cdot)}{\partial N_t^y} + \lambda_t^w\chi\varphi N_t^{\varphi-1}Y_t \tag{39}$$

$$+\left[\lambda_t^N\frac{\partial^2 F(\cdot)}{\partial N_{z,t}^2} + \sum_{i=g,f}\lambda_t^{E^i}\frac{\partial^2 F(\cdot)}{\partial E_{z,t}^i\partial N_t^y}\right][1-D(S_t)]A_t,$$

$$\chi N_t^{\varphi} = \lambda_t^i A_t^i + \lambda_t^w\chi\varphi N_t^{\varphi-1}Y_t, \tag{40}$$

$$0 = \lambda_t^y[1-D(S_t)]A_t\frac{\partial F(\cdot)}{\partial E_t^i} - \lambda_t^i - \mathbf{1}_{i=f}\mathbb{E}_t\{\sum_{s=0}^{\infty}\beta^s(1-d_s)\xi\zeta_{t+s}\} \tag{41}$$

$$+\left[\lambda_t^N \frac{\partial F^2(\cdot)}{\partial N_{z,t}\partial E_t^i} + \lambda_t^{E^i}\frac{\partial F^2(\cdot)}{\partial\left(E_{z,t}^i\right)^2} + \lambda_t^{E^{j\neq i}}\frac{\partial F^2(\cdot)}{\partial E_{z,t}^i \partial E_{z,t}^{j\neq i}}\right][1-D(S_t)]A_t,$$

$$\zeta_t = \lambda_t^y D'(S_t) A_t F(\cdot) + \left[\lambda_t^N \frac{\partial F(\cdot)}{\partial N_{z,t}} + \sum_{i=g,f}\lambda_t^{E^i}\frac{\partial F(\cdot)}{\partial E_{z,t}^i}\right] D'(S_t) A_t, \tag{42}$$

$$\lambda_t^w + \lambda_t^N = \sum_{i=g,f} \lambda_t^{p^i} / A_t^i, \tag{43}$$

$$0 = \lambda_t^{p^i} + \lambda_t^{E^i}, \tag{44}$$

$$0 = \left[\lambda_t^N \frac{\partial F(\cdot)}{\partial N_{z,t}} + \sum_{i=g,f} \lambda_t^{E^i} \frac{\partial F(\cdot)}{\partial E_{z,t}^i} \right] [1 - D(S_t)] A_t + \lambda_t^{num}, \tag{45}$$

$$\lambda_t^y Y_t = \beta\theta E_t \lambda_{t+1}^{\Delta} - \lambda_t^{\Delta}, \tag{46}$$

$$\lambda_t^{p^*} = -(1-\theta) \left[\lambda_t^{\Delta} \epsilon + \lambda_t^{\pi} (\epsilon - 1) \right],$$

$$0 = \theta \left[\lambda_t^{\Delta} \epsilon + \lambda_t^{\pi} (\epsilon - 1) \right] + \lambda_{t-1}^{num} \theta \epsilon V_t^{num} + \lambda_{t-1}^{den} \theta (\epsilon - 1) V_t^{den},$$

$$\lambda_t^{num} = \lambda_t^{p^*} \frac{1}{V_t^{den}} + \lambda_{t-1}^{num} \theta,$$

$$\lambda_t^{den} = -\lambda_t^{p^*} \frac{1}{V_t^{den}} + \lambda_{t-1}^{den} \theta.$$

It is trivial to show that the solution to the last four equations is

$$\lambda_t^{p^*} = \lambda_t^{num} = \lambda_t^{den} = 0,$$

$$\lambda_t^{\pi} = -\lambda_t^{\Delta} \frac{\epsilon}{\epsilon - 1}. \tag{47}$$

We conjecture that

$$\lambda_t^w = \lambda_t^N = \lambda_t^{p^i} = \lambda_t^{E^i} = 0,$$

for $i = f, g$. It is trivial to show that equations (43) to (45) are then satisfied. Also, equations (38) to (42) become

$$\lambda_t^y = \frac{1}{Y_t}, \tag{48}$$

$$\chi N_t^{\varphi} = \lambda_t^y [1 - D(S_t)] A_t \frac{\partial F(\cdot)}{\partial N_t^y},$$

$$\chi N_t^{\varphi} = \lambda_t^i A_t^i, \quad i = g, f$$

$$\lambda_t^i + \mathbf{1}_{i=f} E_t \{ \sum_{s=0}^{\infty} \beta^s (1 - d_s) \xi \zeta_{t+s} \} = \lambda_t^y [1 - D(S_t)] A_t \frac{\partial F(\cdot)}{\partial E_t^i}, \quad i = g, f \tag{49}$$

$$\zeta_t = \lambda_t^y D'(S_t) A_t F(\cdot).$$

The latter seven equations are identical to equations (30) to (35) in the social planner problem. Using (46) and (48), we can solve for λ_t^{Δ} as follows,

$$\lambda_t^{\Delta} = \beta\theta E_t \lambda_{t+1}^{\Delta} - 1 = \frac{-1}{1 - \beta\theta},$$

such that, from (47), we obtain

$$\lambda_t^{\pi} = \frac{1}{1 - \beta\theta} \frac{\epsilon}{\epsilon - 1}.$$

This completes our proof.

C. Calibration procedure

Assume that the economy is initially characterized by a zero inflation policy, such that, under our maintained assumption that there is no initial price dispersion ($\Delta_0 = 1$), it replicates the flexible-price equilibrium. Under our additional maintained assumption that sales subsidy offsets the monopolistic distortion, $(1+\tau^y)\frac{\epsilon-1}{\epsilon} = 1$, it follows that $mc_0 = (1+\tau^y)\frac{\epsilon-1}{\epsilon} = 1$. Assume also no carbon taxation initially ($\tau_0^f = 0$). Assume N_0 is normalized to 1, such that $w_0 = \chi(1)^{\varphi} C_0 = \chi Y_0$. Then the equilibrium conditions (19) and (20) become

$$\frac{\chi Y_0}{A_0^i} = \alpha\omega_i \frac{Y_0}{E_0}\left(\frac{E_0}{E_0^i}\right)^{1-\rho} \Leftrightarrow \frac{\chi}{A_0^i} = \frac{\alpha\omega_i}{E_0}\left(\frac{E_0}{E_0^i}\right)^{1-\rho}, \tag{50}$$

$i = f, g$, with $\omega_g = \omega = 1-\omega_f$, where E_0 is computed from equation (23) evaluated at the target levels for $E_0^i, i = f, g$. Equation (50) can be used to solve for $\{A_0^i\}_{i=f,g}$ as

$$A_0^i = \frac{\chi E_0}{\alpha\omega_i}\left(\frac{E_0^i}{E_0}\right)^{1-\rho}.$$

We can then solve for $\{N_0^i\}_{i=f,g}$ as $N_0^i = E_0^i/A_0^i, i = f, g$. Total energy sector labor then equals

$$\sum_{i=f,g} N_0^i = \frac{\alpha}{\chi}\sum_{i=f,g}\omega_i\left(\frac{E_0^i}{E_0}\right)^{\rho} = \frac{\alpha}{\chi},$$

where the second equality follows from (23). The equilibrium condition (18) can be expressed as

$$\chi Y_0 = (1-\alpha)\frac{Y_0}{N_0^y} \Leftrightarrow N_0^y = \frac{1-\alpha}{\chi}. \tag{51}$$

Total labor input then equals $N_0^y + \sum_{i=f,g} N_0^i = 1/\chi$, which equals 1 (as per our normalization) if and only if $\chi = 1$. Finally, we solve for initial output as

$$Y_0 = [1 - D(S_0)]\, A_0 F\left(N_0^y, E_0\right),$$

for given initial values of atmospheric carbon concentration (S_0) and TFP in final goods production (A_0, which we normalize to 1).

D. Optimal monetary policy problem with QE

Compared to the Lagrangian in the baseline model (Appendix B), the Lagrangian in the model with QE changes as follows,

$$\begin{aligned}&\sum_{t=0}^{\infty}\beta^t E_0\{\ldots + \sum_{i=f,g}\lambda_t^{p^i}\left[\frac{[1+\psi(R_t^i-1)]w_t}{A_t^i} + \mathbf{1}_{i=f}\tau_t^f - p_t^i\right]\\ &-\sum_{i=f,g}\mu_t^{R^i_{\min}}\left[1-R_t^i\right] - \sum_{i=f,g}\mu_t^{R^i_{\max}}\left[R_t^i - 1 - \kappa_i\left(\psi w_t N_t^i - \bar{B}^i\right)\right]\},\end{aligned}$$

where $\mu_t^{R^i_{\min}}$ and $\mu_t^{R^i_{\min}}$ are the Kuhn-Tucker multipliers associated to the inequality constraints $1 \le R_t^i$ and (28), respectively. The FOCs wrt $\{N_t^i\}_{i=f,g}$ and w_t are now given by,

$$\chi N_t^{\varphi} = \lambda_t^i A_t^i + \lambda_t^w \chi\varphi N_t^{\varphi-1} Y_t + \mu_t^{R_{\max}^i} \kappa_i \psi w_t, \qquad (N_t^i, i = f, g)$$

$$\lambda_t^w + \lambda_t^N = \sum_{i=g,f} \lambda_t^{p^i} \frac{1 + \psi(R_t^i - 1)}{A_t^i} + \sum_{i=f,g} \mu_t^{R_{\max}^i} \kappa_i \psi N_t^i, \qquad (w_t)$$

whereas the new FOCs wrt $\{R_t^i\}_{i=f,g}$ read

$$0 = \lambda_t^{p^i} \frac{\psi w_t}{A_t^i} + \mu_t^{R_{\min}^i} - \mu_t^{R_{\max}^i}, \qquad (R_t^i, i = f, g)$$

$i = f, g$. Under the solution for corporate interest rates conjecture above, i.e. $R_t^g = 1$ and equation (29), the constraint $1 \leq R_t^g$ binds in periods in which the first-best equilibrium cannot be replicated, such that $\mu_t^{R_{\min}^g} > 0$, and therefore the constraint (28) for $i = g$ is slack, such that $\mu_t^{R_{\max}^g} = 0$. It follows that

$$\mu_t^{R_{\min}^g} = -\lambda_t^{p^g} \frac{\psi w_t}{A_t^g} > 0 \Leftrightarrow \lambda_t^{p^g} < 0,$$

which has to be verified ex post. Once the first-best equilibrium becomes feasible, it is still the case that $R_t^g = 1$, but this is actually an *interior* solution, because the central bank would *not* want to have a lower value of R_t^g even if that was feasible. Therefore, both $1 \leq R_t^g$ and constraint (28) for $i = g$ are slack after that, such that $\mu_t^{R_{\min}^g} = \mu_t^{R_{\max}^g} = 0$, which in turn requires $\lambda_t^{p^g} = 0$.

Regarding R_t^f, under our conjectured solution, the constraint $1 \leq R_t^f$ is always slack (such that $\mu_t^{R_{\min}^f} = 0$), whereas the constraint (28) for $i = f$ binds (such that $\mu_t^{R_{\max}^f} > 0$) in periods in which the carbon tax gap is large enough that the first-best allocation is not feasible. In those periods, it follows that

$$\mu_t^{R_{\max}^f} = \lambda_t^{p^f} \frac{\psi w_t}{A_t^f} > 0 \Leftrightarrow \lambda_t^{p^f} > 0,$$

which must also be verified ex post. Finally, in periods in which $\tau_t^{f*} - \tau_t^f$ is small enough that the first-best is feasible, the constraint (28) is slack too, such that $\mu_t^{R_{\max}^f} = 0$, which can only be true if $\lambda_t^{p^f} = 0$.

Summary. To summarize, in periods in which $\frac{1}{\psi} \frac{\tau_t^{f*} - \tau_t^f}{w_t / A_t^f} > \kappa_f(\psi w_t N_t^f - \bar{B}^f)$, we have

$$R^g = 1, R^f = \kappa_f(\psi w_t N_t^f - \bar{B}^f), \mu_t^{R_{\min}^g}, \mu_t^{R_{\max}^f} > 0, \mu_t^{R_{\max}^g} = \mu_t^{R_{\min}^f} = 0,$$

whereas in periods in which $\frac{1}{\psi} \frac{\tau_t^{f*} - \tau_t^f}{w_t / A_t^f} \leq \kappa_f(\psi w_t N_t^f - \bar{B}^f)$, we have

$$R^g = 1, R^f = \frac{1}{\psi} \frac{\tau_t^{f*} - \tau_t^f}{w_t / A_t^f}, \mu_t^{R_{\min}^g} = \mu_t^{R_{\max}^f} = \mu_t^{R_{\max}^g} = \mu_t^{R_{\min}^f} = 0.$$

BANCO DE ESPAÑA PUBLICATIONS

WORKING PAPERS

2215 JOSÉ MANUEL CARBÓ and SERGIO GORJÓN: Application of machine learning models and interpretability techniques to identify the determinants of the price of bitcoin.

2216 LUIS GUIROLA and MARÍA SÁNCHEZ-DOMÍNGUEZ: Childcare constraints on immigrant integration.

2217 ADRIÁN CARRO, MARC HINTERSCHWEIGER, ARZU ULUC and J. DOYNE FARMER: Heterogeneous effects and spillovers of macroprudential policy in an agent-based model of the UK housing market.

2218 STÉPHANE DUPRAZ, HERVÉ LE BIHAN and JULIEN MATHERON: Make-up strategies with finite planning horizons but forward-looking asset prices.

2219 LAURA ÁLVAREZ, MIGUEL GARCÍA-POSADA and SERGIO MAYORDOMO: Distressed firms, zombie firms and zombie lending: a taxonomy.

2220 BLANCA JIMÉNEZ-GARCÍA and JULIO RODRÍGUEZ: A quantification of the evolution of bilateral trade flows once bilateral RTAs are implemented.

2221 SALOMÓN GARCÍA: Mortgage securitization and information frictions in general equilibrium.

2222 ANDRÉS ALONSO and JOSÉ MANUEL CARBÓ: Accuracy of explanations of machine learning models for credit decisions.

2223 JAMES COSTAIN, GALO NUÑO and CARLOS THOMAS: The term structure of interest rates in a heterogeneous monetary union.

2224 ANTOINE BERTHEAU, EDOARDO MARIA ACABBI, CRISTINA BARCELÓ, ANDREAS GULYAS, STEFANO LOMBARDI and RAFFAELE SAGGIO: The Unequal Consequences of Job Loss across Countries.

2225 ERWAN GAUTIER, CRISTINA CONFLITTI, RIEMER P. FABER, BRIAN FABO, LUDMILA FADEJEVA, VALENTIN JOUVANCEAU, JAN-OLIVER MENZ, TERESA MESSNER, PAVLOS PETROULAS, PAU ROLDAN-BLANCO, FABIO RUMLER, SERGIO SANTORO, ELISABETH WIELAND and HÉLÈNE ZIMMER. New facts on consumer price rigidity in the euro area.

2226 MARIO BAJO and EMILIO RODRÍGUEZ: Integrating the carbon footprint into the construction of corporate bond portfolios.

2227 FEDERICO CARRIL-CACCIA, JORDI PANIAGUA and MARTA SUÁREZ-VARELA: Forced migration and food crises.

2228 CARLOS MORENO PÉREZ and MARCO MINOZZO: Natural Language Processing and Financial Markets: Semi-supervised Modelling of Coronavirus and Economic News.

2229 CARLOS MORENO PÉREZ and MARCO MINOZZO: Monetary Policy Uncertainty in Mexico: An Unsupervised Approach.

2230 ADRIAN CARRO: Could Spain be less different? Exploring the effects of macroprudential policy on the house price cycle.

2231 DANIEL SANTABÁRBARA and MARTA SUÁREZ-VARELA: Carbon pricing and inflation volatility.

2232 MARINA DIAKONOVA, LUIS MOLINA, HANNES MUELLER, JAVIER J. PÉREZ and CRISTOPHER RAUH: The information content of conflict, social unrest and policy uncertainty measures for macroeconomic forecasting.

2233 JULIAN DI GIOVANNI, MANUEL GARCÍA-SANTANA, PRIIT JEENAS, ENRIQUE MORAL-BENITO and JOSEP PIJOAN-MAS: Buy Big or Buy Small? Procurement Policies, Firms' Financing and the Macroeconomy*.

2234 PETER PAZ: Bank capitalization heterogeneity and monetary policy.

2235 ERIK ANDRES-ESCAYOLA, CORINNA GHIRELLI, LUIS MOLINA, JAVIER J. PÉREZ and ELENA VIDAL: Using newspapers for textual indicators: which and how many?

2236 MARÍA ALEJANDRA AMADO: Macroprudential FX regulations: sacrificing small firms for stability?

2237 LUIS GUIROLA and GONZALO RIVERO: Polarization contaminates the link with partisan and independent institutions: evidence from 138 cabinet shifts.

2238 MIGUEL DURO, GERMÁN LÓPEZ-ESPINOSA, SERGIO MAYORDOMO, GAIZKA ORMAZABAL and MARÍA RODRÍGUEZ-MORENO: Enforcing mandatory reporting on private firms: the role of banks.

2239 LUIS J. ÁLVAREZ and FLORENS ODENDAHL: Data outliers and Bayesian VARs in the Euro Area.

2240 CARLOS MORENO PÉREZ and MARCO MINOZZO: "Making text talk": The minutes of the Central Bank of Brazil and the real economy.

2241 JULIO GÁLVEZ and GONZALO PAZ-PARDO: Richer earnings dynamics, consumption and portfolio choice over the life cycle.

2242 MARINA DIAKONOVA, CORINNA GHIRELLI, LUIS MOLINA and JAVIER J. PÉREZ: The economic impact of conflict-related and policy uncertainty shocks: the case of Russia.

2243 CARMEN BROTO, LUIS FERNÁNDEZ LAFUERZA and MARIYA MELNYCHUK: Do buffer requirements for European systemically important banks make them less systemic?

2244 GERGELY GANICS and MARÍA RODRÍGUEZ-MORENO: A house price-at-risk model to monitor the downside risk for the Spanish housing market.

2245 JOSÉ E. GUTIÉRREZ and LUIS FERNÁNDEZ LAFUERZA: Credit line runs and bank risk management: evidence from the disclosure of stress test results.

2301 MARÍA BRU MUÑOZ: The forgotten lender: the role of multilateral lenders in sovereign debt and default.

2302 SILVIA ALBRIZIO, BEATRIZ GONZÁLEZ and DMITRY KHAMETSHIN: A tale of two margins: monetary policy and capital misallocation.

2303 JUAN EQUIZA, RICARDO GIMENO, ANTONIO MORENO and CARLOS THOMAS: Evaluating central bank asset purchases in a term structure model with a forward-looking supply factor.

2304 PABLO BURRIEL, IVÁN KATARYNIUK, CARLOS MORENO PÉREZ and FRANCESCA VIANI: New supply bottlenecks index based on newspaper data.

2305 ALEJANDRO FERNÁNDEZ-CEREZO, ENRIQUE MORAL-BENITO and JAVIER QUINTANA: A production network model for the Spanish economy with an application to the impact of NGEU funds.

2306 MONICA MARTINEZ-BRAVO and CARLOS SANZ: Trust and accountability in times of pandemic.

2307 NATALIA FABRA, EDUARDO GUTIÉRREZ, AITOR LACUESTA and ROBERTO RAMOS: Do renewable energies create local jobs?

2308 ISABEL ARGIMÓN and IRENE ROIBÁS: Debt overhang, credit demand and financial conditions.

2309 JOSÉ-ELÍAS GALLEGOS: Inflation persistence, noisy information and the Phillips curve.

2310 ANDRÉS ALONSO-ROBISCO, JOSÉ MANUEL CARBÓ and JOSÉ MANUEL MARQUÉS: Machine Learning methods in climate finance: a systematic review.

2311 ALESSANDRO PERI, OMAR RACHEDI and IACOPO VAROTTO: The public investment multiplier in a production network.

2312 JUAN S. MORA-SANGUINETTI, JAVIER QUINTANA, ISABEL SOLER and ROK SPRUK: Sector-level economic effects of regulatory complexity: evidence from Spain.

2313 CORINNA GHIRELLI, ENKELEJDA HAVARI, ELENA MERONI and STEFANO VERZILLO: The long-term causal effects of winning an ERC grant.

2314 ALFREDO GARCÍA-HIERNAUX, MARÍA T. GONZÁLEZ-PÉREZ and DAVID E. GUERRERO: How to measure inflation volatility. A note.

2315 NICOLÁS ABBATE, INÉS BERNIELL, JOAQUÍN COLEFF, LUIS LAGUINGE, MARGARITA MACHELETT, MARIANA MARCHIONNI, JULIÁN PEDRAZZI and MARÍA FLORENCIA PINTO: Discrimination against gay and transgender people in Latin America: a correspondence study in the rental housing market.

2316 SALOMÓN GARCÍA: The amplification effects of adverse selection in mortgage credit suply.

2317 METTE EJRNÆS, ESTEBAN GARCÍA-MIRALLES, METTE GØRTZ and PETTER LUNDBORG: When death was postponed: the effect of HIV medication on work, savings and marriage.

2318 GABRIEL JIMÉNEZ, LUC LAEVEN, DAVID MARTÍNEZ-MIERA and JOSÉ-LUIS PEYDRÓ: Public guarantees and private banks' incentives: evidence from the COVID-19 crisis.

2319 HERVÉ LE BIHAN, DANILO LEIVA-LEÓN and MATÍAS PACCE: Underlying inflation and asymmetric risks.

2320 JUAN S. MORA-SANGUINETTI, LAURA HOSPIDO and ANDRÉS ATIENZA-MAESO: The numbers of equality regulation. Quantifying regulatory activity on non-discrimination and its relationship with gender gaps in the labour market.

2321 ANDRES ALONSO-ROBISCO and JOSÉ MANUEL CARBÓ: Analysis of CBDC Narrative of Central Banks using Large Language Models.

2322 STEFANIA ALBANESI, ANTÓNIO DIAS DA SILVA, JUAN F. JIMENO, ANA LAMO and ALENA WABITSCH: New technologies and jobs in Europe.

2323 JOSÉ E. GUTIÉRREZ: Optimal regulation of credit lines.

2324 MERCEDES DE LUIS, EMILIO RODRÍGUEZ and DIEGO TORRES: Machine learning applied to active fixed-income portfolio management: a Lasso logit approach.

2325 SELVA BAHAR BAZIKI, MARÍA J. NIETO and RIMA TURK-ARISS: Sovereign portfolio composition and bank risk: the case of European banks.

2326 ANGEL-IVAN MORENO and TERESA CAMINERO: Assessing the data challenges of climate-related disclosures in european banks. A text mining study.

2327 JULIO GÁLVEZ: Household portfolio choices under (non-)linear income risk: an empirical framework.

2328 NATASCHA HINTERLANG: Effects of Carbon Pricing in Germany and Spain: An Assessment with EMuSe.

2329 RODOLFO CAMPOS, SAMUEL PIENKNAGURA and JACOPO TIMINI: How far has globalization gone? A tale of two regions.

2330 NICOLÁS FORTEZA and SANDRA GARCÍA-URIBE: A Score Function to Prioritize Editing in Household Survey Data: A Machine Learning Approach.

2331 PATRICK MACNAMARA, MYROSLAV PIDKUYKO and RAFFAELE ROSSI: Taxing consumption in unequal economies.

2332 ESTHER CÁCERES and MATÍAS LAMAS: Dividend Restrictions and Search for Income.

2333 MARGARITA MACHELETT: Gender price gaps and competition: Evidence from a correspondence study.

2334 ANTON NAKOV and CARLOS THOMAS: Climate-conscious monetary policy.