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Abstract

The feedback loop between sovereign and financial sector insolvency has been identified as a key driver of the European debt crisis and has motivated an array of policy proposals. We revisit this "doom loop" focusing on governments' incentives to default. To this end, we present a simple 3-period model with strategic sovereign default, where debt is held by domestic banks and foreign investors. The government maximizes domestic welfare, and thus the temptation to default increases with externally-held debt. Importantly, the costs of default arise endogenously from the damage that default causes to domestic banks' balance sheets. Domestically-held debt thus serves as a commitment device for the government. We show that two prominent policy prescriptions – lower exposure of banks to domestic sovereign debt or a commitment not to bailout banks – can backfire, since default incentives depend not only on the quantity of debt, but also on who holds it. Conversely, allowing banks to buy additional sovereign debt in times of sovereign distress can avert the doom loop. In an extension we show that in the context of a monetary union (such as the euro area) similar unintended negative consequences may arise from the pooling of debt (such as European safe bonds (ESBies)). A central bank backstop (such as the ECB's Transmission Protection Instrument) can successfully disable the loop if precisely calibrated.

Keywords: sovereign default, bailout, doom loop, self-fulfilling crises, transmission protection instrument, ESBies.

JEL classification: E44, E6, F34.

Resumen

El bucle de retroalimentación entre la insolvencia soberana y la del sector financiero se ha identificado como un impulsor clave de la crisis de deuda europea y ha motivado una serie de propuestas políticas. Revisamos este «bucle de perdición» centrándonos en los incentivos de los Gobiernos para el impago de la deuda soberana. Con este fin, presentamos un sencillo modelo de tres períodos con *default* soberano estratégico, en el que la deuda la adquieren bancos nacionales e inversores extranjeros. El Gobierno maximiza el bienestar nacional; por lo tanto, la tentación de entrar en *default* aumenta con la deuda en manos de inversores externos. Es importante destacar que los costes del *default* surgen de manera endógena a partir del daño que causa el *default* en los balances de los bancos nacionales. La deuda en manos nacionales sirve como dispositivo de compromiso para el Gobierno. Mostramos que dos recomendaciones políticas destacadas —la reducción de la exposición de los bancos a la deuda soberana nacional o el compromiso de no rescatar a los bancos— pueden tener efectos contraproducentes, ya que los incentivos para el *default* dependen no solo de la cantidad de deuda, sino también de quién la posee. Por el contrario, permitir que los bancos compren deuda soberana adicional en tiempos de crisis soberana puede evitar el bucle de perdición. En una extensión, mostramos que en el contexto de una unión monetaria (como la zona del euro) pueden surgir consecuencias negativas no deseadas similares a partir de un respaldo del banco central (como el Instrumento de Protección de la Transmisión del Banco Central Europeo) si no está calibrado con precisión o de la agrupación de deuda (como los bonos seguros europeos, también conocidos como «ESBies»).

Palabras clave: *default* soberano, rescate financiero, bucle de perdición, crisis de expectativas autorrealizadas, Instrumento de Protección de la Transmisión, ESBies.

Códigos JEL: E44, E6, F34.

1 Introduction

The "doom loop" or "sovereign-bank nexus" has been identified as a key driver of the European debt crisis and has come back into the spotlight recently, as the response to the public health and the Russian crises have caused sovereign debt to skyrocket. The core of the argument is that problems of sovereign debt sustainability and financial sector stability reinforce each other due to the mutual exposures of the two sectors. If sovereign debt loses value due to deteriorating creditworthiness of the public sector, financial sector balance sheets, highly exposed to domestic public debt, suffer. Weakened financial institutions, in turn, force the government to bailout the financial system – banks for short. Bailouts entail expenses for the government and hence a further deterioration of its fiscal capacity.[1] This vicious circle can amplify fundamental shocks (Acharya et al., 2014, Farhi and Tirole, 2016) or even give rise to crises that are entirely generated by self-fulfilling pessimistic expectations (Brunnermeier et al., 2016, 2017, Cooper and Nikolov, 2018), hence explaining how sovereign crises can develop suddenly and spiral out of control easily.

The existing doom loop theory explains how the banks' sovereign exposure affects the government's incentives to bail out banks. However, it abstracts from the effect this exposure has on the government's incentives to repay. Indeed, increased financial sector holdings of sovereign debt may strengthen the incentives to repay (e.g. Bolton and Jeanne, 2011, Gennaioli et al., 2014). This raises the question if the banks' sovereign exposure may actually be a stabilizing factor rather than a destabilizing one, and thus how to regulate it.

To answer this question, we extend the doom-loop theory to incorporate how the government's incentives to honor its debt are shaped by the exposure of the domestic banks to their sovereign. We identify two channels through which the exposure of banks to sovereign debt shapes the default incentives. The *temptation channel* establishes that the higher the exposure, the lower the foreign creditors' holding of debt and consequently the less tempting it is to default. The *commitment channel* captures that the higher the exposure, the larger the disruption of the financial sector and consequently the larger the cost of a default.

We use our setup to evaluate two prominent policy proposals that have been put forward to address the doom loop. In particular, Brunnermeier et al. (2016, 2017) and Cooper and Nikolov (2018) suggest to reduce the exposure of domestic banks to the government, by either reducing their public debt holdings and by pooling and tranching debt of several sovereigns, or by incentivizing banks to back their domestic sovereign debt with bank equity.. Our theory puts the desirability of these policies to break the doom loop into question: By increasing default incentives such policies may come at too high a cost. By contrast, policies that may appear counterproductive at first, such as allowing banks to stack up on domestic debt in times of distress, may turn out to be beneficial.

We develop these arguments using a simple three-period model of sovereign debt and banks with multiple equilibria, similar to the ones in the papers cited above. In period 0, the govern-

[1] As Brunnermeier et al. (2016) argue, weaker balance sheets also affect the public sector indirectly by causing a credit crunch, which leads to lower output and hence a reduction in the tax base.

ment has to finance a fixed expenditure by issuing debt, which is bought by foreign investors and domestic banks. Furthermore, banks also make loans to entrepreneurs, financed through deposits and equity. In period 1, a sunspot shock may hit the economy. If it does, the government bond price drops to a lower value, causing banks to fail. Bank failure entails a cost, because a fraction of the banks' loans gets destroyed in that event, so that the government has an incentive to intervene and bail banks out. To do so, it needs to issue more debt, which, in our baseline model, is bought by foreign investors. In period 2, all debt matures. The government defaults strategically, but does not discriminate across creditors. Sovereign default is costly in two ways: first, default causes domestic output to drop; second, if default drives banks to bankruptcy, the ensuing financial crisis causes another drop in output. The model features a sunspot equilibrium in which the sunspot variable triggers the doom loop: a drop in the bond price causes a bank bailout, which increases debt and hence makes default more likely, thus validating the initial drop in bond prices.

We then analyze several policy options under our novel theoretical framework, proceeding in three steps. In the first step, we consider policies that rule out the doom loop but may nevertheless be undesirable. In particular, we show that increasing banks' equity ratios or rebalancing their portfolio away from domestic sovereign debt does disable the doom loop – but on the other hand eliminates the commitment device that domestic banks' exposure provides to the government. Therefore, such policies undermine sovereign debt sustainability and make sovereign debt more costly, potentially reducing welfare as a consequence. This argument also applies to the case in which the increase in banks' equity ratios is the result of a no-bailout commitment, as in Cooper and Nikolov (2013).[2] This analysis highlights the *commitment channel:* banks' exposure increases the costs of default thus providing additional commitment.

In the second step, we show that if the bailout is carried out by recapitalizing banks by directly giving them sovereign bonds, then the doom loop ceases to exist. This result is due to the fact that the additional debt issued to implement the bailout is held domestically and therefore it does not increase the default incentives. This equilibrium can be decentralized, as long as regulation doesn't stop banks from buying sovereign debt with the bailout funds. The key to the results in this step is what we refer to as the *temptation channel:* incentives to default on foreign debt are higher than on domestic debt.

In a third step, we consider three extensions. First, while our baseline analysis focuses on multiplicity of equilibria, we show that the same model also generates amplification. We illustrate this for the example of a news shock, a case that may resemble the onset of the health crisis. We show that all the policy conclusions derived before apply to this case too. Reducing banks exposure eliminates amplification through the doom loop, but comes at a cost; a domestically financed bailout achieves the same at no such cost. Then we turn to policy proposals that are specific to the monetary union context of the Euro area. In the second extension, we consider the intervention of the monetary unions' central bank that commits

[2]Cooper and Nikolov (2013) argue in favor of such a commitment not because it is effective per se, but because it incentivizes banks to self-insure against sovereign default by increasing their equity sufficiently.

to enforce an upper bound on the sovereign spread, as a simple way to model the ECB's transmission protection instrument. We show that the doom loop disappears. However, if the upper bound is too low, a new panic equilibrium can emerge where private agents sell off their holding of sovereign debt to the ECB, which increases default incentives, and is thus a self-fulfilling equilibrium. To guarantee its success, this instrument thus should be carefully calibrated. Second, we consider a multi-country extension of the model, to show that bundling and tranching bonds of many countries belonging to a union, as suggested by Brunnermeier et al. (2017) in the context of the ESBies scheme, does not necessarily resolve the doom loop and can be detrimental for welfare.

The model we propose is attractive because, despite its simplicity, it is able to illustrate how a significant exposure of the domestic financial sector to the sovereign may not be as problematic as suggested in previous theory; debt re-nationalization in adverse times may be just what is needed to prevent a market turmoil from developing into a full-blown crisis. This argument is particularly relevant today, as government debt skyrockets across Europe and beyond. We thus provide an argument against policies that restrict the financial sectors exposure to domestic debt, which was prominently advocated by a group of German and French economists (Benassy-Quere et al., 2018). This proposal was soon criticized by Messori and Micossi (2018). Indeed, our model provides a formalization of their critique.

Related literature Our paper builds a bridge between two strands of literature. The first is the literature on the doom loop. Brunnermeier et al. (2016, 2017) and Cooper and Nikolov (2013) propose three-period models that are very similar to ours. In their models multiplicity arises through the exact same mechanisms. Leonello (2017) shows that the doom loop can exist even if banks hold no explicit claims to the government on their balance sheets (bonds or debt) but enjoy government guarantees (deposit insurance, bailouts) and resolves the multiplicity of equilibria through global games. Acharya et al. (2014), Abad (2019) and Farhi and Tirole (2016) provide a slightly different notion of the doom loop. Instead of generating multiple equilibria, the doom loop serves as an amplifier, so that small fundamental changes can lead to large changes in the equilibrium. Our paper incorporates both notions of the doom loop. What distinguishes our model is that in these models the default incentives increase in total sovereign debt, while in our model the default incentives increase only in foreign-held debt but not in bank-held debt, which rather serves as a commitment device. This leads us to arrive to contrary policy conclusions.[3]

The second strand regards the commitment role of domestic exposure to sovereign debt. This idea has been developed both in three-period models (Balloch 2016, Basu 2010, Bolton and Jeanne 2011, Brutti 2011, Erce 2012, Gennaioli et al. 2014 and Mayer 2011) as well as in quantitative dynamic models (Boz et al. 2014, Balke 2023, Engler and Grosse Steffen 2016, Mal-

[3] There is also a strand of the literature that proposed the possibility of twin crisis as the result of an alternative "doom loop". In this literature (see Burnside et al. (2000)) the source of insolvency of banks is driven by currency devaluation instead of repricing of sovereign debt, and the key decision of the govenrment is if to mantain a fixed exchange rate or not (instead of sovereign default). This paper does not tackle the specificities of this alternative "doom loop".

lucci 2014, Sosa-Padilla 2018, Perez 2015, Thaler 2019). It is furthermore backed by empirical evidence. E.g. Gennaioli et al. (2014) show that sovereign default premia decrease in domestic bank's exposure to their sovereign.[4] Relative to this literature we contribute by adding a notion of the doom loop.

2 Model

2.1 Setup

We consider a three period economy $t = 0, 1, 2$ with five types of agents: Bankers, households, entrepreneurs, the government and international creditors. The key player is the government, which has to decide on bailouts and debt repayment.

First we present a brief overview of the timing and sequence of events and then we move on to describe in detail the players, strategies and contracts available.

At $t = 0$ the government sells bonds to international creditors and domestic banks in order to finance an exogenous level of expenditure. Entrepreneurs require external financing to undertake risky investment projects and households have initial resources and decide on savings. Banks issue deposits and use these funds, together with their equity, to provide loans to entrepreneurs and to buy sovereign debt.

At $t = 1$ the realization of a sunspot variable is revealed $s \in \{n, p\}$. $s = p$ constitutes a panic state without any effect on fundamentals that can coordinate beliefs on a lower expected repayment by the government. For simplicity, $s = p$ is assumed to be a zero-probability event. The banks might become insolvent if the price of sovereign debt falls and the sovereign doesn't bail the banks out. In that case its assets are liquidated and a fraction of projects financed by the banks fail. The government has the possibility to bailout the banks by issuing sovereign debt.

At $t = 2$ the aggregate productivity of the investment projects is revealed. The government decides if to repay or default on all of its outstanding debt. Default has a direct cost in terms of output and an indirect cost through the banking sector: Again, banks could become insolvent, which causes costly liquidation of loans. At the end of the period, the surviving projects yield their output and agents consume.

The households

There is a continuum of measure 1 of households with an initial endowment at $t = 0$ of Y_0^h. Households are risk neutral and have no time preference. Households can save by placing deposits D_0 at a bank. The deposit contract has a face value of 1 and a price at issuance of p_0^D. Consequently if the price of the deposit equal its expected repayment, the household

[4] For similar evidence see Sturzenegger and Zettelmeyer (2007), Acharya et al. (2014), Bolton and Jeanne (2011), Reinhart and Rogoff (2011), and Balteanu and Erce (2017)

is indifferent between saving and consuming. In that case we assume he places the entire endowment in a deposit account such that

$$p_0^D D_0 = Y_0^h$$

The bankers

There is a continuum of measure one of bankers with an initial endowment of Y_0^b. Bankers derive utility from consumption at $t = 2$ and are risk neutral. At $t = 0$ bankers can issue deposits D_0, buy sovereign debt B_0^h and provide loans to entrepreneurs L_0.

The loans to entrepreneurs are assets with price p_0^L and face value 1 that mature at $t = 2$ after production. If the bank has to early liquidate this asset then a fraction θ of funded projects fail and default, otherwise all loans are fully repaid. Government bonds are risky claims with a face value 1 that mature at $t = 2$ after production. They are traded at price q_0.

The balance sheet of the bank at $t = 0$ then satisfies

$$Y_0^b + p_0^d D_0 = p_0^L L_0 + q_0 B_0^h$$

where on the left hand side we have the liabilities (equity and deposits), and on the right hand side the assets.

At $t = 1$ a bailout may happen. In that case banks receive S_1 units of a safe asset with payoff 1 at the end of period 2.

At $t = 1$ or $t = 2$ depositor may withdraw their deposits. If they all do so, a bank run occurs and the bank is dissolved. Depositors receive the bank's assets, but a fraction θ of projects is destroyed in the liquidation process. We assume a bank run occurs if the value of the bank's asset, evaluated at liquidation prices, is below that of the deposit liabilities, that is if the following condition holds

$$D_0 > (1 - \theta)L_0 + q_t B_0^h + S_1 \tag{1}$$

In Appendix A we show that this assumption indeed corresponds to individually rational behavior by the depositors. Note that a run implies that depositors are paid below face value and bankers, which are protected by limited liability, have zero consumption. Since such a run leads to inefficient disintermediation, the government may want to avert it by engineering a bailout in period 1 or by avoiding default in period 2. We will get to that below.

In case the bank is not liquidated, then the consumption of bankers at the end of period 2 is given by

$$C^b = (1 - d)B_0^h + S_1 + L_0 - D_0$$

where d is a dummy variable that takes the value of 1 in case of government default. In case of liquidation, bankers get nothing.

Entrepreneurs

There is a measure K_0 of entrepreneurs, each of them with an investment project. Each project requires an initial investment of 1 that is financed by bank loans. Each project provides a risky payoff y at $t = 2$ of

$$y = \omega\,(1 - \vartheta d)$$

where ω is TFP, common to all projects, that is revealed at $t = 2$ and has a c.d.f. given by $F(\omega)$. $F(\omega)$ is continuous and has bounded support. Output is reduced proportionally by a factor ϑ in case a sovereign default takes place. This is aimed to capture the costs of default on the economy on top of those that happen because of early loan liquidation. We assume that the lowest value of ω is high enough to guarantee that the loan can always be repaid and is hence safe.

If the bank that issued the loan is liquidated before project termination (be it in period 1 or 2), then a fraction θ of projects fail and output drops accordingly.

International creditors

International creditors are risk neutral investors and have deep pockets. The opportunity cost of their funds is a zero net rate of return. Therefore they will buy sovereign debt at $t = 0$ and at $t = 1$ as long as the expected net return is greater or equal than zero. Since the face value of the sovereign debt is equal to 1, then as long as the international creditors are the marginal investors, the price of sovereign debt q is given by the repayment probability:

$$q_t = \mathbb{E}_t(1 - d)$$

The government and sovereign debt

At $t = 0$ the government has to cover a fix level of expenses G, for which the only possible source of funding is the issuance of zero-coupon bonds B_0 that mature at $t = 2$. Therefore the initial debt issuance satisfies

$$B_0 = \frac{G}{q_0} \tag{2}$$

where q_0 is the price of sovereign debt.

The government takes into account that the price of debt can be a function of the issuance $q_0(B_0)$. In case there are multiple values of B_0 that satisfy equation (2), then the government selects the minimum of those.[5] The initial issuance is acquired by domestic banks B_0^h and foreign creditors B_0^f.

At $t = 1$ banks may become insolvent. In that case, the government can bail out banks by providing them with just enough safe assets S_1, such that the solvency condition 1 is restored.

[5] We sidestep here an alternative source of multiplicity by having the government decide on the level of debt for a given bond price menu. The multiplicity at this stage is the more "classical" type of multiplicity in debt markets analyzed in Calvo (1988). We focus here on the multiplicity generated by the doom loop.

To finance the bailout, the government issues further debt, which increases the debt stock by

$$\Delta B_1 \equiv B_1 - B_0 = \frac{S_1}{q_1}$$

where q_1 is the price of debt at $t = 1$. As we argue below, it will generally be optimal for the government to bail the banks out at this stage, since letting banks go into liquidation is socially costly.

At $t = 2$, after productivity ω is realized, the government decides if to repay the outstanding debt ($d = 0$) or default in full ($d = 1$). The government takes this decision to maximize domestic consumption, that is aggregated over entrepreneurs, bankers and households. The consumption at $t = 2$ is given by

$$C = Y_2 + S_1 - (1 - d)B_1^f$$

where Y_2 is output at $t = 2$, S_1 the safe assets held by locals and $(1 - d)B_1^f$ the repayment to foreign creditors; where in case of default ($d = 1$) is zero. Output is given by

$$Y_2 = (1 - \vartheta d)\omega K_2$$

where K_2 are the surviving projects when production takes place, that is equal to K_0 if the banks were not liquidated or is given by $K_2 = (1 - \theta)K_0$ else.

Default is hence costly for the government in two way: First there is an the exogenous proportional output losses ϑ. Second, if default renders banks insolvent in period 2, a bank run occurs with the associated additional output loss θ. In sum, the proportional output loss of a joint sovereign and banking crisis at $t = 2$ is given by $\Theta \equiv 1 - (1 - \vartheta)(1 - \theta) \leq 1$. This parameter will play a central role in our analysis.

2.2 Equilibrium and assumptions

We are now ready to define an equilibrium. We focus on symmetric sub-game perfect equilibria, where all agents of the same type take the same decision. In our definition we exploit the fact that a bank run happens iff condition (1) is satisfied.

Definition 1. An equilibrium is given by the initial issuance of debt B_0; banks' balance sheet $\left\{B_0^h, L_0, D_0\right\}$; foreign bond holdings B_0^f; a bailout decision rule $b(s)$; a default decision rule $d(\omega, s)$; asset prices $\left\{q_0, q_1^s, p_0^L, p_0^D\right\}_{s=\{n,p\}}$ such that

1. The initial debt issuance raises enough revenue to cover for expenses G at price q_0,

2. The bank's initial balance sheet maximizes the banker expected consumption at $t = 2$ taking as given the asset prices; the government decision rules for bailout and default; and the bank run condition.

3. The household's $t = 0$ deposit choice is optimal.

4. The bailout decision maximizes the expected domestic consumption at $t = 2$ taking as given the default policy function (no commitment), asset pricing functions and the bank solvency condition. The default policy function maximizes consumption at $t = 2$, taking as given the pricing functions and the bank solvency condition.

5. International creditors demand schedule is satisfied

6. Bond markets clear, in particular $B_0 = B_0^h + B_0^f$

To make the model interesting, we focus on the part of the parameter space in which banks are fragile and the doom loop can arise. In particular we make the following assumptions.

Assumption 1. *The household's endowment is large enough to satisfy* $Y_0^h > (1-\theta)K$

Assumption 2. *The bankers capital is large enough to satisfy* $Y_0^b > \theta K$

The first condition guarantees that deposits are sufficiently plentiful to render banks subject to run risk, while the second assumption in turn makes sure that banks have enough equity to be solvent, as long as no panic in the bond markets occurs.

Discussion of key assumptions Above we assumed that default is full (100% haircut). This simplifies the analysis but is not crucial for our results, which can be extended for an arbitrary haircut, adjusting assumptions 1 and 2 accordingly so that banks are exposed to default and solvent in case of no panic.

We rule out the possibility of selective default. In particular, the government cannot decide to discriminate across debt issued at different periods or by the identity of the creditor. This assumption is empirically plausible – governments in default have rarely discriminated by residence of claim holder or issuance date.[6] Furthermore, it is theoretically plausible in a context of secondary markets. As Broner et al. (2010) show, secondary markets allow creditors to circumvent any discriminatory repayment scheme that the government may want to impose.

3 The doom loop

This model allows for several equilibria. We start by describing an equilibrium in which the bond price is unaffected by the sunspot and banks are solvent in period 1. Then we describe an equilibrium where banks are solvent in normal times ($s = n$), but insolvent in panic times ($s = p$). In the second equilibrium, the allocation in normal times coincides with the allocation of the first equilibrium.

[6] As the few instances of repayment discrimination were carried across other dimensions, such as the currency of issuance and the legal jurisdiction.

3.1 The benign equilibrium

Proposition 1. *There exists an equilibrium where the price of debt does not depend on s, the banks are not exposed to bank runs in $t = 1$, and no bailout is implemented.*

The initial issuance of debt held by foreigners is the minimum level B_0^f that solves

$$B_0^f = \frac{G + K - \left(Y^h + Y^b\right)}{1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K}\right)} \tag{3}$$

and the equilibrium price of debt is given by

$$q_0 = q_1 = 1 - F(\omega^n)$$

where

$$\omega^n = \frac{1}{\Theta}\frac{B_0^f}{K} \tag{4}$$

is the productivity threshold above which the government decides to repay in period $t = 2$.

The proof (Appendix C.1) is simply the solution of the model by backward induction. First we solve for the optimal default decision at $t = 2$. The government defaults for TFP below a threshold $\omega^n = \frac{1}{\Theta}\frac{B_0^f}{K_0}$. Given this optimal default strategy by the government, we then proceed backward in time to the initial period. Sovereign debt is priced by the foreign investors such that the bond price is equal to the repayment probability $q_0 = 1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K}\right)$. The resources raised from abroad are then given by $B_0^f q_0$. Together with the bankers' and the households resources $Y_0^b + Y_0^h$, they must cover the expenses $G + K$:

$$G + K = Y_0^b + Y_0^h + B_0^f\left(1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K}\right)\right)$$

that is condition (3) in Proposition (1). Finally, we verify that at price $q_1 = q_0$ banks are solvent, so that no run occurs in period 1.

We stress two features of this equilibrium. First, the default threshold (4) is intuitive and in line with much of the literature on sovereign default: The larger the foreign debt burden B_0^f, the larger the incentives to default. Conversely, the higher TFP ω and the greater the number of productive assets K_0 available at the last period, the lower the incentives of default. After all, the "punishment" for default is a proportional loss of the output produced by the productive asset.

Second, by inspecting the default threshold (4) we can identify two channels through which domestic banks' exposure to sovereign debt affect the default probability. On the one hand, when domestic banks hold more foreign debt, then the government has to issue less foreign debt B_0^f. This reduces the *temptation* to default. On the other hand, because domestic banks hold enough sovereign debt to become insolvent in case of sovereign default, the costs of default Θ include those related to a bank run at $t = 2$ (θ). Banks' exposure to sovereign debt hence increases the *commitment* of the government not to default. Domestic banks' bond holdings thus reduce the default probability through what we call the *temptation channel* and the *commitment channel.*

3.2 The sunspot equilibrium

Besides this benign equilibrium, there is another equilibrium in which the doom loop kicks in. The doom loop is driven by a perceived higher sovereign default probability that lowers sovereign debt prices to the point that the bank becomes insolvent if liquidated. In that case, the government has the possibility to bailout the bank and avoid a bank run. The bailout transfer S_1 necessary to restore banks' solvency is

$$S_1 = D_0 - (1-\theta)L_0 - q_1^p B_0^h$$

It is financed by issuing debt to international creditors, and consequently the increase in debt held by foreigners is given by

$$\Delta B_1^f = \frac{S_1}{q_1^p}$$

Even after a bailout in period 1, a default at $t = 2$ generates bank insolvency. Thus, the trade-off between default and repayment faced by the government in $t = 2$ is essentially the same, regardless of whether a panic has taken place in $t = 1$ or not, only that the outstanding debt held by foreigners is larger. The default threshold is thus given by

$$\begin{aligned}\tilde{\omega}^p &= \frac{B_0^f + \Delta B_1^f}{\Theta K_0} \\ &= \omega^n + \frac{1}{\Theta K_0}\frac{S_1}{q_1^p}\end{aligned}$$

Investors pricing the debt at $t = 1$ anticipate this and demand compensation for this heightened default risk such that $q_1^p = 1 - F(\tilde{\omega}^p)$. If this price is low enough to render banks insolvent, we have found an equilibrium. The next proposition establishes that under our assumptions such an equilibrium indeed exists.

Proposition 2. *For a small enough ϑ, a sunspot equilibrium exists where in case of panic $s = p$ the price of sovereign debt drops in $t = 1$ and banks are bailed out.*

The price of debt in case of panic is given by the solution to the system

$$q_1^p = 1 - F\left(\tilde{\omega}^p\right) \tag{5}$$

$$\tilde{\omega}^p = \omega^n + \frac{1}{\Theta K_0}\frac{\overbrace{D_0 - (1-\theta)L_0 - q_1^p B_0^h}^{Bailout}}{q_1^p} \tag{6}$$

where $\tilde{\omega}^p$ is the default threshold in case of panic. The equilibrium variables at $t = 0$ and in case $s = n$ coincide with Proposition 1.

The proof (Appendix C.2) shows that as long as banks are exposed to sovereign risk in the sense that they become insolvent if their sovereign bond holdings are written off (Assumption 1) and the productivity distribution has bounded support, there is an equilibrium where panic can emerge.[7] The first condition implies that a low enough bond price triggers a run, and the bounded support condition guarantees that repayment probability can be as low as necessary for a finite level of foreign held debt.

Figure (1) illustrates this equilibrium. Let $q(\omega)$ represent the mapping from the default threshold to prices given by equation (5) (the blue curve). And be $\omega(q)$ the mapping from prices to the default decision given by equation (6) (the red curve). This second mapping is valid only if prices are low enough to render banks insolvent, represented by the orange shaded region represents the region. Finally, the green line marks the default threshold if there is no run in period 1. This value is only valid if there is no run (above the shaded region). The two intersections represent the equilibrium conditional on the two values of the sunspot.

Note that this is the infamous doom loop at play. Just as in Brunnermeier et al. (2016) and Cooper and Nikolov (2013), pessimistic expectations become self-fulfilling. If agents happen to coordinate on the lower bond price, banks become insolvent, forcing the government to increase its debt to finance a bailout, which makes it more likely that the government defaults later on (red curve). A higher default probability in turn justifies a lower bond price (blue curve). The pessimistic expectations are hence validated.

In this discussion of the sunspot equilibrium we focused on the case where bailout is the government's optimal choice. This is guaranteed by the condition that the non-financial costs of default ϑ are small enough (see Appendix C.2).

For the bailout to be optimal, it also has to be feasible. If creditors at $t = 1$ would expect a default with certainty and consequently the price of sovereign debt were zero, then the government could not bailout banks by issuing further debt – a bailout would be unfeasible. However, as long as there is some probability mass above ω^p (the default threshold in case of a panic without bailout) this panic is not self-fulfilling and consequently not an equilibrium. We focus on this case here.

Finally, note that the panic equilibrium, which we depict in the figure, is not locally stable under best response dynamics. However, as Cooper and Nikolov (2013) show, it is easy to obtain a stable panic equilibrium by putting adequate restrictions on the c.d.f. $F(\omega)$, which modify the mapping $q(\omega)$ (the blue curve) to have multiple crossings with $\omega(q)$ (the red curve). All of our analysis goes through if we were to restrict our attention to such a stable equilibrium.

4 Reducing exposure to break the doom loop?

Since the doom loop arose from the banks' fragility, the doom loop can be avoided by reducing the banks' exposure to sovereign debt sufficiently to make them immune to fluctuations in the value of sovereign debt. As Brunnermeier et al. (2016) show, this can be achieved by either raising the banks' equity ratios or by reducing their sovereign bond holdings. Indeed, Cooper

[7] The bounded support is a sufficient but not a necessary condition. All we require is a sufficiently thin right tail.

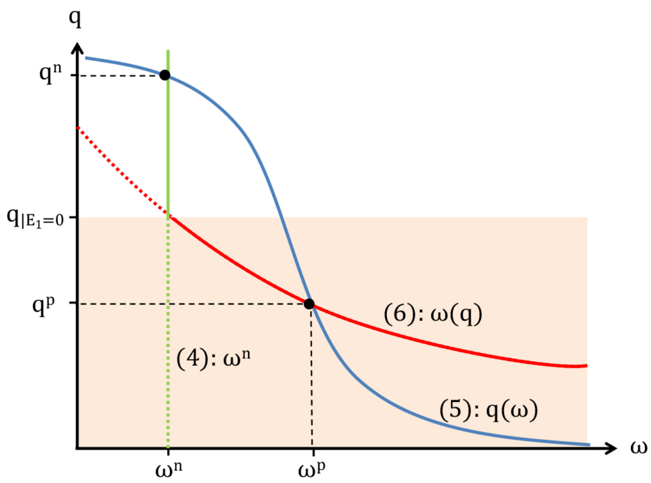


Figure 1: Equilibrium

and Nikolov (2013) use this insight to argue for a no bailout commitment: In their model such a commitment induces banks to self-insure and hold enough equity to never be in need of a bailout.[8] We now revisit these policy proposals.

Consider an alternative version of our economy where the banks are solvent even if the bonds loose all their value. We call this a *no exposure economy*. Motivated by the above cited literature, we consider two variants of the no exposure economy. First, we consider the case that the banks adjust their *liabilities structure*, i.e. increase their equity ratio. This is achieved by shifting resources from households Y^h to bankers Y^b while maintaining the same total endowments for the domestic economy. This allocation is characterized by lower level of deposits $D_0^{ne,E} < D_0$. Second, we consider the case that bank adjusts its *asset structure* by buying less domestic debt and instead purchasing the safe asset $S_0^{ne,S}$. In each case we consider the minimal deviation from the baseline model that delivers *no exposure*. The superscripts E and S refer to the two variants, no exposure by larger equity E or by higher holding of safe assets S. We think about these two alternative economies as regulatory changes aimed at killing the doom loop.

The following proposition characterizes the equilibrium in the no exposure economy and establishes that there is no doom loop any longer.

Proposition 3. *In the no exposure economy, for ϑ large enough, there exists a no-sunspot equilibrium where banks are solvent in $t = 1$. This equilibrium is unique.*

If exposure is avoided by requiring banks to hold safe assets we have that $S_0^{ne,S} = Y_0^h - (1-\theta)K$ while if exposure is achieved with higher bank equity then $Y_0^{h,ne,E} = (1-\theta)K$.

The initial issuance of debt held by foreigners in each case is the minimum level $B_0^{f,i}$ that solves

[8] We do not model the banks' funding choice. However, since the no bailout commitment is irrelevant if the banks has enough equity, we can mimic their policy proposal by simply assuming a high enough equity ratio.

$$B_0^{f,i} = \frac{R + K_0 + S_0^{ne,i} - \left(Y^h + Y^b\right)}{1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{K_0}\right)} \quad \textit{for } i \in \{S, E\}$$

and the equilibrium price of debt is given by

$$q_0^{ne,i} = q_1^{ne,i} = 1 - F(\omega^{ne,i})$$

where $\omega^{ne,i} = \frac{1}{\vartheta}\frac{B_0^{f,i}}{K_0}$ *is the productivity threshold for which the government decides to repay in period* $t = 2$.

Consider first the no exposure economy with a higher bank equity ratio. The amount of funds that the government has to raise from international creditors is the same as in the baseline economy. However, the banks now have enough equity to remain solvent even if the government defaults. So the costs of default are lower ($\vartheta < \Theta$). Banks exposure no longer serves as a commitment device. Thus, the default threshold $\omega^{ne} = \frac{1}{\vartheta}\frac{B_0^f}{K_0}$ is higher and the bond price lower $q_0^{ne,E} < q_0$ through the *commitment channel*. That in turn raises the sovereign debt necessary to finance the expenditures R $\left(B_0^{f,ne,E} > B_0^f\right)$. The latter makes default even more attractive through the *temptation channel* mentioned in the introduction, which amplifies the initial drop in the bond price.

Now consider the no exposure economy where banks reduce their holdings of sovereign debt. This forces the government to raise more funds from the international creditors. This implies that default becomes even more tempting through the *temptation channel*, over and above the loss of commitment already discussed for the previous case.

The economies with no exposure thus feature larger default premia than the baseline economy in normal times, as the government bonds are sold at discount to compensate for the lower repayment probability. The next proposition establishes this:

Proposition 4. *Assume* $\theta > 0$. *Bond prices are lower in the economies with no exposure than the economy with exposure* $q_0^{ne} \leq q_0$. *Prices are lower if exposure is avoided by substituting domestic sovereign debt for the safe asset than if exposure is avoided by increasing equity* $q_0^{ne,S} \leq q_0^{ne,E}$. *The equalities are strict if* $F'(\omega^n) > 0$.

In sum, no matter how banks' exposure is eliminated, the foreign debt burden increases and the cost of default decrease such that default becomes more likely. If there is enough probability mass that the productivity draw can fall in between the default thresholds $\omega^{ne,E}$ and $\omega^{ne,S}$ determining the bond prices in Proposition 4, then the ordering of prices translates into an ordering of welfare: the lower the price, the lower expected consumption and thus welfare. The next proposition formalizes this claim:

Proposition 5. *Assume* $\theta, \vartheta > 0$. *If* $f(\omega)\omega$ *is non-decreasing in the interval* $\left[0, \omega^{ne,S}\right]$ *then ex-ante welfare is lower in the economy with no exposure than in the economy with exposure. If the exposure of banks is avoided by substituting domestic sovereign debt for the safe asset then welfare falls more than if exposure is avoided by increasing equity.*

The condition that $f(\omega)\omega$ is non-decreasing in the interval $\left[0, \omega^{ne,S}\right]$ is sufficient but not necessary. It guarantees that the probability of default in the economy with no exposure is sufficiently higher such that the expected default costs increase despite of facing a lower cost in case of default.[9]

Note further that the existence of the "normal times" equilibrium is no longer guaranteed. If ϑ is too small – which we have ruled out in the proposition –, then the government simply doesn't have enough (exogenous) commitment to finance its expenditures. That is the *commitment channel* would kick in so strongly that it would be unable to finance the expenditures R. This should certainly have some significant welfare costs, but they are outside our simple model.

We have compared prices and welfare assigning the panic a probability of zero for simplicity. As the probability of the panic increases, prices and welfare in the sunspot equilibrium of the exposure economy decrease and could fall below those of the no exposure economies. However, by continuity our results continue to hold if the probability of the panic occurring is small enough.

In sum, under certain conditions it is undesirable to kill the doom loop by ex-ante restricting banks exposure to the sovereign. This is true for both policies considered here: higher bank capital ratios and substitution towards safe assets, such as ESBies. Furthermore, the latter policy is more harmful than the first. By extension, a no-bailout commitment would also be undesirable, even if it causes banks to increase their capital ratios as in Cooper and Nikolov (2013).[10]

Our findings conflict with the policy conclusions of Brunnermeier et al. (2016) and Cooper and Nikolov (2013). The reason for these different conclusions lies in the modeling of default incentives. In their models the incentives to default do not depend on the bank's exposure, only on total debt. Hence ruling out the sunspot equilibrium comes free of any cost for public debt sustainability. On the contrary, in our model policies that kill the doom loop by reducing banks' exposure affect default incentives negatively: They increase the temptation to default and reduce the commitment to repay through the two channels defined before, thus increasing default probabilities.[11]

5 Bailout financing and the doom loop

The *no exposure* policy options we just discussed were successful at disabling the doom loop, but came at the cost of worsening the default incentives in normal times. In this section we

[9]Note that a uniform distribution satisfies this condition, as does a bell shaped distribution to the extent that default is a tail risk. Even if the density is decreasing over this interval the condition can be satisfied as long as $f'(\omega) > -f(\omega)/\omega$.

[10]As we discuss in the working paper version of this paper, in our setup a no bailout commitment by itself (keeping banks liabilities structure fixed) only modifies the doom loop, without disabling it. In that case, a panic in $t = 1$ leads to a banking crisis, which reduces the productive capacity of the economy and thus increases default incentives.

[11]In Brunnermeier et al. and in Cooper and Nikolov's baseline model, default is non-strategic and driven directly by an exogenous "tax capacity" process. Cooper and Nikolov consider strategic default in an extension, but the default incentives are modeled as independent of the bank's balance sheet. Brunnermeier et al. (2016) do however not analyze welfare or claim desirability.

analyze a different policy option, which also succeeds at ruling out the doom loop, but without affecting the default incentives in normal times. Contrary to the previous option, this option consists in *increasing* the banks' exposure to the sovereign.

Consider the baseline model as in section 2.1, but now assume that in a bailout the government directly provides the bank with additional sovereign bonds, instead of borrowing abroad to provide banks with safe assets. In this case a bailout no longer increases the foreign debt burden, which remains at its initial value B_0^f. Hence the benefit of default no longer increases in the size of the bailout, and thus the bond price q_1^p has no reason to fall. Unlike before, the *temptation channel* is mute.[12] That means that the doom loop, which leads to multiplicity of equilibria in the baseline model, is no longer active and we can rule out the sunspot equilibrium.

Proposition 6. *When the bank is bailed out with domestic bonds, the sunspot equilibrium ceases to exist. The benign equilibrium is unique and remains unchanged as in the baseline model.*

Thus increasing the exposure of banks by issuing additional debt in times of self-fulfilling expectations driven crises is benign and, in this simple model, in fact rules out such crises altogether. This is so because such a bailout does not interact with the default incentives like a foreign debt financed bailout does: The temptation to default (foreign debt) doesn't increase. Models where only total debt determines repayment, but not the composition of bond holders do not share this feature.

Furthermore, this policy does not interact with the default incentives in normal times, such that it has all the benefits of the *no exposure* policy but none of the costs.

As we show next the result in Proposition 6 could also be decentralized.

Decentralization

So far our bank was extremely passive in period 1; it had no decision to take. Now we extend the model to allow the bank to choose what to do with the bailout funds in period 1: whether to invest them into domestic debt ΔB_1^h or whether to hold the safe asset S_1, subject to a regulatory exposure constraint.

Just as in the basic setup, the value of the bailout is given by $Bailout = \max\left\{D_0 - L_0 - q_1^p B_0^h, 0\right\}$ and it is financed with the issuance of new debt such that $\Delta B_1 = \frac{Bailout}{q_1^p}$. However now the new debt is sold to foreign investors or domestic banks. Market clearing requires $\Delta B_1 = \Delta B_1^f + \Delta B_1^h$.

As already discussed for period 0, in period 1 the bank again maximizes its expected period 2 value under limited liability $\mathbb{E}\left[max\left\{C^b, 0\right\}\right]$, subject to the budget constraint $Bailout + q_1^p B_0^h \leq S_1 + q_1^p B_1^h$ and the regulatory exposure constraint $q_1^p B_1^h \leq \bar{B}$. The bank is atomistic and thus takes all prices and the government's actions as given. Due to limited liability the bank has an incentive to buy as much debt as possible.[13] It thus invests all the bailout funds into sovereign

[12] The cost of default does not change with the bailout or the bond price either, because banks are bankrupt in case of default and solvent in case of repayment no matter how many bonds they got in the course of the bailout in period 1.

[13] Andreeva and Vlassopoulos (2019) find that banks that had its risks more highly correlated with the domestic sovereign increased more their demand for domestic sovereign debt during the European sovereign

debt, if the exposure limit permits, or up to the limit $\bar{B}$ otherwise, and invests the rest of the bailout funds in the safe asset. Thus the change in sovereign debt holdings and the safe asset holdings by the bank are given by:

$$\begin{aligned} \Delta B_1^h &= \min\left\{\bar{B}/q_1^p - B_0^h, \Delta B_1\right\} \\ S_1 &= Bailout - q_1^p \Delta B_1^h \end{aligned}$$

It is immediately evident that for $\bar{B} \to \infty$ the equilibrium of this economy coincides with that of the economy analyzed before in Proposition 6, where the government bails out the bank with sovereign debt but no trading is possible after the bailout.

On the other hand, for finite $\bar{B}$ the economy could feature multiple equilibria. The following proposition characterizes for which values of $\overline{B}$ this is the case.

Proposition 7. *If $\overline{B} \geq D_0 - (1-\theta)L_0$ there is no sunspot equilibrium and the equilibrium corresponds to the equilibrium described in Proposition 1. If $\overline{B} < D_0 - (1-\theta)L_0$ there is a sunspot equilibrium.*

This result establishes that to rule out the doom loop it is sufficient that the regulatory maximal exposure in the panic state $\overline{B}$ is greater than the equity shortfall in case of default. By Assumption 2 the exposure to sovereign debt in normal times is higher than this threshold as $q_1^n B_0^h > D_0 - (1-\theta)L_0$. Consequently, the doom loop only arises if banks face constraints that force them to lower bond holdings *at market value* sufficiently during panics. In such scenario, the panic is self-fulfilling as the lower exposure of banks weakens the incentives of the sovereign to repay. On the contrary, if banks in panic times are allowed to hold bonds up to a value not too much lower than in normal times, the doom loop ceases to exist.

6 Extension: Amplification

As Cooper and Ross (1998) show, strategic complementarities – such as the one between regarding the bond price which our model considers – typically lead not only to multiplicity of equilibria, but also to amplification. In Appendix B we show that our main policy results carry over to a version of the model where the doom loop amplifies fundamental shocks – such as for example a bad news shock – which is the effect highlighted by Farhi and Tirole (2016) and Acharya et al. (2014).

7 Extension: Policy options in a monetary union

In this section we consider two additional policy options that have been proposed specifically for the context of the European Monetary Union.

debt crisis. The motive they propose to rationalize their finding is the same as here, risk-shifting. There are other reasons why banks may have a higher valuation of government bonds than foreign investors, especially in times of crisis, such as: Regulatory reasons, financial repression, or non-atomistic behavior of banks.

7.1 ECB Transmission Protection Instrument

Besides the regulator, the central bank may also consider policies to curb the doom loop. Indeed, the ECB has recently announced its readiness "*to make secondary market purchases of securities issued in jurisdictions experiencing a deterioration in financing conditions not warranted by country-specific fundamentals*" (ECB press release on the Transmission Protection Instrument, 21 July 2022).[14]

Our model provides a theory of such a non-fundamental deterioration of financing conditions. We can thus use it as a labroatory to evaluate the ECB's policy. In this section we show that such a backstop by the ECB has the potential to work. However, if the ECB is too aggressive in keeping prices close to "fundamentals" it can generate a new source of multiplicity (panic). The new type of panic is driven by a sell-off of sovereign debt from the private sector to the ECB that weakens the incentives for debt repayment.

We model the ECB's intervention as follows: Consider our economy as described before to be a member of the Euro. The ECB sets a lower bound on the sovereign debt price, q^{ECB}, and stands ready to buy as much debt (by issuing safe ECB reserves) as necessary to implement it. As is done in practice, there is limited risk sharing. This arrangement specifies that the the ECB absorbs a fraction ϕ of any losses (or profits) arising from the bond holdings of an individual member state, while the rest is to be covered by that state's national central bank. This implies that in case of default, ECB-held debt is partially domestic and partially foreign. $\phi = 0$ would imply no risk sharing. Besides defaulting within the framework of the Eurosystem, we also allow the country to exit the monetary union upon default, in which case it does not honor the risk sharing arrangement. ECB held debt then becomes entirely foreign, but the country suffers a larger output loss $\vartheta^{Exit} > \vartheta$ in that case.

Assume that $q^{ECB} > q^p$ such that the lower bound is above the price in case of the doom loop panic. Clearly the panic equilibrium in Proposition 2 ceases to exist. Figure 3 panel (a) illustrates this intervention. The policy is effective because it provides a backstop to the vicious circle between expectations of default and the sovereign debt price, by making the price unresponsive once it hits the lower bound. While the original doom loop is thus successfully disabled, the policy introduces a new sort of panic.

To see this, assume for now that a realization of the panic sunspot in period 1 has triggered a debt sell-off, by which we mean that all private agents (domestic banks and foreign investors) sell all their sovereign bonds to the ECB in period 1 in exchange for reserves of a total value of $q^{ECB}B_0$. In $t = 2$, the government thus has 3 options: First, the government may repay in full. Then the ECB makes profits. Second, it can default while abiding by the risk sharing rule. In that case it has to pay the ECB $(1-\phi)q^{ECB}B_0$, such that the losses of the ECB are $\phi q^{ECB}B_0^{ECB}$. The default costs ϑ accrue. Third, the government may decide to break the limited risk sharing rule and exit the monetary union. Then the repayment of debt is zero and the default costs ϑ^{Exit}. As before, the government chooses the option that maximizes domestic consumption.

[14] Press release downloaded Jan 20, 2023 https://www.ecb.europa.eu/press/pr/date/2022/html/ecb.pr220721 973e6e7273.en

Consumption in case of repayment is given by

$$C^r = \underbrace{\omega K_0}_{\text{Output}} + \underbrace{q^{ECB} B_0^h}_{\text{Reserves}} - \underbrace{B_0}_{\text{Debt repayment}} + \underbrace{(1-\phi)\left(B_0 - q^{ECB} B_0\right)}_{\text{ECB profits}}$$

where the second term are the reserves that the domestic banks acquired during the selloff in period 1.

Consumption in case of default is

$$C^d = \underbrace{\omega(1-\vartheta)K_0}_{\text{Output}} + \underbrace{q^{ECB} B_0^h}_{\text{Reserves}} - \underbrace{(1-\phi)q^{ECB} B_0}_{\text{Partial debt repayment to ECB}}$$

Note that default is now effectively partial.

And in case of exit the consumption is

$$C^{Exit} = \underbrace{\omega(1-\vartheta^{Exit})K_0}_{\text{Output}} + \underbrace{q^{ECB} B_0^h}_{\text{Reserves}}$$

Let ω^{ECB} be the productivity level for which the government is indifferent between repayment and (partial) default ($C^r = C^d$), and let ω^{Exit} be the productivity level for which the government is indifferent between partial default and full default ($C^d = C^{Exit}$). Assuming for tractability that ϑ is small enough,[15] the optimal optimal strategy of the government is a threshold policy as follows: Full repayment if $\omega \geq \omega^{ECB}$; default if $\omega \in \left(\omega^{Exit}, \omega^{ECB}\right)$; and exit if $\omega < \omega^{Exit}$; where

$$\omega^{ECB} = \frac{\phi}{\vartheta} \frac{B_0}{K_0}$$

$$\omega^{Exit} = \frac{(1-\phi)q^{ECB}}{\vartheta^{Exit} - \vartheta} \frac{B_0}{K_0}$$

Now consider period 1. From the point of view of domestic banks and international creditors, it is optimal to sell-off the sovereign debt if their expected repayment is lower than the price offered by the ECB. Therefore there will be a sell-off if

$$q^{ECB} > 1 - F\left(\omega^{ECB}\right)$$

in that case the expected losses by the ECB are

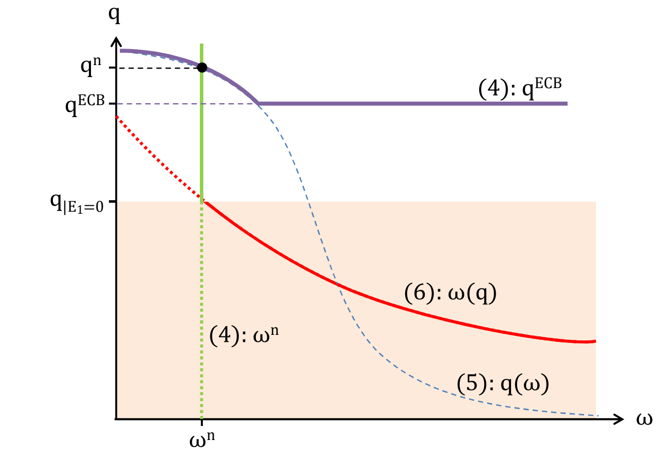
$$\mathbb{E}\left[losses\right] = q^{ECB} B_0 \left(F\left(\omega^{Exit}\right) + \phi\left(F\left(\omega^{ECB}\right) - F\left(\omega^{Exit}\right)\right)\right)$$

where $\left(F\left(\omega^{ECB}\right) - F\left(\omega^{Exit}\right)\right)$ is the probability of a default without exit.

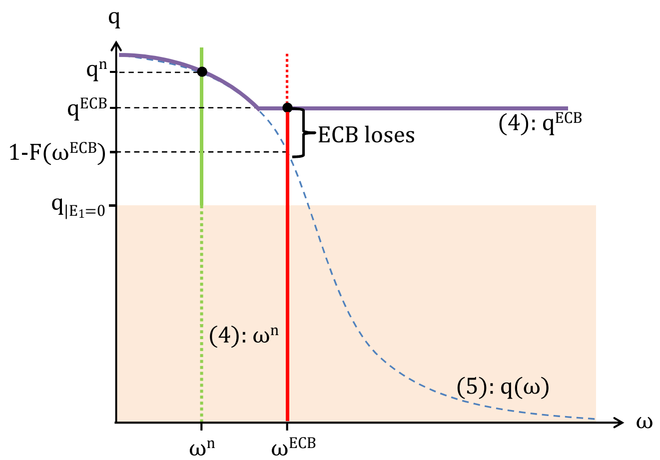
We collect these results in the following proposition. We restrict attention to the case where the ECB sets the lower bound high enough to avoid insolvency of domestic banks (above $q_{E_1=0}$) but we allow it to set it even higher than the fundamental price q^n.

[15] This assumption guarantees that default without exit is a possible outcome (such that $\omega^{Exit} < \omega^{ECB}$). It can be relaxed at the expense of having to consider more case distinctions.

(a) ECB intervention avoiding the doom loop



(b) ECB intervention with a selloff in case of panic



(c) ECB intervention without a selloff in case of panic

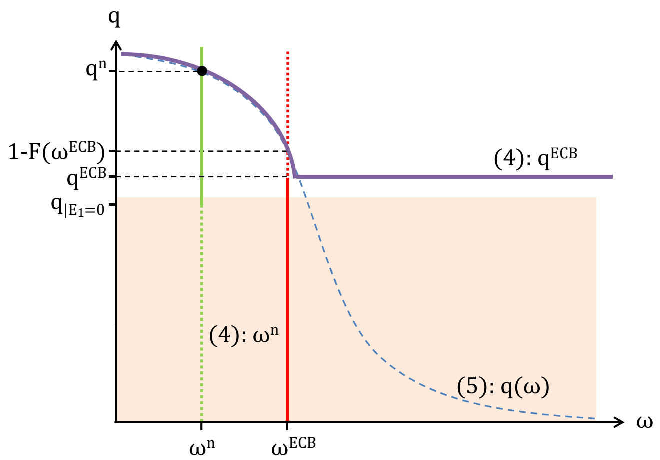


Figure 2: Equilibrium with ECB intervention

Proposition 8. *If the ECB intervenes in case of the panic state* ($s = p$) *at* $t = 1$ *setting a lower bound on sovereign debt* $q^{ECB} > q_{E_1=0}$, *and assuming that* ϑ is small, *there are three possible equilibrium configurations, that depend on the level of the price bound* q^{ECB} *relative to two thresholds* q_1^n *and* $\tilde{q}_1^{ECB}$, *where* q_1^n *is given in Proposition 1 and* $\tilde{q}_1^{ECB} = 1 - F\left(\omega^{ECB}\right)$

1. *If* $q_{E_1=0} < q_1^{ECB} < \tilde{q}_1^{ECB}$ *the only equilibrium is the no sunspot equilibrium in Proposition 1.*

2. *If* $\tilde{q}_1^{ECB} < q_1^{ECB} < q_1^n$ *there is a sunspot equilibrium where in normal times* $s = n$ *the ECB intervention is not required, the default probability is low* $F(\omega^n)$ *and* $q_1 = q^n$, *and where in case of panic* $s = p$ *domestic banks and foreign investors sell-off their sovereign debt holdings to the ECB at* q^{ECB} *and the default probability is higher* $F\left(\omega^{ECB}\right)$. *This sunspot equilibrium coexists with the no sunspot equilibrium from case 1.*

3. *If* $q^n < q^{ECB}$ *then the sunspot equilibrium described in case 2 is the unique equilibrium.*

This proposition stresses the risks of the ECB intervention. The lower bound on the price of sovereign debt q^{ECB} can be effective in avoiding the doom loop panic if the price cap is low enough to satisfy condition 1 (see panel (c) in Figure 3). Nevertheless, if the policy is not well calibrated and the ECB steps in too quickly (condition 2), it can generate a new panic equilibrium (see panel (b) Figure 3). Intuitively, if the ECB is too aggressive and sets a lower bound for the price (upper bound to the risk premium) too close to the fundamental, then a panic may happen, where banks prefer to sell off their bonds to the ECB at q^{ECB}, because they anticipate a higher than normal default probability. This panic is self-fulfilling because the sell-off itself lowers the commitment of the government to repay. This self-fulfilling sell-off cannot happen at a low enough q^{ECB}, since the sell-off doesn't increase default incentives enough to make it profitable to sell the bonds to the ECB at the low price. Finally if the ECB were to put a price floor above the fundamental value (condition 3), the sell-off is individually optimal regardless of what other investors do, and it is thus the only possible outcome.

In Figure 3 panel (a) we show the three possible regions described in proposition 8. To be effective, the ECB has to set the price in the green region. Setting it too low fails to guarantee bank solvency, setting it to high risks generating the sell-off loop. Importantly the sizes of these regions are determined by the degree of risk sharing ϕ, as panel (b) of figure 3 illustrates. Lower risk sharing shrinks the multiplicity region ($\tilde{q}_1^{ECB}$ increases) because the temptation to default is lower when a higher fraction of debt repayment corresponds to the national central bank. However, there is a limit to how much lower risk sharing can shrink the multiplicity region. If risk sharing drops below $\phi^* = \left(1 + \frac{\vartheta^{Exit} - \vartheta}{q^{ECB}\vartheta}\right)^{-1}$ then the government decides only between repayment and default with exit; the default-without-exit option becomes irrelevant. In this case the temptation to default is less responsive to ϕ, since there is only profit sharing in case of repayment but there is no loss sharing in case of default.[16]

[16] Even for the case of no risk sharing $\phi = 0$, there can still exist a multiplicity region if the cost of exit ϑ^{Exit} is close enough to the cost of default and bank insolvency Θ.

(a) Equilibrium regions for a given level of risk sharing (b) Equilibrium regions in the policy space (ϕ, q^{ECB})

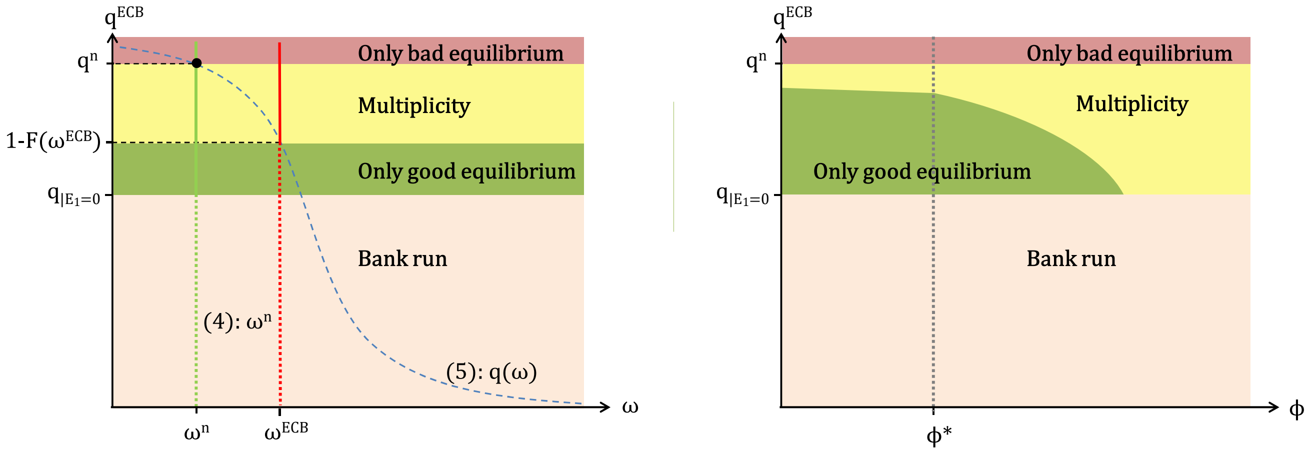


Figure 3: Equilibrium regions with ECB intervention

In sum, if well calibrated the ECBs back stop can work as intended and rule out the doom loop as a pure off-equilibrium threat, without ever being triggered, and without any consequences for the default incentives in normal times. But if the ECB sets the cap on the default premium too low and steps in to early, a new panic can arise. Given the uncertainty that surrounds both the fundamental level of the default premium and the appropriate level of the cap in practice, the ECB may inadvertently end up in this situation. Limits on risk sharing can help to make the task of setting the cap easier, but due to the option to exit the monetary union, they are no panacea either.

7.2 Diversification, ESBies and the doom loop

Another popular proposal to break the doom loop in response to the European debt crisis has been the creation of European Safe Bonds (ESBies) (see Brunnermeier et al. 2017). This proposal consists in creating a European safe asset by tranching a bundle of European sovereign debt. The senior tranche of this collateralized security would constitute a safe asset, and by restricting banks to hold only the senior tranche, as opposed to domestic sovereign debt, banks would become less exposed to the domestic sovereign and the doom loop would be avoided. If this policy is successful at creating a safe asset, then the doom loop would indeed disappear, as we have shown in section 4. However, it would also lead to more debt being held by foreigners and thus the commitment value of banks' exposure would be lost. This would come at the costs of higher spreads in normal times and possibly cause welfare losses, as we have shown in the same section.

In this section we turn to another issue that can arise if this policy is implemented, and that renders it even less beneficial: If banks hold a diversified portfolio of sovereign debt, the doom loop may still be present, even if the bundle is tranched (ESBies). Just that the panic happens at the European level, and not at the level of a single country.

To make this point, we extend our single country model to a continuum of identical countries. We show that if the countries in isolation are exposed to the doom loop, then diversification and tranching do not remove that risk. The doom loop persists in the ESBies economy, and its mechanism is closely related to the original doom loop: If investors expect a surge in default rate among European sovereigns in $t = 1$, then the value of the sovereign debt bundle falls, causing bank insolvency and the need for bailouts in all countries. Since bailouts are financed by additional debt issuance, more countries end up defaulting in $t = 2$, validating the initial beliefs. The main two differences with the single country doom loop are: (i) a larger fraction of debt is held by foreigners; (ii) the default decision of a given country has no impact on the domestic financial system. Thus banks' sovereign debt holdings no longer serve as a commitment device and the temptation to default is larger – these are again the two channels highlighted throughout the paper.

Consider a continuum of measure one of ex-ante identical countries, which are each characterized as in the baseline model. The total amount of debt issued by the continuum of countries is split between the amount held to create the asset bundle and the amount held directly by non-European foreign investors as follows

$$\int_0^1 B_0^i di = \mathcal{B}_0 + \int_0^1 B_0^{i,f}$$

where B_0^i is the debt issued at $t = 0$ by country i, $\mathcal{B}_0$ is a bundle of sovereign debt and $B_0^{i,f}$ is the bonds issued by country i held by foreign investors. The bundle $\mathcal{B}_0$ is tranched into a senior tranche $\mathcal{B}_0^s$ (the ESBies) and a junior tranche $\mathcal{B}_0^j$ where the subordination level is given by ς. If $\varsigma = 0$ then there is no tranching, only diversification. We normalize the face value of a unit of the senior and the junior tranche equal to one, consequently the total issuance of the senior and junior tranches are given by $\mathcal{B}_0^s = (1 - \varsigma)\mathcal{B}_0$ and $\mathcal{B}_0^j = \varsigma\mathcal{B}_0$. Since we assume all countries to be identical, we focus on a symmetric equilibrium and from now on omit the superscript i.

Assume that banks can only hold the senior tranche of the bundle and, for simplicity, that the volume of the senior tranche $\mathcal{B}_0^s$ is set to exactly satisfy European banks' demand for sovereign debt. The junior tranche will be held by non-European investors. Let Q_t^s be the price of the senior tranche and Q_t^j of the junior tranche.

The timing and decisions are as in the baseline model. However, now at $t = 1$ a sunspot $S = \{N, P\}$ is revealed that affects *all* the countries, where N refers to normal and P to Paneuropean panic. After S is revealed the governments have to decide if to bailout banks in case they violate the solvency condition, in order to avoid the destruction of a fraction θ of projects. The bailout is financed with the issuance of additional sovereign debt sold to non-European investors. At $t = 2$ countries learn about their idiosyncratic productivity, which is iid distributed. Then each government decides if to repay the outstanding debt. Note that banks' solvency does not depend on the domestic default decision, since they hold the ESBies whose return is certain by the law of large numbers. Therefore only the exogenous output cost ϑ matter.

The following proposition states that an equilibrium with the doom loop exists.

Proposition 9. *For θ sufficiently large, a subgame perfect sunspot equilibrium exists and is characterized by the following:*

1. *The initial debt issuance is given by the minimum level of debt B_0 that solves*

$$B_0 = \frac{G}{1 - F\left(\frac{1}{\vartheta}\frac{B_0}{K_0}\right)}$$

   (a) *The price at issuance is given by*

$$q_0 = 1 - F(\omega^N)$$

   where $\omega^N = \frac{1}{\vartheta}\frac{B_0}{L_0}$.

   (b) *The price of the senior tranche is given by*

$$Q_0^s = \min\left\{1, \frac{1 - F(\omega^N)}{1 - \varsigma}\right\}$$

   and of the junior tranche

$$Q_0^j = \max\left\{0, \frac{\varsigma - F(\omega^N)}{1 - \varsigma}\right\}$$

2. *For $S = N$ (normal times) no bailout is necessary. The prices are the same as at issuance and the default threshold is ω^N*

3. *For $S = P$ (panic) a bailout is implemented and the default threshold and the price of debt are the solution to the system*

$$\begin{aligned} \omega^P &= \frac{1}{\vartheta}\frac{B_0}{K_0} + \frac{1}{\vartheta}\frac{\frac{Y_0^h - (1-\theta)K_0}{q_1^P} - B_0}{K_0} \\ q_1^P &= 1 - F\left(\omega^P\right) \end{aligned}$$

   where ω^P is the default threshold. The prices of the tranches are given by

$$\begin{aligned} Q_1^{P,j} &= 0 \\ Q_1^{P,s} &= \frac{1 - F(\omega^P)}{1 - \varsigma} \end{aligned}$$

The conditions for the existence of a panic equilibrium are the same as in Proposition 2. As long as banks are exposed to sovereign debt $D_0 > L_0$, bailouts are desirable and TFP ω has bounded support such an equilibrium exists. Diversification and tranching do not remove this equilibrium.

So how does the sunspot equilibrium without ESBies from Proposition 2 compare to the sunspot equilibrium with ESBies here? One evident difference is that the default threshold in normal times now depends on the ratio of total debt to exogenous default costs B_0/ϑ, as opposed to the ratio of foreign held debt and the sum of the exogenous costs of default and the financial disruption B_0^f/Θ. This outcome is similar to that in the *no exposure economy* in Proposition 4, just that here we assume that all debt is pooled and thus held by foreigners, whereas there only a part was sold to foreigners. ESBies just provide a way to create the safe asset, which before we simply assumed to exist.[17] Extending Propositions 4 and 5 it is straightforward to rank the bond prices q^n and q^N from Propositions 2 and 9 and the associated levels of welfare.[18] Conditional on the normal state, the bond price is lower with ESBies than without and the default probability higher. Furthermore, welfare is lower with ESBies under the conditions on F in proposition 5.

In sum, the introduction of ESBies increases the default probability in normal times and does not rule out the doom loop. However, ESBies change the nature of the loop, as now the panic affects all the countries at once. This result holds for any level of subordination. What a higher level of subordination does is to increase the spreads and consequently the fraction of defaults observed in the panic equilibrium. The higher the subordination, the greater the panic needs to be to make the banks insolvent and consequently the higher the cost of the bailouts. This translates into welfare (conditional on the panic) decreasing as subordination goes up.

Our results are in stark contrast with Brunnermeier et al. (2016, 2017). In those papers, diversification and tranching effectively rules out the doom loop. In particular for a high enough level of bank capitalization, the introduction of ESBies removes the risk of the doom loop. There are several differences between the two setups,[19] but the two key differences that explain why ESBies are effective in their setup are the following: First, they assume that bailouts are financed by issuing senior government debt that is paid back with certainty even if the remaining debt is partially defaulted upon and that is thus sold at face value. By contrast, in our model the government finances the bailout by issuing additional sovereign debt that has no preferential treatment with respect to previously issued debt and consequently is valued at market prices. Second, in our model default is strategic such that bond holders may end up getting nothing, while in theirs the government mechanically pays bond holders as much as it can given an exogenous tax capacity, such that they always get something. The first difference strengthens the strategic complementarities in our model, the second ensures the existence of a sunspot equilibrium with a nonzero bond price.

Since the panic is driven by a sunspot, our model of course remains silent about the probabilities of the panic in the ESBies and the baseline model. Furthermore, symmetry may not

[17] Safe in the sense that there is no uncertainty about the payoff of the asset, even if the payoff is below the face value as some countries do default.

[18] The ESBies economy in normal times (N) resembles the no exposure economy (ne, S) from Propositions 4 and 5 with the twist that $B_0^{h,ne,S} = 0$.

[19] The default is not strategic in Brunnermeier et al. (2017) and the government repayment is restricted by the primary surplus that is a random variable with a binary distribution. Therefore the composition of debt holders is irrelevant for repayment as opposed to our setup, where repayment incentives depend on how much debt is held by domestic banks and foreign investors. The underlying structure to generate the doom loop is basically the same: there is a sunspot variable that generates debt repricing and if banks become insolvent then a fraction of the loans are destroyed.

be a reasonable assumption, since ESBies may help less solvent countries to benefit from more solvent countries. Yet the results that normal times get worse (through the *temptation and commitment channels)* and that panics may still happen serve as warning.

8 Conclusion

Banks' exposure to sovereign debt gives rise to the doom loop: A fall in the price of debt can necessitate a bailout, which raises debt and hence the default probability, justifying the fall in the price of debt. However, the same exposure also provides commitment to the government to repay, thus sustaining sovereign debt. This paper combines these two views to challenge a major policy conclusion that can be derived from looking at the doom loop in isolation: that banks' exposure to their government should be reduced.

We show that an increase in the bank equity ratio – which may be a response to the no bailout commitment – or a reduction of bank bond holdings that is sufficiently large to mute the doom loop comes at a cost: Sovereign spreads rise and welfare drops because the commitment value of banks' exposure disappears and the temptation to default rises. Rather, we argue that it is desirable that banks expand their exposure to public debt in times of sovereign distress, thus acting as lenders of last resort and breaking the doom loop.

These result may serve as a warning to the policy makers, which often express discomfort especially about banks high exposures to domestic sovereign debt. Maybe such exposure has more upsides than downsides after all.

Furthermore, we argue that taking into account the incentives to default can also affect the effectiveness of policies such as the ECB's transmission protection instrument or ESBies. While ruling out the original doom loop, both may introduce new types of self-fulfilling crises. However, in the case of the TPI, a careful calibration of the instrument can mitigate this risk.

While our model is no doubt stylized, it is straightforward to extend our analysis along several dimensions. First, for simplicity we assumed that the doom loop is perceived as a zero probability event. Yet by continuity our results would hold as long as it is sufficiently unlikely. Second, in our analysis banks' exposure did not affect the government's default cost *at the margin*. This allowed us to clearly separate between the negative and positive effects that foreign and domestically held debt have on repayment incentives. Allowing bank's exposure to also have a positive effect at the margin would strengthen our mechanism further.

Finally one caveat is in place. Our government is benevolent and maximizes national welfare. Thus there is no role for asset markets to discipline undesirable overspending by self-interested politicians, which might reduce the benefits of the additional commitment that banks' exposure provides.

References

Abad, J. (2019). "Breaking the sovereign-bank nexus". Working Paper.

Acharya, V., I. Drechsler and P. Schnabl. (2014). "A pyrrhic victory? Bank bailouts and sovereign credit risk". *The Journal of Finance*, 69(6), pp. 2689-2739. https://doi.org/10.1111/jofi.12206

Andreeva, D. C., and T. Vlassopoulos. (2019). "Home Bias in Bank Sovereign Bond Purchases and the Bank-Sovereign Nexus". *International Journal of Central Banking*, 15(1), pp. 157-197. https://doi.org/10.2139/ssrn.2910883

Balke, N. (2023). "The employment cost of sovereign default". Technical Report 2023-55. https://doi.org/10.2139/ssrn.4424032

Balloch, C. M. (2016). "Default, commitment, and domestic bank holdings of sovereign debt". Working Paper.

Balteanu, I., and A. Erce. (2017). "Linking Bank Crises and Sovereign Defaults: Evidence from Emerging Markets". Working Papers, 22, European Stability Mechanism. https://doi.org/10.2139/ssrn.2972818

Basu, S. (2010). *Sovereign Debt and Domestic Economic Fragility*. https://doi.org/10.20955/es.2010.22

Benassy-Quere, A., M. Brunnermeier, H. Enderlein, E. Farhi, M. Fratzscher, C. Fuest, P.-O. Gourinchas, P. Martin, J. Pisani-Ferry, H. Rey, I. Schnabel, N. Veron, B. Weder di Mauro and J. Zettelmeyer. (2018). "Reconciling risk sharing with market discipline: A constructive approach to euro area reform". Working Paper.

Bolton, P., and O. Jeanne. (2011). "Sovereign Default Risk and Bank Fragility in Financially Integrated Economies". *IMF Economic Review*, 59(2), pp. 162-194. https://doi.org/10.3386/w16899

Boz, E., P. D'Erasmo and B. Durdu. (2014). "Sovereign Risk and Bank Balance Sheets: The Role of Macroprudential Policies". Working Paper.

Broner, F., A. Martin and J. Ventura. (2010). "Sovereign Risk and Secondary Markets". *American Economic Review*, 100(4), pp. 1523-1555. https://doi.org/10.1257/aer.100.4.1523

Brunnermeier, M. K., L. Garicano, P. R. Lane, M. Pagano, R. Reis, T. Santos, D. Thesmar, S. V. Nieuwerburgh and D. Vayanos. (2016). "The Sovereign-Bank Diabolic Loop and ESBies". *American Economic Review*, 106(5), pp. 508-512. https://doi.org/10.3386/w21993

Brunnermeier, M. K., S. Langfield, M. Pagano, R. Reis, S. V. Nieuwerburgh and D. Vayanos. (2017). "ESBies: safety in the tranches". *Economic Policy*, 32(90), pp. 175-219. https://doi.org/10.1093/epolic/eix004

Brutti, F. (2011). "Sovereign defaults and liquidity crises". *Journal of International Economics*, 84(1), pp. 65-72. https://doi.org/10.1016/j.jinteco.2011.02.001

Burnside, C., M. Eichenbaum and S. T. Rebelo. (2000). "On the Fundamentals of Self-Fulfilling Speculative Attacks". NBER Working Papers, 7554, National Bureau of Economic Research. https://doi.org/10.3386/w7554

Calvo, G. A. (1988). "Servicing the Public Debt: The Role of Expectations". *American Economic Review,* 78(4), pp. 647-661. https://doi.org/10.1016/B978-0-444-87387-3.50017-7

Cooper, R., and K. Nikolov. (2013). "Government Debt and Banking Fragility: The Spreading of Strategic Uncertainty". NBER Working Papers, 19278, National Bureau of Economic Research. https://doi.org/10.3386/w19278

Cooper, R., and K. Nikolov. (2018). "Government Debt and Banking Fragility: The Spreading of Strategic Uncertainty". *International Economic Review,* 59(4), pp. 1905-1925. https://doi.org/10.1111/iere.12323

Cooper, R., and T. W. Ross. (1998). "Bank runs: Liquidity costs and investment distortions". *Journal of Monetary Economics,* 41(1), pp. 27-38. https://doi.org/10.1016/S0304-3932(97)00070-6

Engler, P., and C. Grosse Steffen. (2016). "Sovereign risk, interbank freezes, and aggregate fluctuations". *European Economic Review,* 87, pp. 34-61. https://doi.org/10.1016/j.euroecorev.2016.02.012

Erce, A. (2012). "Selective sovereign defaults". Globalization and Monetary Policy Institute Working Papers, 127, Federal Reserve Bank of Dallas. https://doi.org/10.24149/gwp127

Farhi, E., and J. Tirole. (2016). "Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops". Working Papers, 164191, Harvard University OpenScholar. https://doi.org/10.3386/w21843

Gennaioli, N., A. Martin and S. Rossi. (2014). "Sovereign Default, Domestic Banks, and Financial Institutions". *Journal of Finance,* 69(2), pp. 819-866. https://doi.org/10.1111/jofi.12124

Leonello, A. (2017). "Government guarantees and the bank-sovereign nexus". *Research Bulletin,* 35.

Mallucci, E. (2014). "Sovereign Defaults, Wholesale Funding and Banking Crises". Working Paper.

Mayer, M. (2011). "Sovereign Credit Risk and Banking Crises". SSRN Scholarly Paper ID 1785893, Social Science Research Network, Rochester, NY. https://doi.org/10.2139/ssrn.1785893

Messori, M., and S. Micossi. (2018). "Counterproductive Proposals on Euro Area Reform by French and German Economists". Working Paper.

Perez, D. (2015). "Sovereign Debt, Domestic Banks and the Provision of Public Liquidity". Working Paper, Department of Economics, Stanford.

Reinhart, C. M., and K. S. Rogoff. (2011). "From Financial Crash to Debt Crisis". *American Economic Review,* 101(5), pp. 1676-1706. https://doi.org/10.1257/aer.101.5.1676

Sosa-Padilla, C. (2018). "Sovereign Defaults and Banking Crises". *Journal of Monetary Economics,* 99, pp. 88-105. https://doi.org/10.1016/j.jmoneco.2018.07.004

Sturzenegger, F., and J. Zettelmeyer. (2007). "Creditors' Losses Versus Debt Relief: Results from a Decade of Sovereign Debt Crises". *Journal of the European Economic Association,* 5(2-3), pp. 343-351. https://doi.org/10.1162/jeea.2007.5.2-3.343

Thaler, D. (2019). "Sovereign default, domestic banks and exclusion from international capital markets". Documentos de Trabajo, 1824, Banco de España. https://repositorio.bde.es/handle/123456789/8825

Appendix

A Bank runs and the solvency condition

Here we show how the solvency condition we use for the banking sector can be rationalized by the possibility of a bank run by depositors. We focus only on the depositors bank run decision and take as given prices and their possible dynamics implied by our baseline model. By equilibrium we will refer in this section to the strategies of the depositors only, where they play a coordination game if to run on the bank or not.

There is a continuum of households of measure one, each of whom has an endowment given by Y_0^h. The households only enjoy consumption at period $t = 2$ and have available a storage technology with zero net return. Households are offered also the possibility to buy deposit contracts from the bank. These contracts have a price of p_0^D and promise a payment of 1 upon withdrawal at any period. Nevertheless, early withdrawals at $t = 1$ or $t = 2$ entail a cost λ for the depositor as a fraction of its deposits. It is a cost that captures the costly monitoring of the liquidated assets by the bank that are transferred to the household, since resources are only available after production.

Households problem is then to decide how many deposit contracts to acquire at $t = 0$ and if to early withdraw at $t = 1$ or at $t = 2$. The new information that the household has available at each t is the prices of the assets held by the bank and if others depositors are running. We assume that households expect that prices will not change at $t = 1$ and at $t = 2$ the price of sovereign debt becomes zero with probability F_ω. In the bigger context of the baseline model this states that the sunspot panic is unexpected at $t = 1$ and at $t = 2$ the perceived default probability is F_ω.

We now show that given our assumptions 1 and 2 a bank run at $t = 1$ is an equilibrium response to an unexpected change in the price of sovereign only if the following insolvency condition is satisfied

$$D_0 > (1 - \theta)L_0 + q_1 B_0^h$$

and a bank run is also an equilibrium response at $t = 2$ only if the government defaults. Otherwise, if the price of sovereign debt does not satisfy the insolvency condition at $t = 1$, and if the government does not default there is no equilibrium bank run.

The initial portfolio of the bank is characterized by

$$p_0^D D_0 + Y_0^b = q_0 L_0 + q_0 B_0^D$$

where we have set the price of loans and deposits equal to that of sovereign debt as it holds in equilibrium in the baseline model. We also set the price of sovereign debt to $q_0 = 1 - F_\omega$, the expected repayment probability, having that households share the same beliefs as international creditors and the bank. Then we have that $Y_0^h = q_0 D_0$ and from the baseline model $K = q_0 L_0$.

We can solve the model by backward induction. At $t = 2$ after no panic in $t = 1$ we have that the government announces if it repays or not. If the government repays in full then we have that if the asset value of the bank if liquidated satisfies

$$(1-\theta)L_0 + B_0^D > D_0$$

this follows from assumption 2 and $p_0^D = q_0 = p_0^L$ as follows.

$$\begin{aligned}
(1-\theta)L_0 + B_0^D &> D_0 \\
\Leftrightarrow (1-\theta)\frac{K}{q_0} + \left(\frac{Y_0^b + Y_0^d - K}{q_0}\right) &> \frac{Y_0^h}{q_0} \\
\Leftrightarrow (1-\theta)K + \left(Y_0^b - K\right) &> 0 \\
\Leftrightarrow Y_0^b &> \theta K
\end{aligned}$$

So if all depositors run in this case, the bank has enough assets to liquidate all deposits at face value and still have some assets left. In this case the marginal depositor would be better off by waiting and obtain the same payment but avoid the liquidation cost λ.

On the other hand if there is default by the government the bank becomes insolvent in case of a run since

$$(1-\theta)L_0 < D_0$$

from our assumption 1. Then in this case there is a run equilibrium, since if all depositors are running there will be no assets left at the bank.

At $t = 1$ the households have to decide if to withdraw or not. If the insolvency condition is violated we have that

$$D_0 < (1-\theta)L_0 + q_1 B_0^h$$

so again if all depositors run in this case, the bank has enough assets to liquidate all deposits at face value and still have some assets left. Then it is not optimal to run because the a single depositor could decide not to withdraw and the be paid with certainty with the repayment of the loans that were not liquidated without having to incur in the liquidation cost λ.

On the other hand, if the price of sovereign debt changes and satisfies that

$$D_0 > (1-\theta)L_0 + q_1 B_0^h$$

then the liquidation value of the bank is lower than its liabilities. So if there is a bank run, each depositor finds it optimal to run too, as not running would entail a payoff with zero with certainty. In this case there is no value of $\lambda \in (0, 1]$ that could prevent a run.

B Amplification

As Cooper and Ross (1998) show, strategic complementarities – such as the one between regarding the bond price which our model considers – typically lead not only to multiplicity of equilibria, but also to amplification. We illustrate this next and show that our policy results carry over, thus highlighting how our model relates to the doom loop's amplifying role described by Farhi and Tirole (2016) and Acharya et al. (2014).

To this end, we remove the sunspot shock and rule out the self fulfilling sunspot equilibrium, and add a fundamental shock to agents expectations about the distribution of future productivity. That is, we replace the sunspot shock by a news shock in period 1. We also assume this shock to be perceived with probability zero at $t = 0$. This shock may capture diverse negative developments, including the outbreak of a global pandemic that shifts down the distribution of expected GDP. We choose this shock to illustrate how the doom loop amplifies fundamental shocks, in this case a shock to future productivity. The nature of the shock is irrelevant. A contemporaneous shock to the asset quality of banks or the world interest rate, for example, would be amplified in the same way.

There are now again two states $s \in \{n, r\}$ in period t=1, where n stands for normal as before, and r for recession. In state r the distribution of expected productivity in $t = 2$ shifts to the left. To parametrize this shift in a simple way, we add probability mass ϵ at the lower bound of the support of ω and scale the pdf to satisfy the probability axioms. This implies that the CDF in case $s = r$ is given by

$$F^r(\omega) = F(\omega)(1-\epsilon) + \epsilon \quad \text{for } \omega \in (\underline{\omega}, \overline{\omega}), \epsilon > 0$$

.

To make the shock interesting, we assume that the shock is big enough to threaten bank solvency.

Assumption 3. *Banks are not solvent if the productivity distribution is F^r, or equivalently*

$$\frac{Y_0^b - \theta K_0}{\left(Y_0^h + Y_0^b - K_0\right)} < \epsilon$$

This assumption guarantees that banks are insolvent in case of a recession: $B_0^h q_1^r + (1-\theta)L_0 < D_0$ where $q_1^r = F^r\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$. That is, we assume the opposite for the normal and the recession state: By assumption 2 banks are solvent in normal times, but by assumption 3 they are insolvent in recessions times. The equilibrium now depends on the fundamental shock, but otherwise closely resembles the sunspot equilibrium in proposition 2:

Proposition 10. *A fundamental equilibrium exists where in case of recession $s = r$ the price of sovereign debt falls in $t = 1$ and banks are bailed out by the new issuance of debt by the government.*

The price of debt in case of recession is given by the solution to the system

$$q_1^r = (1 - F(\tilde{\omega}^r))\,(1-\epsilon) \tag{7}$$

$$\tilde{\omega}^r = \omega^n + \frac{1}{\Theta}\frac{\frac{1}{q_1^r}\left(D_0 - (1-\theta)L_0\right) - B_0^h}{K_0} \tag{8}$$

where $\tilde{\omega}^r$ *is the default threshold in case of recession. The equilibrium variables at* $t = 0$ *and in case* $s = n$ *coincide with Proposition 1.*

Due to the doom loop the solution of the system ((7))-((8)) is not unique. Since we focus on amplification in this section, we restrict our attention to the equilibrium with the highest q_1^r.

To understand how the doom loop amplifies the news shock, consider an alternative version of the model where negative bank equity in period 1 is inconsequential such that the government would never bail out banks. In that case the equilibrium default threshold for ω would be always the same, no matter whether good or bad news arrive. The bond price would however reflect the relevant distribution of future productivity.

$$q^{r*} = 1 - F^r\left(\omega^n\right) \tag{9}$$

$$q^n = 1 - F\left(\omega^n\right) \tag{10}$$

$$\text{where } \omega^n = \frac{1}{\Theta}\frac{B_0^f}{L_0} \tag{11}$$

Comparing the baseline and this alternative model, it is clear that nothing changes in good times. In bad times however the doom loop matters. Even if it is absent, the bond price drops in bad times, but if it is present, the drop is larger ($q_1^r < q^{r*}$) since $\tilde{\omega}^r > \omega^n$. That is, the doom loop amplifies the drop in bond prices caused by the fundamental shock. The same holds for the associated drop in welfare.

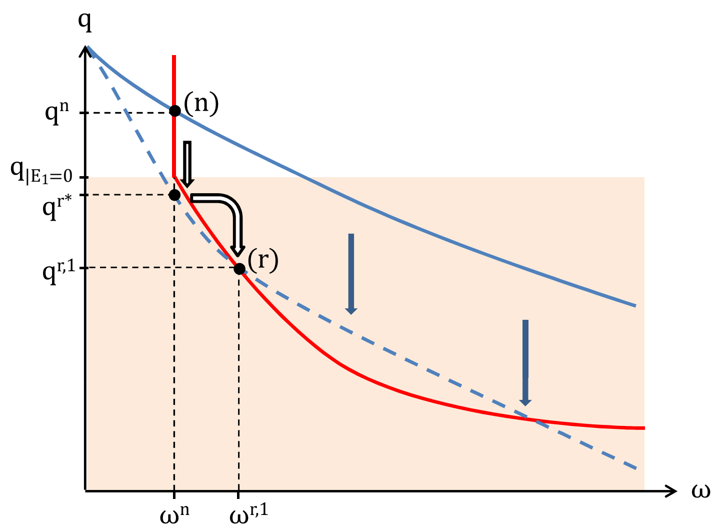


Figure 4: Amplification of a news shock

Figure 4 illustrates this graphically. When the bad state materializes, the bond price drops from q_1^n to q_1^{r*} in the absence of the doom loop. The doom loop then amplifies this initial drop and pushes the bond price further down to q_1^r.

Specifically, as in section 4, reducing the bank's exposure sufficiently by either increasing its equity ratio or decreasing its domestic bond holdings rules out the doom loop. This applies here too. However, propositions 4 and 5 apply as well, that is the success of these policies to rule out amplification has a cost in normal times: reducing banks exposure to sovereign debt to the point that they are solvent regardless of the price of sovereign debt reduces the bond price in normal times and reduces welfare, conditional on normal times and hence if the recession state is sufficiently unlikely. Furthermore, as in section 5, a bailout financed by domestic bonds – or equivalently a bailout with active secondary markets and a loose enough limit on bank bond holdings – disable the doom loop and hence its amplifying effect. We summarizes these points in the following proposition:

Proposition 11. *Define amplification as a situation where $q_1^r < q^{r*}$. Then:*

(i) If exposure is avoided by requiring banks to hold safe assets there is no amplification.

(ii) Bond prices are lower in the economy with no exposure.

(iii) Ex-ante welfare is lower in the economy with no exposure.

(iv) When the bank is bailed out with domestic bonds, there is no amplification.

C Proofs of propositions

C.1 Proposition 1

Proof. First we solve for the optimal default decision of the government at $t = 2$ for the case when default by the government triggers bank insolvency. We do it for some asset levels at $t = 1$ given by $\{B_0^h, B_1^f, L_0, D_0, S_1\}$ and K_0 projects financed.

The consumption if the government repays is given by

$$C^r = \omega K_0 - B_1^f + S_1$$

while if the government defaults we have

$$C^d = \omega(1-\vartheta)(1-\theta)K_0 + S_1$$

then the optimal default strategy is characterized by a productivity threshold

$$\tilde{\omega} = \frac{1}{\Theta}\frac{B_1^f}{K_0}$$

where $\Theta = 1-(1-\vartheta)(1-\theta)$ so if the productivity draw is above this threshold the government repays and otherwise defaults. In the particular case where no bailout was implemented at

$t=1$ then we have that $S_1=0$ and $B_1^f=B_0^f$ so consequently this threshold for the "no panic" event is given by

$$\tilde{\omega}^n=\frac{1}{\Theta}\frac{B_0^f}{K_0}$$

and the corresponding price of sovereign debt is then given by

$$q_0=q_1=1-F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$$

We assume for now that at that price the solvency condition holds for $t=1$ and will only hold at $t=1$ if debt is repaid. We will verify this after finding the asset prices compatible with this guess.

At $t=0$, since the banker expects that the repayment probability of sovereign debt is $q_0=1-F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$ and that in case of default he will be liquidated. The expected payment to the banker of providing loans is also q_0 and consequently $p_0^L=q_0$. Furthermore, as long as $p_0^D>q_0$ the bank has a positive intermediation margin and will leverage the maximum possible. So in equilibrium it holds that $p_0^D=q_0$ and in that case the banker is indifferent across the different levels of leverage.

We have then that at $t=0$, the portfolio of the bank satisfies

$$Y_0^D=p_0^D D_0$$
$$K_0=p_0^L L_0$$

and

$$Y_0^b+p_0^d D_0=p_0^L L_0+q_0 B_0^h$$

where prices satisfy

$$q_0=p_0^L=p_0^D=1-F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$$

The government the budget constraint implies that

$$B_0^f+B_0^h=\frac{G}{1-F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)}$$

and replacing B_0^h from the bank balance sheet we have

$$B_0^f=\frac{G+K_0-Y_0^b-Y_0^h}{1-F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)}$$

the minimum value of B_0^f that solves this equation is the level of debt held by foreigners in equilibrium.

We are left to verify that the bank satisfies the solvency condition at $t = 1$ and also it becomes insolvent at $t = 2$ if there only if there is a default.

Now, at $t = 2$ if the government defaults we have that the banks are insolvent if

$$D_0 > (1 - \theta)L_0$$

and this follows directly from assumption 1 and $p_0^D = p_0^L$. On the other hand if the government repays the bank is solvent if

$$D_0 < (1 - \theta)L_0 + B_0^h$$

or equivalently

$$\begin{aligned} Y_0^h &< (1 - \theta)K_0 + Y_0^b + Y_0^h - K_0 \\ \theta K_0 &< Y_0^b \end{aligned}$$

that is assumption 2.

For $t = 1$ we will show that there cannot be a bank run if for an F such that q_0 is close enough to 1. First, at $t = 1$ the solvency condition is

$$D_0 < (1 - \theta)L_0 + q_0 B_0^h$$

and using that $p_0^d D_0 = Y_0^h$, $p_0^L L_0 = Y_0^h$ we have that the solvency condition can be written as

$$\frac{Y_0^h}{p_0^d} < (1 - \theta)\frac{K_0}{p_0^L} + q_0 B_0^h$$

and using the period $t = 0$ balance sheet and that $p_0^D = p_0^L = q_0$ of the bank we have

$$\begin{aligned} \frac{Y_0^h}{q_0} &< (1 - \theta)\frac{K_0}{q_0} + Y_0^b + Y_0^h - K_0 \\ Y_0^h(1 - q_0) &< (1 - \theta)K_0(1 - q_0) + q_0 \left(Y_0^b\right) \\ Y_0^h &< (1 - \theta)K_0 + \frac{q_0}{(1 - q_0)}\left(Y_0^b\right) \end{aligned}$$

that is satisfied with q_0 close enough to 1.. □

C.2 Proposition 2

Proof. The variables at $t = 0$ are given in the proof of Proposition 1 (no sunspot) and are the same here since we assume the sunspot is perceived to have probability zero.

We proceed by guessing that in case of panic $s = p$ the price of government debt falls and the banks can remain solvent only with a bailout. Then we verify that the increase of debt to finance the bailout sustains the initial fall in the price of debt. We show that the bailout is optimal given that in the absence of a bailout there would be a bank run at $t = 1$ that implies a lower welfare.

Let the price of sovereign debt at $t = 1$ for $s = p$ with an announced bailout be given by q_1^p, we conjecture that q_1^p is low enough to make the bank insolvent

$$q_1^p B_0^h + (1 - \theta) L_0 < D_0$$

such that the liquidation value of the banks assets is below the resources required to satisfy the early withdrawals. The necessary bailout transfer to make the bank solvent and rule out a bank run is given by a level of safe assets S_1 given by

$$S_1 = D_0 - \left(q_1^p B_0^h + (1 - \theta) L_0 \right)$$

Since this transfer is financed with the issuance of sovereign debt, the new issuance required to finance this transfer is given by

$$\begin{aligned} \Delta B_1^f &= \frac{S_1}{q_1^p} \\ \implies \Delta B_1^f &= \frac{D_0 - (1 - \theta) L_0}{q_1^p} - B_0^h \end{aligned}$$

and consequently the total debt held by foreigners is

$$B_1^f = B_0^f + \frac{D_0 - (1 - \theta) L_0}{q_1^p} - B_0^h$$

The price of debt q_1^p is given by the repayment probability

$$q_1^p = 1 - F(\tilde{\omega}^p)$$

where $\tilde{\omega}^p$ is the repayment threshold, that as shown in the previous proof is a function of foreign held debt as follows

$$\begin{aligned} \tilde{\omega}^p &= \frac{1}{\Theta} \frac{B_1^f}{K_0} \\ &= \omega^n + \frac{1}{\Theta} \frac{\frac{D_0 - (1-\theta) L_0}{q_1^s} - B_0^h}{K_0} \end{aligned}$$

then we have that the equilibrium price of debt q_1^p and the default threshold $\tilde{\omega}^p$ are the solution to the system of equations

$$\begin{aligned} q_1^p &= 1 - F\left(\tilde{\omega}^p\right) \\ \tilde{\omega}^p &= \omega^n + \frac{1}{\Theta} \frac{\frac{D_0 - (1-\theta) L_0}{q_1^p} - B_0^h}{K_0} \end{aligned}$$

This system has a solution, note that the system can be rewritten as

$$
\begin{aligned}
G_1(\omega) &\equiv q_1^p(\omega) = 1 - F\left(\omega\right) \\
G_2(\omega) &\equiv q_1^p(\omega) = \frac{D_0 - (1-\theta)L_0}{B_0^h + \Theta K_0\left(\omega - \tilde{\omega}^n\right)}
\end{aligned}
$$

where we have defined functions $G_1(\omega)$ and $G_2(\omega)$ for convenience. An equilibrium price q_1^p is sustained by a threshold $\tilde{\omega}^p$ if $G_1(\tilde{\omega}^p) = G_1(\tilde{\omega}^p) = q_1^p$.

First note

$$
\begin{aligned}
G_2(\tilde{\omega}^n)B_0^h &= D_0 - (1-\theta)L_0 \\
&= \left(Y_0^h - (1-\theta)K_0\right)\frac{1}{q_0}
\end{aligned}
$$

and

$$
\begin{aligned}
G_1(\tilde{\omega}^n)B_0^h &= \left(1 - F(\tilde{\omega}^n)\right)B_0^h \\
&= Y_0^b + Y_0^h - K_0
\end{aligned}
$$

it follows that for q_0 close enough to one we have that $G_1(\tilde{\omega}^n) > G_2(\tilde{\omega}^n)$.Second note that both functions are non-increasing in ω and both have the limit of 0 as $\omega \to \infty$.

Third since we assume that the support of omega is bounded above, there is a $\bar{\omega}$ for which $F(\bar{\omega}) = 1 \Longrightarrow G_1(\bar{\omega}) = 0$ while $G_2(\omega) > 0$ for all ω as given by assumption 1. So it has to be that the two curves cross at least once. We focus on the equilibrium with the highest price. □

Now we move to show the conditions for the bailout to be optimal.

The value of no bailout does not depend on the panic price q_1^p. While the value of bailout depends on q_1^p. The expected consumption after a bailout is

$$
\begin{aligned}
E(c^{bailout}) &= \int_0^{\omega^p} \left((1-\theta)(1-\vartheta)\omega K_0 + S_1\right)dF(\omega) + \int_{\omega^p}^{\bar{\omega}} \left(\omega K_0 + S_1 - B_0^f - \frac{S_1}{q_1^p}\right)dF(\omega) \\
&= \int_0^{\omega^p} \left((1-\theta)(1-\vartheta)\omega K_0\right)dF(\omega) + \int_{\omega^p}^{\bar{\omega}} \left(\omega K_0 - B_0^f\right)dF(\omega)
\end{aligned}
$$

where

$$
\omega^p = \frac{1}{\Theta}\frac{B_0^f + \left(\frac{D_0 - L_0}{q_1^p} - B_0^h\right)}{K_0}
$$

where q_1^p depends on $F(.)$

While in case of no bailout we have

$$
E(c) = \int_0^{\omega^{run}} \left((1-\theta)(1-\vartheta)\omega K_0\right)dF(\omega) + \int_{\omega^{run}}^{\bar{\omega}} \left(\omega(1-\theta)K_0 - B_0^f\right)dF(\omega)
$$

where

$$\omega^{run} = \frac{1}{\vartheta(1-\theta)} \frac{B_0^f}{K_0}$$

where ω^{run} is the default threshold in case there is no bailout, so banks are liquidated at $t = 1$.

If $\omega^{run} \geq \omega^p$ then with certainty the bailout is optimal. This would be a sufficient condition. A sufficient condition for that to hold is that ϑ is low enough relative to θ. Note that in the extreme case of $\vartheta \to 0$ we have that the limit of $\omega^{run} = 0$, while ω^p is greater than zero. Even for $\vartheta > \theta$ the optimal bailout condition can be derived by placing constraints on F that imply that the price of debt in panic q_1^p is sufficiently high. We omit that proof here but is available upon request.

C.3 Proof Propositions 3 and 4

Proof. **Case i: No exposure (ne) achieved by a required level of safe assets S_0.**

In the no exposure economy banks are required to hold a level of safe assets S_0 such that they do not become insolvent in case of a sovereign default. This condition is given by

$$D_0 - (1-\theta)L_0 \leq S_0 \tag{12}$$

in this case the banks are not exposed and the doom loop is ruled out. Even if $q_1 = 0$ banks are solvent in $t = 1$ if condition 12 is satisfied.

Furthermore since banks are always solvent the return they pay to depositors is equal to the storage technology and consequently $p_0^d = 1$. Also the return charged to creditors becomes equal to 1 since now they are not discounted with the sovereign default probability, as a sovereign default does not imply insolvency. This implies that the minimum level of safe assets that guarantee no exposure is given by

$$S_0 = Y_0^h - (1-\theta)K_0 \tag{13}$$

When banks are not exposed then a sovereign default does not trigger the liquidation of the bank and consequently the output loss after default is only given by the fraction ϑ. Then repayment is optimal if

$$\begin{aligned} C^r &\geq C^d \\ \omega L_0 - B_0^f &\geq (1-\vartheta)\omega L_0 \\ \omega &\geq \frac{1}{\vartheta} \frac{B_0^f}{L_0} \end{aligned}$$

the default threshold in this case is given by $\frac{1}{\vartheta}\frac{B_0^f}{L_0}$ that is smaller than in the baseline case $\omega^n = \frac{1}{\Theta}\frac{B_0^f}{L_0}$ since $\theta > 0$ implies that $\Theta > \vartheta$. Then the system of equation that determine the price of debt and the level of debt issuance are given by

$$B_0^f = \frac{R + K_0 + S_0 - \left(Y^h + Y^b\right)}{q_0}$$
$$q_0 = 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{K_0}\right) \tag{14}$$

that given we have that $S_0 > 0$ and $\Theta > \vartheta$ has a solution with a lower q_0 than the system in Proposition 1. To show this rewrite the system as

$$H_1(B_0^f) = \frac{R + K_0 + S_0 - \left(Y^h + Y^b\right)}{B_0^f}$$
$$H_2(B_0^f) = 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{L_0}\right)$$

where an equilibrium price is the highest q_0 such that there is a B_0^f that satisfies $q_0^{ne,S} = H_1(B_0^{f,ne,S}) = H_2(B_0^{f,ne,S})$; where we have used superscript ne, S to refer to the no exposure case achieved by holding the safe asset S. We have that H_1 crosses from above H_2. First note that even for $S_0 = 0$ the fact that this system has ϑ instead of Θ implies that the curve $H_2(B_0^f)$ is displaced to the left, or formally

$$H_2(B_0^f) = 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{L_0}\right) \leq 1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right)$$

consequently the equilibrium happens at an equal or lower price than in the case with exposure. The equilibrium is at a strictly lower price if there is any mass for $\omega \in \left(\frac{1}{\Theta}\frac{B_0^f}{L_0}, \frac{1}{\vartheta}\frac{B_0^f}{L_0}\right)$ as then the previous inequality becomes a strict inequality. Furthermore by having $S_0 > 0$ lowers further the price of debt.

Case ii: No exposure (ne) achieved by having larger equity in the banking sector (e).

In the case where the no exposure is achieved by increasing bank equity we shift a fraction of households from savers to bankers. Such that the total endowment of the economy is constant $Y_0^b + Y_0^h$ but the initial resources owned by banks is larger up to the point where deposits are low enough such that

$$D_0 = (1 - \theta)L_0$$

so the solvency condition is satisfied even with a price of sovereign debt equal to zero.. In this case a single bank is indifferent between holding sovereign debt or the safe asset, although overall the total debt holdings of the baking system determine the risk premium of bonds. We focus on the case where banks hold $S_0 = 0$ that is the case with the minimum spread for government debt. In that case the system of equations that that determine the price of debt and the level of debt issuance is given by

$$
\begin{aligned}
B_0^f &= \frac{R + K_0 - \left(Y^h + Y^b\right)}{q_0} \\
q_0 &= 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{K_0}\right)
\end{aligned} \tag{15}
$$

where we have used the fact that the default cost is only ϑ as opposed to Θ. The solution to this system is the equilibrium price of debt $q_0^{ne,E}$ and debt issuance $B_0^{f,ne,E}$; where the superscript ne, E refers to the no exposure case by having higher equity.

Following the same argument as in the previous case this implies that the price of sovereign debt is lower than in the exposure case. Also, since $S_0 = 0$, the price is higher than in the case where no exposure is achieved with the safe asset.

□

C.4 Proof Proposition 5

First we show that welfare is lower in the no exposure economy where bank capital is larger (ne, E), compared to economy with exposure. Then we move to show that the welfare is even lower in the no exposure economy that has larger safe assets.

Proof. **Lower welfare in the no exposure and higher bank capital economy.**

Government revenues are the same in both economies, so we have that

$$
q_0^{ne,E}\left(B_0^{f,ne,E} + B_0^{h,ne,E}\right) = q_0\left(B_0^f + B_0^h\right)
$$

and since in both cases the banks invest the same amount of resources, we also have

$$
K_0 + q_0 B_0^h = K_0 + q_0^{ne} B_0^{h,ne}
$$

combining these two equations we get

$$
q_0^{ne,E} B_0^{f,ne,E} = q_0 B_0^f \tag{16}
$$

Next consider the consumption levels in the two economies. Consumption in the baseline economy with exposure - conditional on $s = n$ or the no sunspot equilibrium - is, in case of repayment r or default d, given by:

$$
\begin{aligned}
C^{n,r} &= \omega L_0 - B_0^f \\
C^{n,d} &= \omega(1-\vartheta)(1-\theta)L_0
\end{aligned}
$$

Expected consumption - conditional on $s = n$ or the no sunspot equilibrium - is thus

$$E(C) = \int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega(1-\vartheta)(1-\theta)L_0\partial F(\omega) + \int_{\frac{1}{\Theta}\frac{B_0^f}{L_0}}^{\bar{\omega}} \left(\omega L_0 - B_0^f\right)\partial F(\omega)$$

$$\implies E(C) = E(\omega)L_0 - \left[1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right)\right] B_0^f - \Theta L_0 \int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega f(\omega)d\omega$$

Consumption in the economy with no exposure is given by

$$C^{ne,E,r} = \omega L_0 - B_0^{ne,f}$$
$$C^{ne,E,d} = (1-\vartheta)\omega L_0$$

and consequently expected consumption is given by

$$E(C^{ne,E}) = E(\omega)L_0 - \left[1 - F\left(\frac{1}{\vartheta}\frac{B_0^{ne,f}}{L_0}\right)\right] B_0^{ne,f} - \vartheta L_0 \int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega$$

we can use 16 to write this as

$$E(C^{ne,E}) = E(\omega)L_0 - \left[1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right)\right] B_0^f - \vartheta L_0 \int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega$$

Welfare is lower in the no exposure economy if the following condition holds

$$E\left(C\right) > E\left(C^{ne,E}\right)$$
$$\implies -\Theta L_0 \int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega f(\omega)d\omega > -\vartheta L_0 \int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega$$

that can be rewritten as

$$\frac{\int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^f}{\Theta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^f}{\vartheta L_0}}$$

and using again equation 16 we have

$$\frac{\int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^f}{\Theta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}} \frac{1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right)}{1 - F\left(\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}\right)}$$

and since we have from Proposition 4 that

$$1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right) \geq 1 - F\left(\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}\right)$$

a sufficient condition that guarantees $E\left(C\right) > E\left(C^{ne,E}\right)$ is

$$\frac{\int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^f}{\Theta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,Ef}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}}$$

Note that each side of the inequality are the averages of $\omega f(\omega)$ within an interval. So since $\frac{1}{\Theta}\frac{B_0^f}{L_0} < \frac{1}{\vartheta}\frac{B_0^{fne,E}}{L_0}$ if we have that $\omega f(\omega)$ is not decreasing in $\left[0, \frac{1}{\vartheta}\frac{B_0^{fne,E}}{L_0}\right]$ the inequality is satisfied. This condition implies$f'(\omega) \geq \frac{-f(\omega)}{\omega}$ $\forall \omega \in \left[0, \frac{1}{\vartheta}\frac{B_0^{fne,E}}{L_0}\right]$.

Lower welfare in the no exposure with the safe asset requirement.

Consider the second variation from the previous proposition where banks have more safe assets and less bonds. Government revenues are the same in both economies, so we have that

$$q_0^{ne,E}\left(B_0^{f,ne,E} + B_0^{h,ne,E}\right) = q_0\left(B_0^f + B_0^h\right)$$

and since in both cases the banks invest the same amount of resources, we also have

$$K_0 + q_0 B_0^h = K_0 + q_0^{ne} B_0^{h,ne,S} + S_0^{ne,S}$$

combining these two equations we get

$$q_0^{ne,S}\left(B_0^{ne,S,f}\right) - S_0^{ne,S} = q_0\left(B_0^f\right) \tag{17}$$

and using Equation 13 we get

$$q_0^{ne,E} B_0^{ne,S,f} - \left(Y_0^h - (1-\theta)K_0\right) = q_0 B_0^f$$

Expected consumption in the economy with no exposure by holding safe assets is given by

$$E(C^{ne,S}) = E(\omega)L_0 - \left[1 - F\left(\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}\right)\right] B_0^{ne,S,f} - \vartheta L_0 \int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}} \omega f(\omega)d\omega + S_0^{ne,S}$$

and using equation 17 we obtain

$$E(C^{ne,S}) = E(\omega)L_0 - \left[1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{L_0}\right)\right] B_0^f - \vartheta L_0 \int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}} \omega f(\omega)d\omega$$

and consequently welfare is lower when no exposure is achieved by a safe asset requirement if

$$E\left(C^{ne,E}\right) > E\left(C^{ne,S}\right)$$
$$\implies -\vartheta L_0 \int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega > -\vartheta L_0 \int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}} \omega f(\omega)d\omega$$

that we can rewrite as

$$\frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^f}{\vartheta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^f}{\vartheta L_0}}$$

and using Equations 16 and 17 we have

$$\frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}\frac{q_0^{ne,E}}{q_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^f}{\vartheta L_0}}$$
$$\implies \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^{ne,S,f}}{\vartheta L_0}}\frac{q_0^{ne,E}}{q_0^{ne,S}}$$

as shown in Proposition 4 the ratio $\frac{q_0^{ne,E}}{q_0^{ne,S}} > 1$ and consequently a sufficient condition is that

$$\frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}} \omega f(\omega)d\omega}{\frac{B_0^{ne,S,f}}{\vartheta L_0}}$$

So since $\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0} < \frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}$ if we have that if $\omega f(\omega)$ is not decreasing in $\left(0, \frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}\right)$ the inequality is satisfied. □

C.5 Proposition 6

Proof. In this case the bank receives a bailout with bonds. The transfer of bonds an amount ΔB_1^h that satisfies:

$$q_1^p B_0^h + (1-\theta)L_0 + q^p \Delta B_1^h = D_0$$

The default threshold in this case ends up being the same as in the normal equilibrium

$$\tilde{\omega} = \frac{1}{\Theta}\frac{B_0^f}{L_0}$$

since the additional debt issuance does not distort the default decision. There is still a bank run in case of default and no bank run in case of full repayment. Consequently since in the normal equilibrium the bank is solvent, then there is no need for a bailout and the sunspot equilibrium disappears. □

C.6 Proposition 7

Proof. The newly debt issued to finance the bailout is given by

$$\Delta B_1 = \frac{1}{q_1^p} S_1$$

Starting from the case where the constraint binds, we have that the number of bonds that end up in foreign creditors hands is given by

$$\begin{aligned}
\Delta B_1^f &= \frac{1}{q_1^p} S_1 - \left(\frac{1}{q^{p,1}} \bar{B} - B_0^h \right) \\
&= \frac{1}{q_1^p} \left(D_0 - \left(q^p B_0^h + (1-\theta) L_0 \right) \right) - \left(\frac{1}{q^p} \bar{B} - B_0^h \right) \\
&= \frac{1}{q^p} \left(D_0 - (1-\theta) L_0 - \bar{B} \right) \\
\implies q^p \Delta B_1^f &= D_0 - (1-\theta) L_0 - \bar{B}
\end{aligned}$$

with these new levels of local and foreign debt, the total consumption if the government defaults is

$$\begin{aligned}
C^d &= (1-\vartheta)(1-\theta)\omega K_0 + S_1 - q^{p,1} \left(\frac{1}{q^{p,1}} \bar{B} - B_0^h \right) \\
&= (1-\vartheta)(1-\theta)\omega K_0 - ((1-\theta) L_0 - D_0) - \bar{B}
\end{aligned}$$

and under repayment is

$$\begin{aligned}
C^r &= \omega K_0 + S_1 - q^p \left(\frac{1}{q^p} \bar{B} - B_0^h \right) - B_1^f \\
&= \omega K_0 + S_1 - q^p \left(\frac{1}{q^p} \bar{B} - B_0^h \right) - \left(B_0^f + \frac{1}{q^p} \left(D_0 - (1-\theta) L_0 - \bar{B} \right) \right) \\
&= \omega K_0 + S_1 - q^p \left(\frac{1}{q^p} \bar{B} - B_0^h \right) - \left(B_0^f + \frac{1}{q^p} \left(D_0 - (1-\theta) L_0 - \bar{B} \right) \right)
\end{aligned}$$

so the default threshold is given by the value of ω that guarantees

$$\begin{aligned}
(1-\vartheta)(1-\theta)\omega K_0 + S_1 - q^p \left(\frac{1}{q^{p,1}} \bar{B} - B_0^h \right) &= \omega K_0 + S_1 - q^p \left(\frac{1}{q^p} \bar{B} - B_0^h \right) - \left(B_0^f + \frac{1}{q^p} \left(D_0 - (1-\theta) L_0 - \bar{B} \right) \right) \\
(1-\vartheta)(1-\theta)\omega K_0 &= \omega K_0 - \left(B_0^f + \frac{1}{q^p} \left(D_0 - (1-\theta) L_0 - \bar{B} \right) \right) \\
\omega &= \frac{1}{\Theta} \frac{B_0^f}{K_0} + \frac{\frac{1}{q^{p,1}} \left(D_0 - (1-\theta) L_0 - \bar{B} \right)}{\Theta K_0}
\end{aligned}$$

and consequently the default decision follows a threshold strategy and the system of equation which solution correspond to q^p and ω^p corresponds to

$$\begin{aligned}
q^p =& 1 - F(\omega^p) \\
\omega^p =& \frac{1}{\Theta} \frac{B_0^f}{K_0} + \frac{\frac{1}{q^p} \left(D_0 - (1-\theta) L_0 - \bar{B} \right)}{\Theta K_0}
\end{aligned}$$

The level of $\overline{B}$ to be binding is given by $D_0 - (1-\theta)L_0$, any level above that we have the whole bailout can be implemented with domestic bonds and we are back to proposition 6.

By using that $D_0 = \frac{Y^h}{p_0^d}$ and $L_0 = \frac{K_0}{p_0^L}$ and that $p_0^L = p_0^d = q_0$ we have that the level of $\overline{B}$ to be binding is

$$\frac{1}{q_0}\left(Y_0^h - (1-\theta)K_0\right)$$

. $\square$

C.7 Proof of Proposition 8

Proof. If there is a selloff of all debt to the ECB then the last period the government decides contingent on the productivity draw if to repay, default or exit the monetary union. The corresponding payoffs for each option are given by

$$C^r = \underbrace{\omega K_0}_{\text{Output}} + \underbrace{q^{ECB}B_0^h}_{\text{Reserves}} - \underbrace{B_0}_{\text{Debt repayment}} + \underbrace{(1-\phi)\left(B_0 - q^{ECB}B_0\right)}_{\text{ECB profits}}$$

$$C^d = \underbrace{\omega(1-\vartheta)K_0}_{\text{Output}} + \underbrace{q^{ECB}B_0^h}_{\text{Reserves}} - \underbrace{(1-\phi)q^{ECB}B_0}_{\text{Partial debt repayment to ECB}}$$

$$C^{Exit} = \underbrace{\omega(1-\vartheta^{Exit})K_0}_{\text{Output}} + \underbrace{q^{ECB}B_0^h}_{\text{Reserves}}$$

we characterize the solution using two thresholds. Let ω^{ECB} be the threshold for which for any $\omega > \omega^{ECB}$the repayment is preferred to default. This threshold is given by

$$\omega^{ECB} = \frac{1}{\vartheta}\frac{B_0}{K_0}$$

Second let ω^{Exit} be the threshold for which a productivity draw $\omega < \omega^{Exit}$ implies that exit is preferred to only default. The threshold is given by

$$\omega^{Exit} = \frac{(1-\phi)q^{ECB}}{\vartheta^{Exit} - \vartheta}\frac{B_0}{K_0}$$

These two thresholds characterize the optimal decision of the government if $\omega^{Exit} < \omega^{ECB}$ such that the government repays if $\omega \geq \omega^{ECB}$, defaults and stays in the union if $\omega^{Exit} \leq \omega < \omega^{ECB}$, and exits if $\omega < \omega^{Exit}$. The condition $\omega^{Exit} < \omega^{ECB}$ is satisfied for ϑ low enough. In particular the assumption corresponds to

$$\phi \geq \left(\frac{(q^{ECB})}{(q^{ECB}) + (\frac{\vartheta^{Exit}}{\vartheta} - 1)}\right)$$

Consequently the repayment probability in case of a selloff is given by

$$\tilde{q}^{ECB} = 1 - F\left(\frac{\phi}{\vartheta}\frac{B_0}{K_0}\right)$$

We focus on the case where this repayment probability is lower than in normal times $q_1^n = 1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$, this condition is satisfied again by a low enough ϑ such that

$$\phi \geq \frac{1}{\theta/\vartheta + (1-\theta)}\left(\frac{G + K - \left(Y^h + Y^b\right)}{G}\right)$$

Now we can proceed to consider three scenarios. In the first scenario, if the ECB sets their floor price $q^{ECB} < \tilde{q}^{ECB}$ then a selloff is not an equilibrium, because even after a selloff the expected repayment is higher than the floor price of the ECB. It is not optimal for the banks to sell their sovereign debt holdings. There is no panic also because we are considering only the cases where q^{ECB} is large enough such that banks are always solvent.

In the second scenario, $\tilde{q}^{ECB} < q^{ECB} < q^n$, there are two possible equilibria. In normal times banks coordinate on not selling off debt and then the expected repayment is that of normal times q^n for which it is not optimal indeed to sell the debt to the ECB. On the other hand if there is a panic and a selloff of sovereign debt to the ECB, then the expected repayment is $\tilde{q}^{ECB} < q^{ECB}$ and consequently the banks are indeed better off by selling all their sovereign debt holdings to the ECB. The ECB by buying sovereign debt at a price higher than the expected repayment has losses in expectation.

The third scenario $q^n < q^{ECB}$ makes a dominant strategy for banks to sell their sovereign debt irrespectively of what other banks do. The price offered by the ECB is above the expected repayment by the government even in normal times. Therefore in this case there is always a selloff of sovereign debt and an equilibrium with losses on expectation for the ECB. □

C.8 Proof Proposition 9

Proof. We solve for the equilibrium by backward induction. First focusing on the state where $S = N$ (normal state).

Now we have a continuum of countries and the banks are allowed only to hold the senior tranche of a CDO backed by the debt issued by each country. The subordination level is given by ς. Having a continuum of countries, this implies that domestic banks are not exposed to domestic sovereign debt.

We start from the conjecture that for $S = N$ no bailout has to be implemented in $t = 1$, then we verify this is the case. For $t = 2$, and $S = N$ the condition that guarantees that there is never a bank-run in $t = 2$ is given by

$$Q_2^s \mathcal{B}_0^s + (1-\theta)L_0 \geq D_0$$

that we assume holds and then also verify it. So under the two conjectures of no bailout and bank solvency in $t = 2$ even in case of sovereign default, total consumption in case of default is

$$C^d = (1-\vartheta)\omega L_0 + Q_2^s \mathcal{B}_0^s$$

and in case of repayment

$$C^{ne,r} = \omega L_0 + Q_2^s \mathcal{B}_0^s - B_0$$

and consequently the default threshold is given by $\omega^N = \frac{1}{\vartheta}\frac{B_0}{L_0}$. Note that the default cost is only captured in this case by ϑ, since there is there is no need to liquidate banks after a default.

Given this default threshold, the value of the senior tranche at $t = 2$ is given by

$$Q_2^s = \min\left\{1, \frac{1 - F\left(\frac{1}{\vartheta}\frac{B_0}{L_0}\right)}{1 - \varsigma}\right\}$$

and of the junior tranche

$$Q_2^j = \max\left\{0, \frac{\varsigma - F(\omega^N)}{1 - \varsigma}\right\}$$

In this case there is no uncertainty in $t = 2$ with respect to the payoff of the two tranches once the state $S = N$ is revealed in $t = 1$. Furthermore, since we assume that the possibility of panic is not anticipated by agents, the prices at $t = 1$ (for $S = N$) and at issuance $t = 0$ are the same of those in $t = 2$. Furthermore the price of the bond is given by the repayment probability

$$q_0 = 1 - F\left(\frac{1}{\vartheta}\frac{B_0}{K_0}\right)$$

and consequently the debt issuance required to obtain a revenue of G is the minimum value of B_0 that solves

$$G = B_0\left(1 - F\left(\frac{1}{\vartheta}\frac{B_0}{K_0}\right)\right)$$

where we have used the fact that $L_0 = K_0$ since all the projects are financed and at the price $p_0^L = 1$ since banks are not exposed to the sovereign default.

We are left to verify our two conjectures hold at these prices. Since bankers are always solvent the interest rate on loans and deposits is zero. This implies that $D_0 = Y_0^h$ and $L_0 = K_0$. Furthermore, the balance sheet of the bank at $t = 0$ satisfies that

$$Y_0^h + Y_0^b - K_0 = Q_0^S B_0^S$$

and replacing this in condition (2) we have

$$\begin{aligned} Y_0^h + Y_0^b - K_0 + (1-\theta)L_0 &\geq D_0 \\ \Longrightarrow Y_0^h + Y_0^b - K_0 + (1-\theta)K_0 &\geq Y_0^h \\ \Longrightarrow Y_0^b &\geq \theta K_0 \end{aligned}$$

the last expression is precisely the Assumption 2. So we verify that banks are solvent in $t = 1$. Following the same steps for the solvency condition at $t = 2$ we have again

$$Y_0^b \geq \theta K_0$$

So we have verified the two conjectures hold in equilibrium.

Now we move to the case where $S = P$ (world panic). We conjecture that banks become insolvent and require a bailout and then find the equilibrium price of debt to then verify indeed banks are insolvent at that price.

If the bank had to be bailed out in $t = 1$, then the bank got a transfer S_1 and the outstanding debt increased to $B_1 = B_0 + \frac{S_1}{q_1^p}$ where q_1^p is the price of sovereign debt at $t = 1$ in case of panic. In this case the consumption in case of repayment is

$$C^{ne,r} = \omega K_0 + Q_2^{s,p}\mathcal{B}_0^s + S_1 - B_1$$

and in case of default

$$C^{ne,d} = (1-\vartheta)\omega K_0 + Q_2^{s,p}\mathcal{B}_0^s + S_1$$

the default threshold is then given by

$$\begin{aligned}
\omega^P K_0 + Q_2^s\mathcal{B}_0^s + S_1 - B_1 &= (1-\vartheta)\omega^P K_0 + Q_2^s\mathcal{B}_0^s + S_1 \\
\omega^P &= \frac{1}{\vartheta}\frac{B_1}{K_0} \\
\Longrightarrow \omega^P &= \frac{1}{\vartheta}\frac{B_0}{K_0} + \frac{1}{\vartheta}\frac{S_1/q_1^p}{K_0}
\end{aligned}$$

the bailout transfer is set such that the bank is solvent and is given by

$$S_1 = D_0 - (1-\theta)L_0 - Q_1^{s,p}\mathcal{B}_0^s$$

this trivially satisfied that bank are solvent . To finance the bailout the newly issued debt is given by

$$\begin{aligned}
\Delta B_1 &= \frac{S_1}{q_1^p} \\
&= \frac{D_0 - Q_1^{s,p}\mathcal{B}_0^s - (1-\theta)L_0}{q_1^p}
\end{aligned}$$

replacing this in the default threshold we have

$$\omega^P = \frac{1}{\vartheta}\frac{B_0}{K_0} + \frac{1}{\vartheta}\frac{Y_0^h - (1-\theta)K_0 - Q_1^{s,p}\mathcal{B}_0^s}{q_1^p K_0}$$

and since in the panic equilibrium it has to be the senior tranche is partially defaulted we have that

$$Q_1^{s,p} = \frac{q_1^p}{1-\varsigma}$$

and since we defined the senior and junior tranche to have face value of 1, just as the bonds, we have that their total supply is given by $\mathcal{B}_0^s = (1-\varsigma)\mathcal{B}_0$ and in the symmetric equilibrium $\mathcal{B}_0 = B_0$ and replacing that in the default threshold we get

$$\omega^P = \frac{1}{\vartheta}\frac{B_0}{L_0} + \frac{1}{\vartheta}\frac{\frac{D_0-(1-\theta)L_0}{q_1^p} - B_0}{L_0}$$

then $t = 1$ the threshold for default and the price of debt is the solution to the system of equations

$$\begin{aligned} \omega^P &= \frac{1}{\vartheta}\frac{B_0}{K_0} + \frac{1}{\vartheta}\frac{\frac{Y_0^h-(1-\theta)K_0}{q_1^p} - B_0}{K_0} \\ q_1^p &= 1 - F\left(\omega^P\right) \end{aligned} \tag{18}$$

we can rewrite the system using two functions of q in terms of ω as

$$\begin{aligned} H_1(\omega) &= \frac{Y_0^h - (1-\theta)K_0}{\vartheta\omega L_0} \\ H_2(\omega) &= 1 - F(\omega) \end{aligned}$$

and an equilibrium threshold satisfies $q_1^p = H_1(\omega^P) = H_2(\omega^P)$ and q_1^p corresponds to the equilibrium price.

We have that i) both functions are decreasing in ω; ii) evaluated at $\omega^n = \frac{1}{\vartheta}\frac{B_0}{L_0}$ we have that $H_1(\omega^n) < H_2(\omega^n)$, this since $H_2(\omega^n)$ is the price of sovereign debt in the case $S = N$ and which ensures that banks are solvent. While $H_1(\omega^n)$ is the price of debt that makes the solvency condition be satisfied with equality. iii) We assume the support of ω is bounded above and and $F(\omega)$ is continuous.

Follows from i), ii) and iii) that the system of equations 18 has at least one solution with $\omega^P > \omega^N$ and consequently with a bailout.

□

C.9 Proof Proposition 10

Proof. The variables at $t = 0$ are given in the proof of Proposition 1 (no sunspot) and are the same here since we assume the recession is perceived to have probability zero.

First we show by contradiction than in case of recession, the bank becomes insolvent at $t = 1$. Suppose that for $s = r$ the bank is solvent in $t = 1$ and no bailout is requires. Then at $t = 2$ total consumption in case of repayment is given by

$$C^R = \omega K_0 - B_0^f$$

and in case of default

$$C^D = (1-\theta)(1-\vartheta)\omega K_0$$

the default threshold would then coincide with $\omega^N = \frac{1}{\Theta}\frac{B_0^f}{K_0}$. Now evaluating the solvency condition with this default threshold we have that the bank is insolvent and faces a bank run if

$$q_1^r B_0^h < D_0 - (1-\theta)L_0$$

where the price q_1^r is given by the repayment probability $q_1^r = 1 - F^r\left(\omega^N\right) = \left(1 - F\left(\omega^N\right)\right)(1-\epsilon)$ so ϵ determines the proportional fall in the price. Replacing this in the solvency condition we get

$$\begin{aligned} q_1^n B_0^h(1-\epsilon) &< D_0 - (1-\theta)L_0 \\ \left(Y_0^h + Y_0^b - K_0\right)(1-\epsilon) &< D_0 - (1-\theta)L_0 \end{aligned}$$

and replacing the price of loans and deposits

$$\left(Y_0^h + Y_0^b - K_0\right)(1-\epsilon) < \frac{Y_0^h}{p_0^D} - (1-\theta)\frac{K_0}{p_0^L}$$

that for q_0 close enough to one is guaranteed by Assumption (3). Consequently the bank is insolvent.

Let the price of sovereign debt at $t=1$ for $s=r$ and with an announced bailout be given by q_1^r, the necessary bailout transfer to make the bank solvent and rule out a bank run is given by a level of safe assets S_1 given by

$$S_1 = D_0 - \left(q_1^r B_0^h + (1-\theta)L_0\right)$$

Since this transfer is financed with the issuance of sovereign debt, the new issuance required to finance this transfer is given by

$$\begin{aligned} \Delta B_1^f &= \frac{S_1}{q_1^r} \\ \implies \Delta B_1^f &= \frac{D_0 - (1-\theta)L_0}{q_1^r} - B_0^h \end{aligned}$$

and consequently the total debt held by foreigners is

$$B_1^f = B_0^f + \frac{D_0 - (1-\theta)L_0}{q_1^r} - B_0^h$$

and we see how it depends on the price of sovereign debt.

The price of debt q_1^r is given by the repayment probability

$$q_1^r = 1 - F^r(\tilde{\omega}^r) = (1 - F(\tilde{\omega}^r))(1-\epsilon)$$

where $\tilde{\omega}^r$ is the repayment threshold, that as shown in the previous proof is a function of foreign held debt as follows

$$\begin{aligned}\tilde{\omega}^r &= \frac{1}{\Theta}\frac{B_1^f}{K_0} \\ &= \omega^n + \frac{1}{\Theta}\frac{\frac{D_0-(1-\theta)L_0}{q_1^s} - B_0^h}{K_0}\end{aligned}$$

then we have that the equilibrium price of debt q_1^r and the default threshold $\tilde{\omega}^p$ are the solution to the system of equations

$$\begin{aligned}q_1^p &= (1 - F(\tilde{\omega}^r))\,(1-\epsilon) \\ \tilde{\omega}^p &= \omega^n + \frac{1}{\Theta}\frac{\frac{D_0-(1-\theta)L_0}{q_1^p} - B_0^h}{K_0}\end{aligned}$$

Existence and optimality of the bailout follows the same steps as the proof of Proposition (2).. □

C.10 Proof Proposition 11

Proof. Claim (i)

No exposure requires than even after default banks remain solvent. So just as developed in the proof of section C.3 the minimum level of safe assets that guarantee no exposure is given by $S_0 = Y_0^h - (1-\theta)K_0$. Following exactly the same steps as that proof we have that period zero, equilibrium prices and debt issuance held by foreigners are the values that solve

$$B_0^{f,ne} = \frac{R + K_0 + S_0 - \left(Y^h + Y^b\right)}{q_0}$$

$$q_0^{ne} = 1 - F\left(\frac{1}{\vartheta}\frac{B_0^{f,ne}}{K_0}\right)$$

and since banks are not exposed there is no need for a bailout in case of a recession and the price in case of recession is given by

$$q_1^r = 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{K_0}\right) = q_0(1-\epsilon)$$

where since the default threshold in recession and normal times is the same we have no amplification. Amplification is only driven by the bailout and the subsequent increase in foreign held debt.

Claim (ii) and (iii)

Using the same argument in the proof of section C.3 the debt prices at issuance are lower in the no exposure economy and since the recession is also perceived to happen with probability zero the proof in section C.4 also proofs that welfare is lower in the no exposure economy for this case.

Claim (iv)

In this case the bank receives a bailout with bonds. The transfer of bonds is an amount ΔB_1^h that satisfies:

$$q_1^r B_0^h + (1-\theta)L_0 + q_1^r \Delta B_1^h = D_0$$

The default threshold in this case ends up being the same as in the normal equilibrium

$$\tilde{\omega}^r = \frac{1}{\Theta}\frac{B_0^f}{L_0}$$

since the additional debt issuance does not distort the default decision. There is still a bank run in case of default and no bank run in case of full repayment. The price of debt is nevertheless different, since the TFP distribution is shifted, and is given by

$$q_1^r = 1 - F^r\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right) < q_1^n$$

but there is no amplification as $q_1^r = q^{r*}$

$\square$

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