## DISCRETE PROBABILITY FORECASTS: WHAT TO EXPECT WHEN YOU ARE EXPECTING A MONETARY POLICY DECISION

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### DISCRETE PROBABILITY FORECASTS: WHAT TO EXPECT WHEN YOU ARE EXPECTING A MONETARY POLICY DECISION<sup>(\*)</sup>

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#### Abstract

We apply discrete probability forecasts to the expectations of monetary policy rate changes, both in the United States and in the euro area. By using binomial trees from options theory, forecast distributions are derived from the instantaneous forward yield curve, based on interest rate swaps. We then use a non-randomised discrete probability forecast evaluation that confirms the presence of a systematic upward bias, consistent with the presence of a term premium. Consequently, we propose a bias-correction methodology to increase the accuracy of the density forecasts regarding monetary policy expectations. This research provides pivotal insights into understanding and improving predictive tools in monetary policy forecasting.

Keywords: discrete probability forecast, monetary policy decisions, interest rate expectations, binomial tree.

JEL classification: C53, C58, G12, G17.

#### Resumen

En este documento se usan proyecciones de probabilidad discreta para las expectativas de los tipos de política monetaria en los Estados Unidos y el área del euro. Utilizando árboles binomiales a partir de la teoría de opciones, se derivan representaciones para las distribuciones de probabilidad utilizando la curva de rendimiento a plazo instantánea, determinada a partir de los *swaps* de tipos de interés. Posteriormente, evaluamos las proyecciones de probabilidad discreta utilizando una metodología no aleatorizada, que confirma la existencia de un sesgo sistemático al alza, coherente con la presencia de una prima a plazo. En consecuencia, proponemos un método para la corrección de este sesgo que permite aumentar la precisión de las expectativas de la política monetaria. Este trabajo proporciona los conocimientos fundamentales para comprender y mejorar las herramientas predictivas relacionadas con la política monetaria.

Palabras clave: proyecciones de probabilidad discreta, decisiones de política monetaria, expectativas de tipos de interés, árbol binomial.

Códigos JEL: C53, C58, G12, G17.

### 1 Introduction

The realm of central banking hinges on the effective use of monetary policy tools to achieve predefined targets primarily associated with inflation and unemployment (Friedman, 1995). Chief among these tools is the official policy rate, a powerful determinant of the broader economic landscape influencing price levels, bond markets, and the banking system (Svensson, 2012). However, it's not merely the actual decisions concerning policy rates that hold significance, but also the anticipation of future shifts in the official interest rate that shapes medium and long-term rates and have been the objective of the central banks use of Forward Guidance as a non-conventional monetary policy tool (Bernanke, 2020). Consequently, generating precise measures of these expectations is a crucial aspect for monetary policy makers and analysts, who can predict market reactions to monetary policy decisions, assess the efficacy of their communications related to the forward guidance of the monetary policy, and accordingly adapt their messages. This paper address this necessity by employing binomial trees to obtain discrete probability forecasts of monetary policy rates, and address the presence of an unobserved term premium, using the evaluation of those forecasts for generating unbiased predictions. Therefore, we enrich existing literature by offering both an evaluation mechanism and bias-corrected estimates.

Forecasting serves as an indispensable tool for economists and policy makers, aiding in informed decision-making. Historically, point forecasts, providing merely mean or median estimates, were prevalent. However, they offer limited insights, failing to account for forecast uncertainty or potential alternate outcomes. Consequently, literature has expanded beyond these to encompass density forecasts (Diebold et al., 1998) and interval forecasts (Christoffersen, 1998). Density forecasts associate probabilities with various potential outcomes, while interval forecasts allocate a given probability (for instance, 95% predictive intervals) to a range of forecasts.

These sophisticated evaluation techniques for density and interval forecasts have found wide applicability across diverse fields within economics and finance. For example, risk management extensively employs these tools via Value at Risk measures (Berkowitz, 2001; Lopez, 2001; Engle and Manganelli, 2004), contributing to a more comprehensive understanding of risk scenarios. Similarly, in Option Pricing, the full distribution of the underlying asset is essential (Hull and White, 1987), and these forecasts add significant value. Further, they are increasingly being used in inflation forecasting (Gimeno and Ibáñez, 2018; Hilscher et al., 2022), as well as in the burgeoning literature on Growth at Risk (Adrian et al., 2019; Chavleishvili and Manganelli, 2019; Plagborg-Møller et al., 2020; Brownlees and Souza, 2021; Carriero et al., 2022).

2 Although density forecasts offer more comprehensive insights than point forecasts, the evaluation mechanisms appropriate for the latter, such as those proposed by Diebold and Mariano (2002), do not apply to density forecasts. Diebold et al. (1998) have thus introduced a potent tool for assessing the accuracy of density forecasts, the probability integral transform  $(z_t)$ . This corresponds to the cumulative density function of the probability for a specified value  $y_t$ . Employing this transformation allows for testing whether estimated probabilities match the actual ones by verifying if  $z_t$  adheres to a uniform distribution. This method has been widely adopted for evaluating density forecasts across various domains: volatility models (Andersen et al., 2003), exchange rates (Patton, 2006), interest rates (Diebold and Li, 2006), GDP (Caselli et al., 2020), GDP growth and unemployment (Bowles et al., 2011), inflation (Clements, 2006; Galbraith and van Norden, 2012), or oil prices (Mazzeu et al., 2019).

While the evaluation of density forecasts offers substantial benefits, its application requires the forecast densities to be continuous. This requirement often clashes with practical situations where forecast distributions are inherently non-continuous, such as in the case of yield discrete forecasts. For example, questions like 'Who will win the next election?' or 'Will there be a recession next quarter?'. In such scenarios, the probability integral transform framework from Diebold et al. (1998) is not directly applicable. More precisely, the forecast pertains to the probability of occurrence of a particular binary or discrete event. Numerous studies (e.g., Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Chauvet and Potter, 2002; Liu and Moench, 2016) have adopted probit models to estimate economic recessions. Galbraith and van Norden (2011) explored the adequacy of recession forecasts from the Survey of Professional Forecasters (SPF) and determined the continuous conditional expectation function. Similarly, predicting election outcomes, where each candidate is given a probability of victory, serves as another instance of discrete probability forecasting (e.g., Wolfers and Zitzewitz, 2004; Erikson and Wlezien, 2008; Rothschild, 2009). Authors, such as Czado et al. (2009) or Kheifets and Velasco (2017), have developed methodologies to evaluate discrete densities as well as continuous ones, serving as the foundation for the methodology adopted in this paper.

actions, which can be understood as rate cuts or hikes, depending on the current economic environment. The provision of a probability distribution 3function for policy rates, as exemplified by the Federal Reserve Bank of Atlanta, provides a comprehensive view. It yields market-implied probabilities for the three-month average fed funds rate, conveying information beyond just the average expected value. The function also delineates the 5th to 95th percentile region and the density distribution of potential interest rate values. Financial data providers like Bloomberg or Thomson Reuters have also entered this space, generating policy rate probabilities for multiple Central Banks. Specifically, they procure probability estimates from fed fund futures (for the US) and Overnight Indexed Swap (OIS) rates available in each monetary policy area. However, the objective variable forecast (i.e., monetary policy rate changes), follows a discrete distribution, since monetary policy moves in discrete changes on days of monetary policy meetings. Monetary policy expectations hold considerable significance for various economic agents and financial analysts. Consequently, their frequent monitoring becomes crucial for central banks. Bauer and Rudebusch (2016) highlight the importance of monetary policy expectations as they convey information about participants' view of the future path of policy rates. In that sense, it reflects what the market expect about the next monetary policy

This paper makes three significant contributions. Firstly, we devise a clear and consistent methodology to derive the discrete probability forecasts of market participants, implicitly embedded in fixed income market prices. This approach offers a direct insight into the expectations implicit in market dynamics. Our second contribution is the evaluation of the forecasting accuracy of these subjective discrete probability forecasts. Indeed, we applied the methodology employed by other authors such as Czado et al. (2009) for the transformation of continuous forecasts into discrete ones. This examination brings to light the upward bias present as a consequence of the inherent risk aversion of investors and provides an estimate of their magnitude. Providing evaluation of the entire forecast distribution rather than a point or interval is more accurate (Berkowitz, 2001), since it produces additional information about the size or magnitude of the forecast error. Precisely, our last contribution is using this information to propose a correction for these biases in discrete probability forecasts, caused by the term premium. This correction allows for the extraction of unbiased, objective measures of market participants' expectations concerning the trajectory of monetary policy.

### 2 Obtaining Monetary Policy Forecasts

### 2.1 Sources of data

The computation of expectations for monetary policy rates is of extreme importance for financial market participants, economists and central banks, and different type of measures can be used. The first group refers to financial market-based indicators, and the second one to market participants' surveys. as input. Our paper focus on the first group and, more precisely, on the In the first group, one can refer to model-based measures where interest rate expectations are estimated through models that use financial market data use of swaps of policy rates both in the euro area and the United States. This technique is widely used by different market data providers, such as Thomson Reuters or Bloomberg, who provide probabilities of interest rate change expectations using financial derivatives, mainly overnight index swaps (OIS) and options (for the US).

An Overnight Index Swap (OIS) is an interest rate derivative where two agents agree to exchange fixed and floating payments during the life of the contract. The floating leg corresponds to short term rates, such as the effective federal funds rate (EFFR) for the US or the  $\epsilon$ STR for the euro area. It is computed as the accrued interest of investing a notional amount on an overnight rate and repeating this strategy during the life of the contract. The fixed leg is the agreed rate, called OIS rate, so that it represents a good proxy for the expected policy rate evolution (Lloyd, 2018).

OIS rates can be obtained from financial market providers and for different tenors, ranging from the one-week to 30 years' maturity. They are heavily traded by market participants because they are a very useful tool to change the duration risk of the assets and liabilities, without adding credit risk (no initial payment required) and low counterparty risk (further reduced by the use of margin calls). More precisely, bid-ask spreads, commonly used as a measure of liquidity have been relatively low over time, especially for some specific maturities. Figure 1 shows bid-ask spreads distribution along time for given maturities and both the Euro Area and United States. The bid-ask spreads are well below the 2 basis points (bp), with the median spread stays around 0.7 bp in both areas.

for given maturities and both the Euro Area and United States. The bid-ask

OIS rates have been widely used as a proxy for interest rate expectations by several authors. For instance, Taylor and Williams (2009) state that they can be used to measure the average expectation of overnight interest rates during a certain period, equal to the maturity of the swap. Moessner and Rungcharoenkitkul (2019) evaluate how the market reacts to economic news in the United States based on estimated probabilities of hitting the zero lower bound (ZLB) derived from OIS. Additionally, Bauer and Rudebusch (2013) employ overnight index swaps to analyse the signalling effect (i.e., change in expected path of future short-term rates) after different large-scale asset purchase (LSAP) announcements.

as a proxy of the expected interest rates path in any given date. Thus, to derive probabilities from these expected paths of short term rates, we need to transform the instantaneous forward rate into a Binomial Tree (Figure to transform the instantaneous forward rate into a Binomial Tree (Figure<br>2, bottom), which are commonly used for Option Pricing, in order to get probabilities of monetary policy changes from the Forward Curve. 2, bottom), which are commonly used for Option Pricing, in order to get Therefore, in this paper we use OIS rates to obtain the spot yield curve for each maturity, using the approach of Gimeno and Nave (2009) to obtain the Svensson (1994) yield curve (Figure 2, top). This way, we can obtain the forward instantaneous yield curve, that we are going to use (in a first stage) probabilities of monetary policy changes from the Forward Curve.

#### 2.2 From forward rates to binomial trees 2.2 From forward rates to binomial trees

 $C_1$  et al. (1979) introduced a simple discrete-time model for values  $C_1$  $\frac{C_{\text{OX}}}{C_{\text{OX}}}$  et al. (1979) introduced a simple discrete-time model for valuing financial options. The value of the options is related to underlying price evolution, which is unknown, but the authors assume the stock price follows a multi-<br>which is unknown, but the authors assume the stock price follows a multipricative binomial process over discrete periods. Following this approach, the price of the asset  $(\nu_t)$  in the following period  $(t+1)$  can take two possible values:  $S_{t+1} = u \cdot S_t$  (where  $u > 1$ ) implying that the underlying price S would go up), with probability p and  $S_{t+1} = d \cdot S_t$  (where  $d < 1$ , implying that the go up), with probability p and  $D_{t+1} - a \cdot D_t$  (where  $a \leq 1$ , implying that the underlying price S would go down), with probability  $1 - p$ . The multiplicaunderlying price S would go down), with probability  $1 - p$ . The multiplicative model implies repeating the binomial process at each period, to ensure a process at each period, to ensure<br> $\frac{1}{2}$  so  $d \cdot y = 1$ . That way possible that the tree is recombinant (i.e.,  $d = \frac{1}{u}$ , so  $d \cdot u = 1$ ). That way, possible outcomes for asset prices will be increasing over time (i.e., there would be<br>three possible values for  $S_{t+2} \in \{2d \cdot S, S, 2u \cdot S\}$ , four possible values for stated possible values for  $D_{t+2} \in \{2a \cdot D_t, D_t, 2a \cdot D_t\}$ , but possible values for  $S_{t+3} \in \{3d \cdot S, d \cdot S, u \cdot S, 3u \cdot S, 3u \cdot S\}$  and so on). When used for stocks, this  $S_{t+3} \in \{3d \cdot S_t, d \cdot S_t, u \cdot S_t, 3u \cdot S_t\}$ , and so on). When used for stocks, this framework implies transforming a continuous process (stock prices evolution) into a discrete one, by arbitrarily reducing the time intervals, and to simplify evolution of prices into two only possible outcomes (up and down, regardless In the magnitude of mose changes).<br>In the monetary policy rates, we have the monetary policy rates, we have the huge three huges plicative binomial process over discrete periods. Following this approach, the price of the asset  $(S_t)$  in the following period  $(t+1)$  can take two possible values:  $S_{t+1} = u \cdot S_t$  (where  $u > 1$ , implying that the underlying price S would go up), with probability p and  $S_{t+1} = d \cdot S_t$  (where  $d < 1$ , implying that the outcomes for asset prices will be increasing over time (i.e., there would be three possible values for  $S_{t+2} \in \{2d \cdot S_t, S_t, 2u \cdot S_t\}$ , four possible values for of the magnitude of those changes).

In the case of movements in the monetary policy rates, we have three huge advantages of using such procedure, relatively to stock prices. First, we know exactly when the changes will be (i.e., every time there is a monetary policy meeting), so the time intervals are not arbitrary, but known in advance. Second, the instantaneous forward rate provides with the expected (i.e., mean) evolution of the underlying asset. And third, changes in the interest rates are of discrete magnitudes. Although this magnitude has changed with time, we can approach market expectations with the size of the changes based on previous decisions.<sup>1</sup> More precisely, in our assessment, we will assume that the expected size of the change is equal to the last observed rate change.<br>We can change is equal to the last observed rate change. We can see this assumption is reasonable as the size of the changes tend to the can see this assumption is reasonable as the size of the enanges tend to be somewhat constant over certain periods of time, related to the monetary policy stance (Figure 3). we come that concern over correlate periods or this, related to the monetary be somewhat constant over certain periods of time, related to the monetary We can see this assumption is reasonable as the size of the changes tend to be somewhat constant over certain periods of time, related to the monetary<br>relievations (Eiguns 2)

The computation of such binomial tree (Figure 2, bottom) can be done as follows. First, one need to get the expected variations derived from the as follows. First, one need to get the expected variations derived from the change in the forward curve between each monetary policy meeting. This arises from the definition for the forward curve at a future meeting  $(f_{t+$ which can be understood (under a risk neutral perspective that we will relax change in the forward curve between each monetary policy inceting. This arises from the definition for the forward curve at a future meeting  $(f_{t+1})$ ,  $\frac{1}{2}$  is a fact of  $\frac{1}{2}$  are commonly used for  $\frac{1}{2}$ , in order to  $\frac{1}{2}$ , in order to  $\frac{1}{2}$ , in order to  $\frac{1}{2}$ , in a later section) as the sum of all possible values of the monetary policy m a fact section, as the same of an possible values of the monetary pency<br>rates  $i \in \{1, T\}$  on that meeting  $(u_{n+1})$  multiplied by the probability  $(u_{n+1})$  $\sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{j} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{$  $r_{\text{1}}$  is called by the probability ( $\frac{1}{100}$   $\frac{1}{100}$ ) multiplied by the probability  $\frac{1}{100}$ which can be understood (under a risk neutral perspective that we will relax in a later section) as the sum of all possible values of the monetary policy rates  $i \in \{1:T\}$  on that meeting  $(y_{t+1,i})$  multiplied by the probability  $(p_{t+1,i})$ rates  $i \in \{1 : T\}$  on that meeting  $(y_{t+1,i})$  multiplied by the probability  $(p_{t+1,i})$  associated to each rate (Equation 1), associated to each rate (Equation 1),

$$
f_{t+1} = \sum_{i}^{I} y_{t+1,i} \cdot p_{t+1,i} \tag{1}
$$

To transform equation 1 into the first step in a binomial tree, we need to identify two assumptions. First, that there are only two potential outcomes possible, which in the monetary policy case will be either maintaining the possible, which in the monetary policy case win be entire maintaining the<br>monetary policy rates unchanged or moving them. For the following meeting, monetary policy rates unchanged or moving them. For the following meeting, with a horizon of less than six weeks, markets have a clear expectations of the direction of this change in case there is one, either hike or cut, and the direction of this change in case there is one, either hike or cut, and<br>they never find in a situation where they give options to both hike and out rates. Thus our two options will be hike/maintain or cut/maintain, depending on the direction of the change. We will obtain the direction of depending on the direction of the didinge. We will obtain the direction of the change for each specific meeting from the slope of the forward curve (i.e., if  $f_{t+1} - f_t > 0 \to \text{change}_{t+1} = \text{hike};$  if  $f_{t+1} - f_t < 0 \to \text{change}_{t+1} = \text{cut}.$ Second, we need to determine the size of the change (i.e.,  $\delta$ , so  $\delta_{t+1} = \delta$  if change<sub>t+1</sub> = hike and  $\delta_{t+1} = -\delta$  if change<sub>t+1</sub> = cut) to the size of the last observed change in monetary policy rates, since monetary policy decisions  $\frac{1}{2}$  for the monetary policy rates, since monetary policy decisions than those changes. Using those assumptions, we can tends to have an inertia in those changes. Come those assumptions, we can<br>simplify equation 1, into equation 2, simplify equation 1, into equation  $\mathcal{Z}$ , cut rates. Thus our two options will be hike/maintain or cut/maintain, depending on the direction of the change. We will obtain the direction of depending on the direction of the change. We will obtain the direction of the change for each specific meeting from the slope of the forward curve (i.e., the change for each specific meeting from the slope of the forward curve (i.e., if  $f_{t+1} - f_t > 0 \rightarrow \text{change}_{t+1} = \text{hike}$ ; if  $f_{t+1} - f_t < 0 \rightarrow \text{change}_{t+1} = \text{cut}$ ).<br>Second, we need to determine the size of the change (i.e.,  $\delta$ , so change<sub>t+1</sub> = hike and  $\delta_{t+1} = -\delta$  if change<sub>t+1</sub> = cut) to the size of the last tends to have an inertia in those changes. Using those assumptions, we can identify two assumptions. First, that there are only two potential outcomes possible, which in the monetary policy case will be either maintaining the change<sub>t+1</sub> = hike and  $\delta_{t+1} = -\delta$  if change<sub>t+1</sub> = cut) to the size of the last observed change in monetary policy rates, since monetary policy decisions simplify equation 1, into equation 2, simplify equation 1, into equation 2,

$$
f_{t+1} = (f_t + \delta_{t+1}) \cdot p_{t+1} + f_t \cdot (1 - p_{t+1}) \tag{2}
$$

where  $f_t$ ,  $f_{t+1}$ , and  $\delta$  are known, while  $p_{t+1}$  (i.e., the probability of a change of monetary policy rate at meeting in  $t + 1$ ) can be directly deduced enarge of monetary poncy rate at meeting in  $v + 1$ ) can be uncerly deduced (Equation 3), Second, we need to determine the size of the change (i.e., δ, so δt+1 = δ if where  $f_t$ ,  $f_{t+1}$ , and  $\theta$  are known, while  $p_{t+1}$  (i.e., the probability of a<br>change of monetary policy rate at mosting in  $t+1$ ) can be directly deduced  $t_{\text{equation 3}}$ , where  $f_t$ ,  $f_{t+1}$ , and  $\delta$  are known, while  $p_{t+1}$  (i.e., the probability of a where  $f_t$ ,  $f_{t+1}$ , and  $\theta$  are known, while  $p_{t+1}$  (i.e., the probability of a<br>change of monetary policy rate at meeting in  $t + 1$ ) can be directly deduced  $\epsilon$  Similarly, the Federal Reserve Board has been adjusting the dimension of changes to dimension of changes to  $\epsilon$ 

bps. Similarly, the Federal Reserve Board has been adjusting the dimension of changes to

to be of 25 bps, with the exception of the accommodating monetary policy period. The accommodating monetary per

 $\frac{1}{1}$ Additionally, it is worth noting that, along history, central banks used to announce rate changes of equal or similar size within each monetary policy stance period. (Figure 3. In the Euro Area, the magnitude of interest rate changes has been mainly of 50 bps during the period before the Global Financial Crisis, but they were lowered afterwards to during the period before the Global Financial Crisis, but they were lowered after wards to<br>25 bps. Later, and coinciding with the inception of Global Financial Crisis, the European Central Bank temporarily increase the size of the changes to 50 bps. Finally, after the introduction of negative rates in 2014, the size of policy rate changes was lowered to 10 6 to be of 25 bps, with the exception of the accommodating monetary policy period. bps. Similarly, the Federal Reserve Board has been adjusting the dimension of changes to monetary policy stance. Therefore, one can observe the most predominant size of changes to be of 25 bps. The exception of the exception of the exception of the accommodation of the accommodation. The exception of the accommodation of the accommodation. The exception of the accommodation of the accommodation.

$$
p_{t+1} = \frac{f_{t+1} - f_t}{\delta_{t+1}} \tag{3}
$$

The same procedure can be repeated for each subsequent meeting (Figure The same procedure can be repeated for each subsequent meeting (Figure 4), 4),  $\mathcal{F}_{\mathcal{F}}$  same procedure can be repeated for each subsequent meeting (Figuree ) for each subsequent meeting (Figuree ). 4),

$$
f_{t+2} = (f_t + \delta_{t+1} + \delta_{t+2}) \cdot p_{t+1} \cdot p_{t+2} +
$$
  
\n
$$
(f_t + \delta_{t+1}) \cdot p_{t+1} \cdot (1 - p_{t+2}) +
$$
  
\n
$$
(f_t + \delta_{t+2}) \cdot (1 - p_{t+1}) \cdot p_{t+2} +
$$
  
\n
$$
f_t \cdot (1 - p_{t+1}) \cdot (1 - p_{t+2})
$$
\n(4)

we obtain a recombining binomial tree with three potential values two meeting a head. That is if  $\delta_{\alpha+1}$  and  $\delta_{\alpha+2}$  have the same sign (i.e.  $\delta_{\alpha+1} = \delta_{\alpha+2}$ )  $\frac{1}{2}$  and  $\frac{1}{2}$  is,  $\frac{1}{2}$  and  $\frac{1}{2}$  is,  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$ ,  $\frac{1}{2}$ By restricting future monetary policy movements to have the same sign  $\mathbf{F}$ ft · (1 − pt+1) · (1 − pt+2) · (1 − pt+2)<br>|-<br>|ings ahead. That is, if  $\delta_{t+1}$  and  $\delta_{t+2}$  have the same sign (i.e.,  $\delta_{t+1} = \delta_{t+2}$ ), ings ahead. That is, if  $\alpha$  have the same sign (i.e.,  $\alpha$ ),  $\alpha$  have the same sign (i.e.,  $\alpha$ ),  $\alpha$ By restricting future monetary policy movements to have the same size,

$$
f_{t+2} = (f_t + 2\delta) \cdot p_{t+1} \cdot p_{t+2} +
$$
  
\n
$$
(f_t + \delta) \cdot (p_{t+1} \cdot (1 - p_{t+2}) + (1 - p_{t+1}) \cdot p_{t+2}) +
$$
  
\n
$$
f_t \cdot (1 - p_{t+1}) \cdot (1 - p_{t+2})
$$

while if  $\frac{1}{\sqrt{1+\frac{1}{\sqrt{1+\frac{1}{2}}}}}$  and  $\frac{1}{\sqrt{1+\frac{1}{2}}}$  is the  $\frac{1}{\sqrt{1+\frac{1}{2}}}$ ft · (1 − pt+1) · (1 − pt+2) · (1 − pt+2)<br>|while if  $\delta_{t+1}$  and  $\delta_{t+2}$  have different sign (i.e.,  $\delta_{t+1} = -\delta_{t+2}$ ),

$$
f_{t+2} = (f_t + \delta) \cdot p_{t+1} \cdot (1 - p_{t+2}) + \n f_t \cdot p_{t+1} \cdot p_{t+2}) + (1 - p_{t+1}) \cdot (1 - p_{t+2}) \n (f_t - \delta) \cdot p_{t+2} \cdot (1 - p_{t+1})
$$

Once we have obtained the value of  $p_{t+1}$  from equation 3, we can solve Once we have obtained the value of  $p_{t+1}$  from equation 3, we can solve equation 4 to obtain  $p_{t+2}$ . Thus, recursively, we can obtain the probability of a change in the monetary policy rate for each meeting  $p_{t+h}$ . This way, we can<br>a change in the monetary policy rate for each meeting  $p_{t+h}$ . This way, we can a change in the monetary policy rate for each meeting  $\{p_h\}_{h=t+1}^T$ , associated to a set obtain a set of probabilities for each meeting  $\{p_h\}_{h=t+1}^T$ , associated to a set of feasible values  $\{\ldots 2\delta, \delta, 0, -\delta, -2\delta, \ldots\}$ , with h referring to the h-meeting of reasonce variety  $\{1, 20, 0, 0, 0, 20, \ldots\}$ , with *n* reterring to the *n*-meeting ahead to be forecast. Therefore, this approach produce a universe of forecasts, evaluated at each point in time  $(t)$  and for each meeting ahead. Annex A provides the pseudo-code for obtaining this sequence of probabilities. obtain a set of probabilities for each meeting  $\{p_h\}_{h=t+1}^T$ , associated to a set ahead to be forecast. Therefore, this approach produce a universe of fore-A provides the pseudo-code for obtaining this sequence of probabilities.<br>This set of probabilities is based on conditional information, which means, which means, which means, which me Once we have obtained the value of  $p_{t+1}$  from equation 3, we can solve equation 4 to obtain  $p_{t+2}$ . Thus, recursively, we can obtain the probability of  $\overline{O}$  is the the value of pt+1 from equation  $\overline{O}$ once we have obtained the value of  $p_{t+1}$  from equation 5, we can solve a change in the monetary policy rate for each meeting  $p_{t+h}$ . I his way, we can<br>obtain a set of probabilities for each meeting  $f_n, \mathcal{X}^T$  associated to a set obtain a set of probabilities for each meeting  $\{p_h\}_{h=t+1}$ , associated to a set<br>of foscible values  $\int$   $\partial \delta \delta = \delta - \partial \delta$ , and with h-referring to the h-meeting of leasible values  $\{ \ldots 20, 0, 0, -0, -20, \ldots \}$ , with *n* referring to the *n*-meeting

This set of predictions is based on conditional information, which means, This set of predictions is based on conditional information, which means, This set of predictions is based on conditional mormation, which means, all forecasts rely on given information until  $t - 1$ . We follow previous literature (e.g., Christoffersen, 1998; Diebold et al., 1998; Engle and Manganelli,  $(2004)$ , who propose the use of conditional density forecasts. This means the assessment of density forecasts for a given period of time  $(t)$  should only  $(2004)$  analyze the way to assess the performance of VaR estimates using variconsider known information until  $t - 1$ . For instance, Engle and Manganelli ables known at time  $t - 1$ . Additionally, they stated that, if the estimated model is equal to the true data, the assessment function should be identically and independently distributed (i.i.d.). Independence property refers to the absence of serial auto-correlation across time, which implies forecast bias do not depend on previous errors estimation. all forecasts rely on given information until  $t - 1$ . We follow previous litera-<br> $\frac{d}{dt}$  showledge of the case all forecasts rely on given information until t − 1. We follow previous literaan forecasts ferrom given information until  $t = 1$ , we follow previous intera- $\frac{1}{2}$ 

### 3 Discrete Probability Forecasts

#### 3.1 Density Forecasts

Once we have a candidate probability forecast as the one we got from previous section 2.2, we need to evaluate the accuracy of such probabilities. Following Diebold et al. (1998), let  $\{\phi_t(y_t|\Omega_t)\}_{t=1}^m$  be the sequence of actual (unobserved) conditional densities governing a series  $y_t$  (in our case, the probabilities assigned by market practitioners of future monetary policy movements); where  $\Omega_t$  is the available information at time t. Let also  ${p_t(y_t|\Omega_t)}_{t=1}^m$  be the corresponding sequence of one-step ahead density forecasts we want to evaluate (the probability estimates described in previous section 2.2); and  $\{y_t\}_{t=1}^m$  the realizations from the process (the changes in the monetary policy rates by central banks).

The null hypothesis we would want to test is that the proposed density forecasts  $(p_t)$  correspond with the actual densities  $(\phi_t)$ , that is,

$$
H_0: \{\phi_t(y_t|\Omega_t)\}_{t=1}^m = \{p_t(y_t|\Omega_t)\}_{t=1}^m
$$

The main challenge of testing this hypothesis is that we only have one observation of  $y_t$  for each moment of time t, and a full density distribution for each moment. Thus, to evaluate this hypothesis, we need to use the probability integral transformation (PIT), e.g. the likelihood of observing a lower probability than the observed one, being the probability measured by the probability forecast (Rossi, 2014),

$$
z_t = \int_{-\infty}^{y_t} p_t(x)dx = P_t(y_t),
$$
\n(5)

where, for each realization  $y_t$ ,  $z_t$  is the cumulative probability according to the density forecast  $p_t$ . This transformation produce a uniformly distributed variable  $z_t$ , from  $y_t$  regardless of the type of original distribution of  $y_t$  (Rosenblatt, 1952). Thus, assuming that the density forecast is continuous (which is a required condition to be able to get  $z_t$ ), Diebold et al. (1998) show  $(H_0 \to \{z_t\}_{t=1}^m \sim \mathcal{U}[0,1]),$  but not under any other alternative distribution. that  $z_t$  is a variable with a uniform distribution under the null hypothesis

Thus, if the null hypothesis holds (i.e., the density forecast corresponds to the actual distribution), we would obtain an outcome of the  $z_t$  transformation following a uniform distribution. By contrast, if the density forecast departs from the true density, we will have  $z_t$  values that are clearly non-uniform. For instance, if we observe a distribution of  $z_t$  where too many values are accumulated in the left side, this would imply that the forecast density is giving lower probabilities to the smaller values than would be needed, thus indicating an upward bias in the distribution. In a similar vein, an accumulation of values on the right side of  $z_t$  would imply a downward bias in the density forecast. A distribution of  $z_t$  that has too many values in the center of the distribution implies that the variance is lower than estimated, while if they are concentrated in the extremes, implies a higher variance or fatter tails than forecast.

One of the great advantages of this density forecast evaluation is, precisely, that in addition to testing the accuracy of the forecast, it also gives information of the causes of the eventual rejection of the null hypothesis of correct forecast (e.g., what are the deviations from the uniform distribution), as we will show in section 4. Furthermore, this information is also useful to correct any bias in the original density forecast, as we will later discuss in section 4.1.

### 3.2 Discrete Probability Forecasts

The density forecast evaluation methodology described in previous section 3.1 is built around the continuity of the cumulative density distribution, that allows the transformation in equation 5 to be a bijective function (i.e.,  $Y_t \leftrightarrow Z_t$  and produce  $z_t$  to have a uniform distribution under the null hypothesis. However, as we showed in section 2, in the case of monetary policy changes, the movements are discrete, and the probability distributions are discontinuous by nature, with several possible values of  $z_t$  for the same value of  $y_t$  (i.e.,  $Y_t \leftarrow Z_t$ , for any given  $z_t$ , we can figure out the value of  $y_t$ , but for a given  $y_t$ , monetary policy rate, there are multiple valid  $z_t$ ). For this reason, in this section we need an alternative transformation to equation 5 for computing the PIT for discrete density forecasts that allows us to perform the evaluation proposed by Diebold et al. (1998) in these discrete cases.

To illustrate the proposed methodology, let's suppose a discrete probability forecast for the distribution of monetary policy interest rates for a given central bank meeting as the one shown in Figure 4. The horizontal axis show each possible realization of  $y_t$  (i.e., the possible values of future interest each  $y_t$ . In this example, two possible rates are shown: e.g., maintaining and hiking the monetary policy rate, with probabilities of  $.6, .4$  respectively. In this situation, if we finally observe an unchanged policy rate, we would have to impute a value to  $z_t$  that could be any value between  $P_{i-1} = .0$  (i.e., the probability of a rate cut) and  $P_i = .6$  (i.e., the accumulated probability of a rate cut) and  $P_i = .6$  (i.e., the accumulated probability of unchanged rates). Thus, under a discrete density forecast,  $z_t \in [P_{i-1}, P_i]$  $\frac{1}{2}$  would be unidentified.<br> $\frac{1}{2}$ ,  $\frac{1}{2}$ , rates) while the vertical axis represents cumulative probability densities for would be unidentified.

Authors, such as Brockwell (2007) or Liesenfeld et al. (2006) have proposed techniques to overcome this problem, obtaining a randomized PIT  $(z_t)$  $using equation 6,$ 

$$
z_t = w_t \cdot (P_i - P_{i-1}) + P_{i-1}, \tag{6}
$$

where  $w_t$  is a random variable that we set to follow a standard uniform distribution  $(w_t \sim \mathcal{U}[0,1])$ . That way, one can transform the discrete probability function into a continuous form and test for the accuracy of the predictions. To increase the precision and avoid that in different evaluations we obtain different outcomes, we need the simulation to be of a large enough magnitude. Nevertheless, this option implies an additional source of noise and is computationally cumbersome.

For those reasons, authors like Czado et al. (2009) or Kheifets and Velasco  $(2017)$  propose a non-randomized and uniform version of the PIT histogram. That means the Conditional Cumulative Density Function (CDF) can be computed based on the observed value of  $y_t$ , for  $\{u\} \in [P_{i-1}, P_i]$ ,

$$
u = \{P_{i-1} + \Delta p, P_{i-1} + 2\Delta p, \dots, P_i\},\tag{7}
$$

where we set  $\Delta p = .001 \cdot (P_i - P_{i-1}))^2$ . This way, we obtain density functions for the forecast referred to each moment in time  $t$  and for a given meeting ahead h. Later on, we obtain the aggregated CDF for all the period. The formula shown in equation 7 let us obtaining the  $z_t$  for the unidentified interval, i.e.,  $z_t \in [p_{i-1}, p_i]$  following an uniform distribution and avoiding randomized bias. In the case of the previous example (i.e., Figure 4), we would obtain the following  $z_t$  values: if we observe an unchanged rate,  $z_t = \{.0006, .0012, \ldots, .6\};$  if there is a rate hike,  $z_t = \{.6004, .6008, \ldots, 1\};$ <br>while if we observe a rate sut, x would be a series of 0. while if we observe a rate cut,  $z_t$  would be a series of 0.<br>With this transformation, where  $z_t$ where we set  $\Delta p = 0.01 \cdot (P_i - P_{i-1})^2$ . This way, we obtain density functions where we set  $\Delta p = 0.01 \cdot (I_i - I_{i-1})$ . This way, we obtain density functions First formula shown in equation 7 fet us obtaining the  $z_t$  for the unique-<br>fied interval, i.e.,  $z_t \in [p_{t-1}, p_t]$  following an uniform distribution and avoiding randomized bias. In the case of the case of the case of the previous example  $f(x)$ ,  $\sum_{i=1}^{\infty}$ ,  $f$ we would obtain the following  $z_t$  values. If we observe an unchanged rate,<br> $z_t = \frac{1}{2}$ .0006, 0012, ..., 6\, if there is a rate hike,  $z_t = \frac{1}{2}$ .6004, 6008, .... 1};  $\omega_t = \{0.0000, 0.0012, \ldots, 0.0\}$ , if there is a rate find,  $\omega_t =$ 

With this transformation, we are back to the situation where  $z_t$  would have a uniform distribution under the null hypothesis that the estimated have a uniform distribution under the null hypothesis that the estimated  $\frac{2W}{\sigma^2}$  based  $\frac{2W}{\sigma^2}$  based  $\frac{2W}{\sigma^2}$  based on  $\frac{2W}{\sigma^2}$  based on  $\frac{2W}{\sigma^2}$  based on  $\frac{2W}{\sigma^2}$  based on  $\frac{2W}{\sigma^2}$ pressed between  $\psi_t(y_t|\omega_t)_{t=1}$ . Using this approach, we can be defined to  $\psi_t(y_t|\omega_t)_{t=1}$ . Using this approach, we can be probability forecast  $\{p_t(y_t|\Omega_t)\}_{t=1}^h$  is equal to the underlying distribution of the variable of interest  $\{\phi_t(y_t|\omega_t)\}_{t=1}^h$ . Using this approach, we can compare the observed function with a uniform distribution  $\mathcal{U}[0, 1]$ . Annex B shows the pseudocode used for this procedure of obtaining the probabilities of monetary relievantes for future meetings. pseudocode used for this proceder<br>policy rates for future meetings. when this transformation, we are back to the situation where  $z_t$  would

Eigure 5 (upper panels) shows potential outcomes for the cumulative dis-The value of  $\alpha$  randomized PIT ( $\alpha$ ) is problem to the probability of maintaining tributions of  $z_t$  in case of an overestimation of the probability of maintaining rates in the previous example (i.e., real probability-green- equal to  $.4$  instead of the forceast red of the miss case, we would be expecting ress finds than observed, so the PIT will go below the diagonal (black) that would be the ideal scenario of perfect forecast. In the case of the central panels of Figure 5 we show the opposite situation, where we have an underestimation of the probability of maintaining rates in the previous example (i.e., real probability-green- equal to .8 instead of the forecast-red-.6). Here, since we would expect more hikes than observed, the PIT will go above the diagonal (black). Finally, in the lower panels we show two extreme outcomes: i) failure to predict rate cuts (green), which would be signaled by an excess of zeros in the distributions, and ii) a failure to predict rate hikes (red line), which will result in an excess of ones. of the forecast-red-.6). In this case, we would be expecting less hikes than

# 3.3 Multi-period Forecasts

The previous discussion on both density forecasts (section 3.1) and discrete probability forecasts (section 5.2), are valid when the observations  $(t)$  are<br>independent. We can assume that this is the case, when we are using one independent. We can assume that this is the case, when we are using one period forecasts (i.e., from meeting to meeting). However, if we want to period.  $T_{\text{total}}$  formula shown in the formula shown in equation  $\left( \frac{1}{2} \right)$  and  $\left($ we cannot use overlapping forecasting norizons (i.e.,  $i \to i+2$ ,  $i+1 \to i+3j$ )<br>and claim that the forecast observations remain independent. Diebold et al. and claim that the forecast observations remain independent. Diebold et al.  $(1000)$ This solution has a cost in terms of the number of observations available to  $\lim_{\epsilon \to 0} \frac{\epsilon}{\epsilon}$ . This solution has a cost in terms of the humber of observations available to<br>perform the forecast evaluation, which can become so critical to make it. perform the forecast evaluation, which can become so critical to make it<br>unforcible for larger harizons. unfeasible for larger horizons. probability forecasts (section 3.2), are valid when the observations  $(t)$  are evaluate longer horizons, we have an additional difficulty, since in that case, we cannot use overlapping forecasting horizons (i.e.,  $t \to t + 2$ ,  $t+1 \to t+3$ ) (1998) propose to use non-overlapping horizons (i.e.,  $t \to t+2$ ,  $t+2 \to t+4$ ).

<sup>&</sup>lt;sup>2</sup>We approach the non-randomized PIT based on 1000 possible realizations of  $u$  compressed between  $P_{i-1}$  and  $P_i$ . Therefore we set  $\Delta$  equal to  $1/1000 \cdot (P_i - P_{i-1})$ . This implies, that instead of trying to probabilities of trying to probabilities of  $\sigma$ 

To overcome this difficulty, we propose to use the conditional forecasts. This implies, that instead of trying to predict the probabilities of rate movements between t and  $t + h$  with the information available at t, we will compute/evaluate the probabilities of rate movements between  $t+h-1$  and  $t+h$ with the information available at  $t$ . According to Diebold et al. (1998), the independence property can be extended to a sequence of conditional densities as forecasts contain past information. In that sense, conditional density estimations for  $\{y\}_{t=1}^h$  can be understood as the joint distribution of density functions from  $t = 1$  to h, where h refers to the meeting ahead date. functions from t = 1 to h, where h refers to the meeting ahead date.

$$
f(y_h, \dots, y_1 | \Omega_1) = f_h(y_h | \Omega_m) \cdot f_{h-1}(y_{h-1} | \Omega_{h-1}) \cdots f_1(y_1 | \Omega_1)
$$
 (8)

Thus, using the binomial trees (section 2.2) we have the probabilities of Thus, using the binomial trees (section 2.2) we have the probabilities of hike, cut, and maintain for each independent horizon  $h$ . So this individual situations (represented in figure  $6$ ) are the ones we will evaluate for horizons greater than one meeting ahead. Figure  $6$  shows the path of interest rate greater than one meeting ahead. Figure  $6$  shows the path of interest rate probabilities based on the binomial tree (red line), the observed monetary possibilities based on the binomial free (red line), the observed monetary<br>policy rate decisions (black line) and the evaluation according to conditional probabilities (green line) for the second and third meeting ahead, respectively. The evaluation of the density forecast for the 2 meeting ahead (left-hand side) considers the observed path until  $t + 1$  where two probabilities arise: cut or maintain. Oppositely, if one follows the binomial tree (non-conditional probabilities), the estimations show 3 possible outcomes. Similarly, the evaluation of the third meeting ahead (right-hand side) will account for known information until the previous period. In this case, the number of possible outcomes is shortened by the observed interest rate direction. The same procedure is shortched by the observed meetings ahead, roughly equivalent to five years. Using these forecasts, we can maintain evaluations of non-overlapping periods, without any sensible loose of observations. periods, without any sensible loose of observations.

### 4 Monetary Policy Forecast and Evaluation 4 Monetary Policy Forecast and Evaluation 4 Monetary Policy Forecast and Evaluation

In this section we evaluate the discrete probability forecasts obtained from In this section we evaluate the discrete probability forecasts obtained from  $\frac{1}{2}$  is computed as the cumulative probability is evaluated as the section 2.2 using the transformation presented in section 3.2 and 3.3. As defined previously, PIT is computed as the cumulative probability evaluated at the observed monetary policy rate change. Our assessment includes interest rate expectations for each of the Monetary Policy Meetings in the period<br>est rate expectations for each of the Monetary Policy Meetings in the period between January 1999 and June 2023 held by both the ECB and the FOMC. Therefore, we compare the estimated probabilities of maintaining, cutting  $\frac{1}{2}$ or hiking with information in  $t$  (i.e., the day of the last observed monetary policy meeting) about the actual decision taken in the meeting in  $t + 1$ . This way, we obtain the probability distributions  $(z_{t+1})$ , following equation 7 detailed in the previous section, and comparing them with the actual decisions produce the PITs.

Top left charts of Figure 7 shows the evaluation of the probability forecasts in both the euro area (blue) and the US (green), for the decisions on the  $\overline{S}$ m both the curo area (orde) and the obs (green), for the decisions on the next meeting. The X axis represents the PIT of the cumulative probabilities of a uniform distribution, the one you would expect in case of a correct probability forecast. The Y axis represents the PIT of the forecast cumulative probabilities. Therefore, a correct probability forecast would imply a line over the bisectrix in the chart. A line below the bisectrix would imply a downward bias in the forecasts, where there would have been given excess probabilities to the cut (vs. maintain) and maintain (vs. hike). By contrast, a line above the bisectrix would imply an upward bias in the forecasts, where the forecasts are assuming too high probabilities for maintain (vs. cut) and hike (vs. maintain). The latter is what we observe both for the euro area and the US.

The same procedure can be applied to longer horizons. The only caveat is that we have to evaluate, not the movement between t and  $t + h$ , but the movement between  $t + h - 1$  and  $t + h$  with the information available in t, to ensure the independence of each of the forecasts. The outcome of the evaluation of these forecasts are presented in Figure 7 for the decisions on  $t+2$  (top right),  $t+4$  (center left),  $t+8$  (center right),  $t+16$  (bottom left) and  $t + 40$ . Taking into account that the meetings have a frequency of roughly 6 weeks, these horizons represents a quarter, a semester, a year, two years and 5 years ahead. As was the case with the next meeting decision forecast, we also observe an upward bias in the probability forecast. More relevant is that the bias slightly but monotonically increase with the horizon. This is specially driven at the beginning of the distribution (i.e., at the zero of the X axis), implying the presence of a substantial amount of cases where the final decision is a cut in the monetary policy rate, while we were predicting a maintain or hike decision.

This result is not unexpected, since it is completely consistence with the presence of a risk premium in the OIS curve to compensate those insuring for the uncertainty on future monetary policy rate evolution. This uncertainty (and, thus, the risk premium) will be higher, the longer the maturity. The main consequence of this risk premium is specially an underestimation of rate cuts, that will be higher, the longer the horizon. The conclusion from this evaluation is that these forecasts cannot be trusted if applied without a correction for the upward bias.

#### 4.1 Bias Correction

In previous sections, we have provided the methodology to compute discrete probability forecasts as well as the assessment of its performance, showing that they produce upward biased predictions, especially for longer horizons (see Section 4). This outcome is reasonable, taking into account that interest rate expectations are obtained directly from OIS curves and, therefore, they include an unobserved term premium component by nature. Thus, we can consider them to be the risk-neutral probabilities of monetary policy changes (i.e.,  $P_Q$ ). Ideally, we would like to transform these  $P_Q$  into objective probabilities (i.e.,  $P_P$ ). (i.e.,  $P$ ). Ideally, we would like to transform these  $P$  into objective prob- $(1.6., 1Q)$ . Ideally,

In fact, the density forecast evaluation originally proposed in Diebold et al. (1998), also allows for an easy estimation of the correction of bias in the distribution. In our case of discrete probability distributions, all we need the distribution. In our case of discrete probability distributions, all we need is using the observed distribution of zt, as a correction for the probability is using the observed distribution of  $z_t$ , as a correction for the probability forecast. That is, In fact,  $IP$ ).<br>Let de density forecast evaluation originally proposed in Diebold et al. (1999), also allows for an extended of the correction of the correction of the correction of the correction of bias in the correction of the correction of bias in the correction of the correction of the correction o

$$
P_{P_t}(y_t) = P_{Q_t}(y_t) \cdot q_t(P_{Q_t}(y_t)),
$$
\n(9)

where  $P_{Q_t}(y_t)$  are the original (risk-neutral) probabilities we have comwhere  $P_{Q_t(y_t)}$  are the original (risk heating) probabilities we have computed in section 3.2,  $P_{P_t}(y_t)$  would be the objective (unbiased) probabilities, while  $q_t(P_{Q_t}(y_t))$  is the function that allows going from the Q measure to the<br>
R measure. Following Disheld et al. (1008), we propose to use  $g(x)$  (where P measure. Following Diebold et al. (1998), we propose to use  $q_t(z_t)$  (where<br>x we the observed enveloping distribution of monetary policy extremes) as  $z_t$  was the observed cumulative distribution of monetary policy outcomes) as a proxy to  $q_t(P_{Q_t}(y_t))$ , where  $\mathbf{P}_{\text{max}}$  $z_t$  was the observed cumulative distribution of monetary policy outcomes) as

$$
q_t(z_t) = \frac{z_t(y_t)}{P_{P_t}(y_t)}.
$$

Thus,  $q_t(z_t)$  is the bias correction based on the distribution of  $z_t$  computed in the earlier section for forecasting evaluation. Therefore, if discrete probability forecasts are correct, and  $z_t$  is equal to a uniform distribution  $\mathcal{U}(0,1)$ , then no correction is needed  $(q_t(z_t) = 1)$  and  $P_{P_t}(y_t) = P_{Q_t}(y_t)$ . Otherwise, if  $PQ_t$  shows and upward bias, then  $q_t(z_t) > 1$ , and  $P_{P_t}(y_t) > P_{Q_t}(y_t)$ , giving more probability for the cut and maintain options. By contrast, in the case of a  $PQ_t$  with a downward bias, then  $q_t(z_t) < 1$ , and  $P_{P_t}(y_t) < P_{Q_t}(y_t)$ , giving the correction as the correction  $q_t(x_t)$  can be  $r_t(y_t)$  can be  $r_t(y_t)$ ,  $\ldots$   $r_t(y_t)$ ,  $\ldots$   $r_t(y_t)$ ,  $\ldots$ 

the correction  $q_t(z_t)$  can be obtained as the likelihood of observing a lower probability than  $p_i$  using (non corrected) density forecasts as plotted in figure 7. probability than  $p_i$  using (non corrected) density forecasts as plotted in figure  $7$ 

This way, we are able to get corrections for any given interest rate forecast based on a sample of density forecast previously evaluated. Additionally, once we use estimated bias correction to get probability forecasts, the same procedures to evaluate the uniformity of the resulting PIT can be applied. Annex C show the prodedure to obtain this bias correction.

Going back to the example in figures  $4$  and  $5$ , we have a case where we have forecast that the monatery policy rate would be maintained with a 60% probability and hiked with a 40% probability (see figure 4). Continuing with the upward bias example (see figure 5 upper row), if we observe that the  $z_t$ for maintaining is 80% then  $q(z) = \frac{8}{6}$ , and  $P_P(\text{maintain}) = .6 \cdot \frac{8}{.6} = .8$ .

Using this process we can correct the upward biases that we originally have. In order to do so, we use a jacknife procedure, were for each monetary policy decision and horizon, we compute  $z_t$  with all the other observations available. Then we use, for each case, the corrected  $P_{P_t}$ . Figures 8 and 9 shows the result of this correction (blue lines) of the original probability estimates (red lines) in the euro area (left) and United States (right) for different meeting-ahead forecasts (1, 2 and 3 in 8, and 8, 16, and 40 in 9). Visually, we can see the bias correction density forecast follow an uniform distribution and this result can be confirmed based on Kolmogorov-Smirnov tests for comparing two distributions both for the euro area and the US (Table 1).

The results suggest our methodology provides more accurate forecasts, in the sense that bias correction densities follow a uniform distribution. Moreover, we are able to enhance the estimates not only for short-term horizon forecasts but, more importantly, for longer horizons. For instance, in the previous section, we highlighted the result that forecasts do not assign any probability to interest rate reductions, especially for long-term horizons in the US. However, the corrections obtained for the 8, 16, and 40 meeting ahead suggest rate cuts are also likely to be observed. We also show that corrected probabilities perform well in both areas.

The improvement of our estimates can be confirmed based on a forecast evaluation along time, which is provided in Figures 10 (for the euro area) and 11 (for the US). These charts show the difference between direct estimations of the PIT and the bias corrected PIT, so that they can be interpreted as the improvement given by of our predictions. The gain after bias corrections is averaged for each meeting ahead and shown in Table 2, pointing to lower errors after correcting for risk premium bias.

In the euro area and for short-period horizons, we can observe downward corrections are higher for the period of the Global Financial Crisis and up to the introduction unconventional monetary policy, meaning that forecasts pointed to higher interest rates than the observed ones. For longer horizons, the upward bias is especially relevant after the Zero Lower Bound (Figures  $10$  and  $11$ ).

In the US, upward bias for short-period horizons are observed at an earlier stage for the Global Financial Crisis compared to the euro area and from 2013 onward following the announcement of Federal Reserve Board normalisation plans. For long term forecasts, we can observe a similar pattern along time but higher bias correction.

As a consequence, we can visually confirm that corrected probabilities are more accurate as they do not assign higher probabilities to lower or higher possible outcomes. Moreover, valid corrections have been obtained region and for each monetary policy ahead. that manage to capture both the sign and the magnitude of the bias in each  $\frac{1}{2}$  all possible rates multiplied by the probability associated to each probability associa

#### 4.2 Practical applications rate. Therefore, we are able to get for a function  $\mathbf{r}$ 4.2 Fractical applications

This section offers an illustration of some indicators of relevance for monetary policy decisions that can be obtained through the proposed methodology. In 2022, central banks started monetary policy normalisation, which conveys an upward sloping expected path of policy rates. In that context, corrected probabilities forecasts offered in the previous sections can be translated into several indicators such as the expected path of interest rates, terminal rates or interest rate probabilities. They provide useful information such as when the hiking cycle could end (i.e., the terminal rate) in a context of tightening monetary policy, which level would be reached or even, the pace for rates going down afterwards.

As stated in equation 1 of section 2.2, forward rates can be defined as the sum of all possible rates multiplied by the probability associated to each rate. Therefore, we are able to get forward curves after applying the bias correction to the risk neutral probabilities, as derived in section  $4.1<sup>3</sup>$  The obtained forward curves can be understood as the projected path of policy rates, i.e., the expected evolution of overnight rates (DFR, for the euro area and EFFR in the US) for a given horizon of time as shown in Figure 12. precisely, to be reached in September (with data as of June) and implying

 $\frac{3}{2}$ For figures 12-17, we have first compute unconditional probabilities based on the binomial tree and later we applied the corrections based on the obtained PIT for each  $A$  and  $A$  monitoring a head. meeting ahead.

Therefore, these projections provide useful additional information for economic agents and central banks, e.g., which are market expectations for policy rates at each point in time, based on known information and more importantly, excluding possible risk bias.

and EFFR in the US) for a given horizon of time as shown in  $\mathbb{R}^n$  as shown in  $\mathbb{R}^n$ 

offered by central bank speeches suggested a later ending of the hiking cycle (November) meaning an increase in the terminal rate of 10 bps (to  $5.41\%$ ). In both areas, premium bias corrections used to obtain risk-neutral forward curves lead to similar expected rates in the very short horizon but differences intensify between risk-neutral and biased curves for medium and long-term 17projections. Additionally, financial markets have been recently monitoring terminal rates. Terminal rates offer an insight of the policy rate to be reached at the ending point of the hiking (or cutting) cycle, mathematically obtained as the peak (or local maximum) of the forward curve. According to Figure 12 and considering OIS rates as of June 14th, 2023 for the euro area, the ending point of the hiking cycle would have been reached in December 2023, pointing to a terminal rate of 3.86%, using direct estimation and 3.76% after correcting for the upward bias. Terminal rates expectations experienced upward pressure later in July, approaching 4%. For the US, the end of the hiking cycle was expected at an earlier stage than for the euro area, more precisely, to be reached in September (with data as of June) and implying a policy rate of 5.31% after bias correction. Later on, hawkish messages

Not only the value or moment of time of the terminal rate is relevant, but the changes in these terminal rates over time are also a relevant indicator to follow. Figure 13 shows its behaviour during 2023 for both direct estimation and corrected forward curves. In the two regions, terminal rates have followed an increasing trend in 2023, with the exception of the Sillicon Valley Bank crisis event in march, which involved a significant reduction (more than 100 bps) in terminal rates.

Interest rate probability densities and fan charts at a given point in time can be also an useful tool, conveying information about the dispersion or uncertainty around estimates. Figures 14 and 15 shows probabilities assigned to each rate and for each meeting ahead according to known information until July 13th, 2023. For the computations, we distinguish between both direct estimates and risk neutral probabilities. According to section 2.2, the range of potential values expand for longer horizon projections and the comparison of bias and corrected density functions highlights two facts previously mentioned. First, corrected probabilities involve a shift to the left of the distributions, which means that corrected estimates give higher probabilities to lower rates. Second, corrected probabilities expand the set of possible values, assigning non-zero probabilities for the three options for rate changes, i.e., cut, hike and maintain.

In a similar vein, Figures 16 and 17 for the euro area and the US, respectively, show the range of all possible values for policy rate expectations based on three different estimation dates: November 2021 (before starting the hiking cycle), June 2023 (before June central decisions) and July 2023. The procedure to construct these fan charts departs from probability density functions (PDF) as shown in Figures 14 and 15 to get the values lying inside the interval defined by percentiles [10-90] of the distribution. The analysis of

different monetary policy contexts, such as, before (i.e., 2021) and during the hiking cycle (i.e., 2023), suggests uncertainty around policy rates increases during the hiking periods both in the short and long term.

### 5 Conclusion

needed. In that sense, we propose a complete methodology which has been The estimation of monetary policy rates is of extreme importance for financial participants and Central Banks. For that reason, one should ensure that forecasts are correctly estimated and calibrated to correct possible bias, if proved to perform well.

First, we described how to generate probability estimates using marketbased models that permit to extract forecasts for a wide range of meetings ahead. Secondly, we employ statistical procedures based on the work by Diebold et al. (1998) to evaluate the performance of such estimates. Thirdly, we have developed a methodology that permits not only assessing whether or not, density forecasts are correctly estimated, but we are also able to compute the sign and the size of the bias. Regarding to this, we managed to overcome one of the assumptions conveyed in Diebold et al. (1998), i.e., the continuity of the probability function. Therefore, we contribute to existing literature applying a new technique to transform a discrete probability function into a continuous one. As far as we know, our work constitutes the first piece of paper that propose such methodology to evaluate and calibrate interest rate expectations. Last, but not least, we produce bias corrected density forecasts that perform well and accomplish our main objective.

This paper contributes to existing literature in several ways. First, we are able to improve probabilities assigned by market practitioners of future monetary policy movements as biased estimates are quantified for different forecast horizons, which permits obtaining risk neutral probabilities. Second, we provide a new methodology for researchers in the field of estimating the decomposition of interest rate into the expectations and term premium factor. Third, as estimating and correcting the upward bias present in monetary policy expectations, we are able to expand the binomial tree approach and make it closer to a trinomial tree.

We are aware our paper has certain limitations. For example, we have assumed that estimation bias is constant over time so that they do not vary depending on the economic environment. However, one could think this is not always the case, and therefore, future work could be developed in order to incorporate dynamic corrections.

Going forward, our paper can be extended in several ways. For instance, the described methodology could be employed to other type of economic and financial forecast that do not follow a continuous function. Additionally, our work does not aim to provide an economic meaning for the estimation bias obtained in this paper. In that sense, further work could be developed to analyze the relationship of the bias with other variables of uncertainty or risk premium.

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### A Binomial tree from forward OIS curves

The following pseudocode is used to generate the probabilities for the monetary policy meetings according to the binomial tree as explained in section 2.2.

```
# Pseudocode Procedure for Obtaining Monetary Policy Forecast
# Step 1: Define values of variables
size_change = last_observed_rate_change
num\_meetings = number_of_fature\_meetingsmeeting dates =vector with dates of future meetings
forward\_rates = vector_of_forward\_rates# Step 2: Compute changes in forward rates between meetings
change forward = forward rates(1:num meetings)-forward rates(0:num meetings-1)
# Step 3: define potential states of the monetary policy rate in the first meeting
rate changes1(1)=size change*sign(change forward(1)
rate changes1(2)=0
# Step 4: Obtain probabilities for each state
probabilities1(1)=change forward(1)/(size change*sign(change forward(1))
probabilities1(2)=1- probabilities1(1)
# Step 5: Repeat steps 2-4 for each future meeting
for meeting in range(2, num meetings):
       marginal_prob\_change = change\_forward(mecting)(size change*sign(change forward(meeting))
        rate changes meeting = { rate changes meeting-1
                                   size_change*sign(change_forward(meeting)) }
        solve [ rate changes meeting* prob changes meeting == change forward meeting]
```
endfor

### B From discrete to continuous PIT

This pseudocode offers the procedure to transform a discrete PIT into a continuous one as explained in section 3.2

```
# Pseudocode Procedure for continuous densities and evaluation Monetary Policy Forecast
# Step 1: Define values of variables
prob cut t m=matrix with probabilities rate cut time t meeting ahead m
prob maintain t m=matrix with probabilities rate maintain time t meeting ahead m
prob_hike_t_m=matrix_with_probabilities_rate_hike_time_t_meeting_ahead_n
change_observed_t_m=matrix_with_observed_rate_change_time_t_meeting_ahead_m
where each row represents a point in time and each column the meeting ahead
u=uniform_vector_of_size_1000_defined_on_the_interval_p_i_to_p_i-1
P_i = probability of observed outcome
P_{i-1} = probability of a lower rate with respect to observed outcome.
i.e., cut if observed outcome is maintain and maintain if observed outcome is hike
PIT_t=CDF_for_one_period
PIT=CDF_for_all_periods
num\_meetings = number_of_futive_meetingsnum\_periods = number_of\_periodsfor meeting in range(1, num meetings):
# Step 2: Compute PIT at time t
# Step 2.1.: Create the vector udifferent options depending on the observed outcome:
i.e. cutting rates refers to negative change_observed_t_m
i.e. maintaining rates refers change_observed_t_m = 0i.e. hiking rates refers to positive change_observed_t_m
# Step 2.2.: Compute PIT for rate cut
if change_observed_t_m<0 and prob_cut_t_m=0
PIT t = 0else if change_observed_t_m<0 and prob_cut_t_m>0
25
if change observed t m=0 and (prob cut t m+prob maintain t m)=0
PIT_t = u = \{0, 0 + delta, 0 + 2delta, ..., p_i\}where delta equal to 1/1000*(p_i)# Step 2.3.: Compute PIT for rate maintain
PIT t = 0else if change_observed_t_m=0 and (prob_cut_t_m+prob_maintain_t_m)>0
PIT_t = u = \{P_{i-1}, P_{i-1} + delta, P_{i-1} + 2delta, ..., p_i\}where delta equal to 1/1000*(p_i)# Step 2.4.: Compute PIT for rate hike
if\ change\_observed\_t\_m\textsubscript{\textit{i}}0PIT_t = u = \{P_{i-1}, P_{i-1} + delta, P_{i-1} + 2delta, ..., p_i\}where delta equal to 1/1000 * (p_i)else if change_observed_t_{.}m_{\lambda}0 and prob_hike_t_{.}m=prob_maintain_t_{.}m)
```

```
PIT_t = u = \{0, 0 + delta, 0 + 2delta, ..., 1\}# Step 3: Compute PIT for all periods
PIT = \{PIT\_1, PIT\_2, PIT\_3, \ldots, PIT\_T\}where T refers to number of periods evaluated
# Step 4: Evaluate PIT
plot PIT and do Kolmogorov-Smirnov test to check if PIT follows a uniform distribution,
as shown in figure 7 and table 1
endfor
```
### C Bias Correction

```
This pseudocode offers the procedure to offer bias corrected probabilities as shown in section 4.1
    # Pseudocode Procedure for Monetary Policy Forecast correction
    # Step 1: Define values of variables
    prob cut t m=matrix with probabilities rate cut time t meeting ahead m
    prob maintain t m=matrix with probabilities rate maintain time t meeting ahead m
    prob hike t m=matrix with probabilities rate hike time t meeting ahead m
    prob cut t m RN=matrix with corrected probabilities rate cut time t meeting ahead m
    prob maintain t m RN=matrix with corrected probabilities rate maintain time t meeting ahead
    prob hike t m RN=matrix with corrected probabilities rate hike time t meeting ahead m
    PIT=CDF_for_all_periods as obtained in Annex2
    PIT corrected=CDF for all periods with bias corrections
    num_meetings = number_of_future_meetings
    num\_periods = number_of\_periods# Step 2: Corrected probabilities
    for meeting in range(1, num meetings):
    prob cut t m RN=sum(PIT<=prob cut t m)/size(PIT)
    prob hike t m RN=sum(PIT>=(prob cut t m+prob maintain t m))/size(PIT)
    prob maintain t m RN=1-prob cut t m RN-prob hike t m RN
    # Step 3: Repeat forecast evaluation to obtain PIT corrected as described in steps 2-4 of
Annex B.
```
endfor

Meetings ahead	Direct estimation		Bias correction	
(euro area)	$h$ -test	$p$ -value	h-test	$p$ -value
1	0.192	0.00	0.00	0.97
$\overline{2}$	0.179	0.00	0.00	0.97
4	0.180	0.00	0.00	0.97
8	0.194	0.00	0.00	0.97
16	0.242	0.00	0.00	0.98
40	0.230	0.00	0.00	0.98
48	0.227	0.00	0.00	0.98
56	0.186	0.00	0.00	0.98
(United States)	$h$ -test	$p$ -value	$h$ -test	$p$ -value
1	0.167	0.00	0.00	0.97
$\overline{2}$	0.134	0.00	0.00	0.97
4	0.190	0.00	0.00	0.97
8	0.218	0.00	0.00	0.97
16	0.269	0.00	0.00	0.98
40	0.245	0.00	0.00	0.98
48	0.316	0.00	0.00	0.98
56	0.417	0.00	0.00	0.98

Table 1: Test for uniform distribution. Direct and bias correction forecasts. Kolmogorov-Smirnov for two distributions

The Kolmogorov-Smirnov test compares to distributions, where the null hypothesis corresponds to the two distributions been equal, while the alternative hypothesis is that the samples comes from different distributions. We compare both the inverse CDF of the bias correction PIT  $(N[\mu_{PIToorrected}, \sigma_{PIToorrected}])$  and the direct estimation PIT  $(N[\mu_{PIT direct}, \sigma_{PIT biased}] )$  with the inverse CDF of a  $N[\mu_{uniform}, \sigma_{uniform}]$ . If the value of the h-test is equal to 0, it means that PIT follows a uniform distribution, and 1 otherwise.

Meetings ahead	Direct estimation	Bias correction	
(euro area)			
1	0.133	0.108	
$\overline{2}$	0.127	0.103	
$\overline{4}$	0.134	0.109	
8	0.149	0.111	
16	0.180	0.119	
40	0.177	0.127	
48	0.167	0.120	
56	0.147	0.114	
(United States)			
1	0.108	0.093	
$\overline{2}$	0.098	0.084	
$\overline{4}$	0.145	0.121	
8	0.165	0.131	
16	0.204	0.156	
40	0.171	0.126	
48	0.176	0.131	
56	0.202	0.137	

Table 2: Comparison between mean error for direction estimation bias and corrections

The table compares the CDF for different meetings ahead for both the biased and the corrected probabilities.





Figure 2: Forward (blue) and spot (red) yield curve (top) and transformation of the forward curve (blue) into a binomial tree of potential monetary policy rate paths (red) where vertical lines (dashed black) represent future monetary policy meetings (bottom) (for June,  $6^{th}$  2019)



Figure 3: Historical size of monetary policy changes for the ECB (top) and the Fed (bottom) from year 2000 to 2021.







Figure 4: Discrete Probability Forecast example.



Figure 5: Potential outcomes for the Discrete Probability Forecast evaluation. In both graphs, black diagonal line represents uniform distribution

Figure 6: Conditional Probabilities and Discrete Probability Forecast. Red lines show possible outcomes from the binomial tree in  $t$ , black lines shows the observed path, and green lines the conditional probabilities to be evaluated. MA refers to Meeting ahead.



Figure 7: Empirical evaluation of discrete probability forecasts







Figure 9: Correction of Discrete Probability Forecasts (cont.)



Figure 10: Prediction corrections over time (euro area)

The chart shows the difference between corrected PIT and direct estimated PIT, where higher values show higher upward bias.



Figure 11: Prediction corrections over time (United States)

chart shows the difference between corrected PIT and direct estimated PIT, where higher values show higher upward bias.



Figure 12: Expected path of monetary policy rates

Figure 13: Terminal rates





Figure 14: Probability distribution (PDF) for each meeting ahead: euro area



Figure 15: Probability distribution (PDF) for each meeting ahead: United States



Figure 16: Fan charts: policy rate expectations for the euro area

The area in light blue show expected policy rate lying in the interval of percentiles 10,90 of the estimated PDF. Dark blue lines show the forward curve, that represents the expected mean rate.



Figure 17: Fan charts: policy rate expectations for the US

The area in light blue show expected policy rate lying in the interval of percentiles 10,90 of the estimated PDF. Dark blue lines show the forward curve, that represents the expected mean rate.

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