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Agustín Maraval1 (*)
and Christophe Planas (**)

(*) Banco de España.
(**) Eurostat.

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Abstract

The paper deals with the problem of identifying stochastic unobserved two-component models, as in seasonal adjustment or trend-cycle decompositions. Solutions based on the properties of the unobserved component estimation error are considered, and analytical expressions for the variances and covariances of the different types of estimation errors (errors in the final, preliminary, and concurrent estimator and in the forecast) are obtained for any admissible decomposition. These expressions are relatively simple and straightforwardly derived from the ARIMA model for the observed series.
It is shown that, in all cases, the estimation error variance is minimized at a canonical decomposition (i.e., at a decomposition with one of the components noninvertible), and a procedure to determine that decomposition is presented. On occasion, however, the most precise final estimator is obtained at a canonical decomposition different from the one that yields the most precise preliminary estimator.
Three examples illustrate the results and the computational algorithms. The first and second examples are based on the so-called Structural Time Series Model and ARIMA Model Based approaches, respectively. The third example is a class of models often encountered in actual time series:

Key words: Seasonal Adjustment: Unobserved Component Models; Signal Extraction; ARIMA Models; Identification; Estimation Error

Contents

0 Introduction and Summary

We consider the problem of decomposing an observed series into the sum of two uncorrelated components, each one the output of a linear stochastic process, which can be parametrized as an ARIMA model. Thus the basic model (presented in Section 1) is that of an observed ARIMA model with unobserved ARIMA components. Examples are the seasonally adjusted series plus seasonal component decomposition of economic series, the trend–plus–cycle decomposition often used in business cycle analysis, and, in general, signal–plus–noise type of decompositions. The analysis centers on Minimum Mean Squared Error (MMSE) estimators of the unobserved components.

It is well known that the general unobserved components model presents an important identification problem, which stems from the fact that, for a given series, there is in general an amount of white–noise variation that can be arbitrarily allocated between the two components (see, for example, Bell and Hillmer, 1984; or Watson, 1987). This identification problem is discussed in Section 2. Broadly speaking, two main approaches have been developed. In one of them, the overall ARIMA model for the observed series is specified following the standard Box and Jenkins (1970) procedure, and the models for the components are derived from the overall model. This approach has been termed the "ARIMA–Model–Based" (AMB) approach; it has been mostly developed in the context of seasonal adjustment, and basic references are Burman (1980) and Hillmer and Tiao (1982). The second approach directly specifies the models for the components; it has been termed "Structural Time Series Model" (STSM) approach and basic references are Engle (1978) and Harvey (1989). This approach has been heavily used in applied econometrics work.

The applied relevance of AMB methods for seasonal adjustment (and trend estimation), even in very large–scale applications, has increased considerably as of lately. In particular, the European statistical Agency (EUROSTAT) is at present, as a result of a study comparing alternative methods (see Fisher, 1995), using, and recommending the use of, a fully AMB method (namely, program SEATS; see EUROSTAT, 1994). Furthermore, the new US Bureau of the Census program, X12, is an hybrid, which now incorporates many AMB features (see Bureau of the Census, 1995, or Findley and Monsell, 1995).

In this paper, the analysis will apply in general to model–based methods, independently of whether they employ an STSM or an AMB approach. The assumptions used to identify a unique decomposition are, in the STSM approach, to restrict the order of the moving average polynomial in the component models, and, in the AMB approach, to assign all possible noise to one of the components, so as to make the other one noninvertible. In this last case, the decomposition is termed "canonical", and the associated noise–free component, a canonical component.

Be that as it may, the fact remains that there is no universally accepted criterion to reach identification in unobserved component models, and the properties of the different admissible decompositions have not been much explored. In this paper, we analyse some of these properties, mostly in connection with the components estimation error. Burridge and Wallis (1985) within the STSM approach, and Hillmer (1985) within the ABM approach, have provided algorithms for computing the variance of the components estimation error. In this paper, an alternative approach, close to the one in Watson (1987), is followed, which permits us to obtain simple analytical expressions for the variances of the components estimation error for different admissible decompositions.

When choosing between two admissible decompositions that only differ in the allocation of white noise to the components, one relevant consideration could be the precision of the associated estimators. There are, however, several types of estimators, depending on the available information. For periods close to the end of the series, preliminary estimators have to be used, which will be revised as new observations become available, until the final or historical estimator is obtained. Since it seems reasonable that an agency producing seasonally adjusted data, for example, would like to provide historical series as precise as possible, we begin by considering (Sections 3 and 4) the historical estimator.

Several properties of the historical estimator and its associated error are derived. In particular, it is shown that the crosscovariance-generating function between the estimators of the two components is identical to the autocovariance-generating function of each component estimation error. Thus the admissible decomposition that minimizes the components estimation error minimizes also the covariance between the two component estimators. Given that the components are assumed orthogonal, this feature seems an additional desirable property of the decomposition that provides the most precise estimator.

For a given overall ARIMA model, the different admissible decompositions can be expressed as a function of a parameter α in the unit interval. The two extreme values, $\alpha = 0$ and $\alpha = 1$, correspond to the two possible canonical decompositions, each one associated with noninvertibility of one of the components. Section 4 expresses the variance of the final estimation error as a second-order polynomial in α, where the coefficients can be determined from the overall ARIMA model. The decomposition that yields the most precise component estimators is derived and it is shown that it will always be a canonical one. Which of the two canonical decompositions it happens to be depends on the stochastic properties of the series, and a simple algorithm to determine which component should be made canonical is provided. Heuristically, the rule can be interpreted as making noninvertible the most stable of the two components (i.e., adding all noise to the most stochastic component).

In Sections 5 and 6 the results are extended to any preliminary estimator and to forecasts of the components. The estimation error is, in this case, equal to the sum of the error in the historical estimator plus the so-called revision error. Since, for an agency involved in short-term policy, minimizing the error in the measurement of the signal for the most recent period seems an important feature, special attention is paid to the error in the concurrent estimator of the components. It is seen how, for all preliminary estimators, the variance of the estimation error is a polynomial of degree 2 in α, with coefficients that are straightforward to derive from the overall ARIMA model; furthermore, this variance is always minimized at a canonical decomposition.

Which one of the two canonical decompositions it is can be determined from the following rule, which applies to historical as well as to preliminary estimation: Specify each component in its canonical form and consider the MMSE estimation filter for the component at time t. Let ν_0 denote the coefficient of x_t in this filter. If the component with smallest ν_0 weight is made canonical, then the estimation error variance (for both components) is minimized; i.e. all noise is then assigned to the component with the largest weight. Thus, if interest centers on having the most precise historical estimator, ν_0 denotes the central weight of the WK filter. If, alternatively, the most precise concurrent estimator is sought, ν_0 denotes the first weight of the one-sided filter. More generally, if interest centers on minimizing the error of the estimator of the component for time t, computed at time $(t+k)$, then ν_0 is the weight of x_t in the truncated filter (i.e., the filter that extends up to x_{t+k}).

It will often be the case that the same canonical decomposition minimizes the variance of the different types of estimators and, broadly, that decomposition will be the one with the most stable component made noninvertible. There are, however, cases, when the components have similar degrees of stability, where the solutions "switch" and, for example, one of the canonical decompositions yields the most precise final estimator, while the other one yields the most precise concurrent estimator. Still, the switching of solutions is seen to happen when the estimation error variances for the two canonical decompositions are relatively close, and hence the choice matters little.

Three examples are discussed in Section 6. The first one is a "trend–plus–cycle" model similar to the ones used by economists in business–cycle analysis. The second example is a quarterly ARIMA model; these two examples illustrate the derivation of the estimation error variances from the parameters of the "observed" model within the STSM and the AMB approaches. The third example consists of a class of models that are often found to approximate reasonably well the stochastic properties of many series: the so–called Airline Model of Box and Jenkins (1970, chapter 9). This example extends the results in Hillmer (1985), and presents some stylized facts often found in actual time series.

1 The Model

We consider the problem of decomposing an observed series x_t into two Unobserved Components, s_t and n_t, as in

$$x_t = s_t + n_t \tag{1.1}$$

The two components are the output of the linear stochastic processes

$$\phi_s(B)\, s_t = \theta_s(B)\, a_{st}, \tag{1.2.a}$$

$$\phi_n(B)\, n_t = \theta_n(B)\, a_{nt}, \tag{1.2.b}$$

where $\phi_\bullet(B)$ denotes a finite polynomial in the lag operator B, having all roots on or outside the unit circle. Letting $\delta_\bullet(B)$ represent the stationary transformation of the component, we shall also use the representation

$$\phi_s(B) = \varphi_s(B)\, \delta_s(B); \quad \phi_n(B) = \varphi_n(B)\, \delta_n(B), \tag{1.3}$$

where $\varphi_\bullet(B)$ contains the roots outside the unit circle and $\delta_\bullet(B)$ contains the unit roots. Finally, $\theta_\bullet(B)$ denotes a finite polynomial in B with the roots on or outside the unit circle. The model consists of equation (1.1)–(1.2) and some additional assumptions.

Assumption 1: The variables a_{st} and a_{nt} are independent normally distributed white–noise innovations in the components. ∎

Assumption 1 implies, of course, that the two components are uncorrelated. Important examples of the decomposition (1.1) are the "trend + detrended series" decomposition often used in business cycle analysis, where the trend may be a random walk and the detrended series a low–order stationary process, and the "seasonal component + seasonally adjusted series" decomposition, where the seasonal component is often modeled as

$$U(B)\, s_t = \theta_s(B)\, a_{st}, \tag{1.4}$$

with $U(B)$ the nonstationary "seasonal" polynomial $U(B) = 1 + B + \ldots + B^{\tau-1}$ (τ denotes the number of observations per year), and the seasonally adjusted series is given by a process of the type:

$$\nabla^d\, n_t = \theta_n(B)\, a_{nt}, \tag{1.5}$$

with d typically 1 or 2. Since, as the examples illustrate, each component is basically characterized by its autoregressive (AR) roots, AR roots associated with different frequencies should be allocated to different components. Thus we specify the following assumption, which also avoids redundant roots in the polynomials of (1.2.a) and (1.2.b).

Assumption 2: The polynomials $\phi_s(B)$ and $\phi_n(B)$ share no root in common. The same holds true for the polynomials $\phi_s(B)$ and $\theta_s(B)$, and for the polynomials $\phi_n(B)$ and $\theta_n(B)$. ∎

Equations (1.1) and (1.2), and Assumptions 1 and 2 imply that the observed series x_t follows the general ARIMA process

$$\phi(B)\, x_t = \theta(B)\, a_t. \tag{1.6}$$

The AR polynomial $\phi(B)$ is given by

$$\phi(B) = \phi_s(B)\, \phi_n(B), \tag{1.7}$$

and hence it can also be factorized as $\varphi(B)\ \delta(B)$, with $\varphi(B) = \varphi_s(B)\ \varphi_n(B)$, and $\delta(B) = \delta_s(B)\ \delta_n(B)$, so that $\delta(B)$ denotes the stationarity–inducing transformation for x_t. The Moving Average (MA) part, $\theta(B)\, a_t$, is determined by the identity:

$$\theta(B)\, a_t = \phi_n(B)\, \theta_s(B)\, a_{st} + \phi_s(B)\, \theta_n(B)\, a_{nt}, \tag{1.8}$$

and the constraint that the roots of $\theta(B)$ lie on or outside the unit circle. Although not strictly needed, for convenience, we shall introduce the following assumption.

Assumption 3: The polynomial $\theta(B)$ is invertible. ∎

In the next section we shall see that Assumption 3 implies no loss of generality. Also without loss of generality, and unless otherwise specified, throughout the paper it will be assumed that $V_a = 1$, where V_a is the variance of a_t in (1.6). It should be kept in mind, thus, that the innovation variances V_s and V_n will be implicitly expressed as a fraction of V_a. Let $F = B^{-1}$ denote the forward operator; it will prove useful to define the inverse (or dual) model of (1.6), given by

$$\theta(B)\, z_t = \phi(B)\, a_t. \tag{1.9}$$

Under Assumption 3, model (1.9) is stationary, with Autocovariance Generating Function (ACGF) given by

$$h(B,F) = \sum_{j=0}^{\infty} h_j(B^j + F^j) = \pi(B)\,\pi(F), \tag{1.10}$$

where $\pi(B)$ contains the coefficients of the AR expansion of (1.6), that is

$$\pi(B) = \phi(B)/\theta(B) = \sum_{j=0}^{\infty} \pi_j\, B^j, \quad (\pi_0 = 1). \tag{1.11}$$

Notice that the variance of the inverse process is given by

$$h_0 = \sum_{j=0}^{\infty} \pi_j^2. \tag{1.12}$$

2 Identification of the Model

Having observations on x_t, model (1.6) can be identified from the data. For the rest of the discussion, we shall assume that the ARIMA model for x_t is known. Given this overall model, there is obviously an infinite number of ways of decomposing x_t as in (1.1)–(1.2) under Assumptions 1–3.

If the only identification restrictions that are considered are restrictions in the orders of the polynomials of (1.2), then the necessary and sufficient condition for model identification is that, for at least one of the components, the order of the AR polynomial be larger than the order of the MA polynomial; see Hotta (1989). Thus, letting p_s, p_n, q_s, and q_n denote the orders of the polynomials $\phi_s(B)$, $\phi_n(B)$, $\theta_s(B)$, and $\theta_n(B)$, respectively, under

Assumption 4a: $p_s > q_s$ or $p_n > q_n$ (or both), ∎

the model consisting of equations (1.1)–(1.2) and Assumptions 1, 2, and 3, is identified.

Be that as it may, one may question whether zero-coefficient restrictions are the most adequate ones. To illustrate the point, we consider a simple UC model similar to the ones used in business cycle analysis (see, for example, Stock and Watson, 1988). The observed (annual) series is the sum of a trend component, s_t, and a detrended series, n_t, where the trend is the random–walk process

$$\nabla\, s_t = a_{st}, \tag{2.1.a}$$

and the detrended series is the stationary ARMA(1, 1) model

$$(1 + .7B)\, n_t = (1 + .2B)\, a_{nt}. \tag{2.1.b}$$

(Since (2.1.a) satisfies Assumption 4a, for a particular observed series $x_t = s_t + n_t$, the model would be identified.) Direct inspection of (2.1.b) shows that the detrended series consists of a stationary cyclical behavior (with period 2) and some random noise. Assumptions 1–3 are assumed to hold, and the equations in (2.1) imply that the observed series x_t can be seen as the output of the ARIMA (1, 1, 2) process:

$$(1 + .7B)\, \nabla\, x_t = \theta(B)\, a_t. \tag{2.2}$$

Setting, for our example, $V_s = 5V_n$, it is easily found that $\theta(B) = (1 + .364B - .025B^2)$. For a time series generated by (2.1), Figures 1a and 1b display the two components, and Figures 2a and 2b exhibit the spectra of x_t and of the two components, which we shall represent as $g_x(\omega)$, $g_s(\omega)$, and $g_n(\omega)$, with ω being the frequency in radians. (To simplify terminology, "spectrum" will also denote the pseudospectrum of nonstationary series; see Harvey, 1989.) Figure 2b shows that $g_s(\omega)$ has a minimum for $\omega = \pi$, which is found to be equal to $g_s(\pi) = V_s/4$. It follows that if a white–noise component u_t, with variance V_u in the interval $[0, V_s/4]$, is removed from s_t and added to n_t, the resulting components also provide an acceptable decomposition of x_t. The only difference would be that the new s_t component would be smoother, while n_t would now be noisier, as evidenced in Figures 1c and 1d for the case $V_u = V_s/5$.

In general, if white noise with variance $0 \leq V_u \leq V_s/4$ is removed from s_t and assigned to n_t, it is straightforward to find that the new s_t and n_t components follow processes of the type:

$$\nabla\, s_t = (1 + \theta_s B)\, a_{st} \tag{2.3.a}$$

$$(1 + .7B)\, n_t = (1 + \theta_n B)\, a_{nt}, \tag{2.3.b}$$

For a given model (2.2) for the observed series, different decompositions of the type (2.3) would provide admissible decompositions that would differ in the way the noise contained in the series is allocated to the two components.

Consider an analyst interested in whatever is in the series that cannot be attributed to the trend. He wishes, thus, to remove the trend and nothing but the trend. He will, consequently, avoid adding noise to the trend component, and would choose the decomposition for which V_u is equal to its maximum value $V_s/4$. (Identification of unobserved components by using the "minimum extraction" principle was first proposed by Box, Hillmer, and Tiao, 1978; and Pierce, 1978.) The spectra of the two components in this case are given in Figures 2c and 2d, where they are compared to the spectra of the components in Figure 1. Since the requirement that it should not be possible to decompose s_t into a smoother component plus white noise implies that $g_s(\pi) = 0$, and since the time domain equivalent of this spectral zero is the presence of the factor $(1 + B)$ in the MA part of the component model, s_t will follow the noninvertible model

$$\nabla\, s_t = (1 + B)\, a_{st},$$

and the model for n_t will be as in (2.3.b).

Alternatively, a similar type of reasoning may lead to the transfer of noise from n_t to s_t. Assume, for example, that model (2.2) holds for a time series observed with a twice–a–year frequency. Then model (2.3.b) represents a seasonal component and, if interest centers on the seasonally adjusted series, one may wish to remove from the series as little as possible, and hence the chosen decomposition would consist of a noninvertible seasonal component n_t, with $g_n(0) = 0$, and an invertible seasonally adjusted series s_t. As a consequence, the seasonal component would follow the model

$$(1 + .7B)\, n_t = (1 - B)\, a_{nt},$$

and the model for s_t would be as in (2.3.a). Therefore, the minimum extraction requirement yields two canonical solutions, both of which can be easily justified; each one is characterized by noninvertibility of one of the two components.

Back to the general case of (1.2), assume, in general, that s_t is an invertible and identified component (i.e., $p_s > q_s$). Then, a white–noise component can be removed from s_t and assigned to n_t. It is easily seen that the new model for s_t has $p_s = q_s$; thus we replace Assumption 4a with the more general one

Assumption 4b: $p_s \geq q_s$ or $p_n \geq q_n$ (or both). ∎

For a given ARIMA model for the observed variable, the class of admissible decompositions is given by the pair of components s_t and n_t satisfying (1.1), (1.2), (1.7), (1.8), and Assumptions 1, 2, 3, and 4b. We require, of course, nonnegative spectra $g_s(\omega)$ and $g_n(\omega)$. In the general case of an infinite number of admissible decompositions, identification of a unique model can then be reached with the following assumption:

Assumption 5: For $\omega \in [0, \pi]$, either $\min g_s(\omega) = 0$ or $\min g_n(\omega) = 0$ (or both). ∎

Identification is, in this case, obtained by forcing a component to be noninvertible. Following Box, Hillmer, and Tiao (1978), a noninvertible component will be denoted a "canonical" component, and the associated decomposition, a canonical decomposition. Since the spectra of the components cannot be negative, in the two-component case there will be two canonical decompositions. One of them puts all additive white noise in the component n_t, the other one, in the component s_t. Any admissible decomposition can be seen as something in between, whereby some noise is allocated to n_t and some to s_t. Notice that, since no additive noise can be extracted from a noninvertible series, if the observed series is noninvertible, the decomposition (if there is one) is necessarily unique, with all components canonical, sharing the same spectral zero. In this case, no identification problem arises, and hence, for our purposes, Assumption 3 implies no loss of generality.

As shown in Hillmer and Tiao (1982), canonical components display some important features. In particular, any other admissible component is equal to the canonical one plus added noise, and hence the canonical requirement makes the component as smooth as possible. On the negative side, Maravall (1986) shows how canonical components can produce large revisions in the preliminary estimators of the component. Besides, the existence of two canonical solutions reflects some basic ambiguity concerning the desirable properties of a component. It seems reasonable, for example, that, in order to avoid noise-induced overreaction, the monetary authority may be interested in a smooth (noise-free) seasonally adjusted series. On the other hand, it sounds also reasonable that the analyst wishes to keep in the series everything but seasonality, in which case the seasonal component would be noise-free. Therefore, both canonical solutions could, in principle, be rationalized.

Some additional suggestions have been made to overcome uncertainty over which admissible decomposition should be chosen. For example, given that different admissible decompositions imply different properties of the estimators, to be on the safe side, Watson (1987) and Findley (1985) propose to select a "minimax" solution (i.e., the decomposition that maximizes the MSE of the MMSE estimators). Still, as a general rule, canonical components (i.e., Assumptions 4b and 5) are used in the AMB approach, while zero-coefficient restrictions (i.e., Assumption 4a) are used in the STSM approach and in econometric applications of UC models. Besides its simplicity, the choice may possibly reflect the tradition in econometrics of identifying models (in particular, simultaneous equation models) by using zero-coefficient restrictions (see, for example, Theil, 1971).

3 MMSE Estimators and Their Properties

3.1 Optimal Estimators of the Components

We have mentioned that the properties of the component estimator will depend on the admissible decomposition selected. Our intention is to explore this dependence. In order to do that, we consider first the case of a complete realization of the process, i.e., the case of a series x_t with t going from $-\infty$ to ∞. Let the series be stationary, and write (1.2) and (1.6) more compactly as

$$x_t = \psi(B)\, a_t; \qquad s_t = \psi_s(B)\, a_{st}; \qquad n_t = \psi_n(B)\, a_{nt}, \tag{3.1}$$

where $\psi(B) = \theta(B)/\phi(B)$, $\psi_s(B) = \theta_s(B)/\phi_s(B)$, and $\psi_n(B) = \theta_n(B)/\phi_n(B)$. The minimum Mean Squared Error (MSE) estimator of s_t is given by

$$\hat{s}_t = \nu(B,F)\, x_t = V_s \frac{\psi_s(B)\, \psi_s(F)}{\psi(B)\, \psi(F)}\, x_t, \tag{3.2}$$

where F is the forward operator $F = B^{-1}$; see Whittle (1963). The symmetric and centered filter $\nu(B,F)$ is the so-called Wiener–Kolmogorov (WK) filter. Letting $A_j(B,F)$ denote the ACGF of component j,

$$A_j(B,F) = \psi_j(B)\, \psi_j(F)\, V_j, \qquad j = s, n,$$

and $A_x(B,F) = \psi(B)\, \psi(F)$, expression (3.2) can be rewritten

$$\hat{s}_t = [A_s(B,F)/A_x(B,F)]\, x_t. \tag{3.3}$$

In terms of the AR and MA polynomials, after simplification, the WK filter can be expressed as:

$$\nu(B,F) = V_s \frac{\theta_s(B)\, \theta_s(F)\, \phi_n(B)\, \phi_n(F)}{\theta(B)\, \theta(F)} \tag{3.4}$$

Expression (3.4) shows that, under Assumption 3 (invertible observed series), the filter will be convergent, independently of the roots of the AR polynomials. The filter (3.4) in fact extends to nonstationary series, with unit roots in $\phi_s(B)$ and/or $\phi_n(B)$; see Bell (1984), and Maravall (1988). Direct inspection shows that the WK filter (3.4) is simply the ACGF of the model

$$\theta(B)\, z_t = \theta_s(B)\, \phi_n(B)\, b_t, \tag{3.5}$$

with b_t white noise with variance V_s. Since $\theta(B)$ is invertible, the model is stationary and its ACGF will converge. The effect on the filter of different admissible decompositions will show up in the MA part of (3.5), through the polynomial $\theta_s(B)$ and the variance V_s.

Unless the model for the series is a pure AR model, the filter (3.4) will extend from $-\infty$ to ∞. Its convergence however guarantees that, in practice, it could be approximated by a finite filter, and it is generally the case that, for the usual series length, the estimator of the component for the central periods of the series can be safely seen as generated by the WK filter (3.4). This estimator, obtained with the complete filter, is often denoted "historical" or "final" estimator; it shall be the one of interest until Section 5.

3.2 Covariance Between Estimators

It is a well-known result that minimum MSE estimators of orthogonal components yield estimators with nonzero crosscovariances. This discrepancy has been the cause of concern (see, for example, Nerlove, 1964; Granger, 1978; and Garcia Ferrer and Del Hoyo, 1992), and hence one could argue that another possible identification criterion could be to select, among the admissible decompositions, the one that minimizes the (lag-0) covariance between the estimators. This covariance is easily found from the following result (proofs of the results not derived in the text are in the Appendix).

Lemma 1: Let $C(B, F)$ denote the CrossCovariance Generating Function (CCGF) for the two estimators $\hat{s}_t$ and $\hat{n}_t$. Then $C(B, F)$ is equal to the ACGF of the model

$$\theta(B)\, z_t = \theta_s(B)\, \theta_n(B)\, b_t, \tag{3.6}$$

where b_t is white noise with variance $(V_s\, V_n)$. ∎

Lemma 1 implies that $C(B, F)$ is symmetric and convergent. Since model (3.6) is stationary, all covariances will be finite. The variance of the model yields the lag-0 covariance between $\hat{s}_t$ and $\hat{n}_t$; this covariance, thus, will always be positive (and hence the variance of the estimator will always underestimate the variance of the component). However, the fact that the covariances between $\hat{s}_t$ and $\hat{n}_t$ are finite implies the following result.

Lemma 2: When the series x_t is nonstationary, the historical estimators $\hat{s}_t$ and $\hat{n}_t$ are uncorrelated. ∎

For nonstationary series (the case of applied interest) minimum MSE estimation of the components preserves, thus, the orthogonality assumption, and, for example, the statement in García Ferrer and Del Hoyo (1992) that "whereas the theoretical components are uncorrelated, the estimators will be correlated in general" is only correct for stationary series. Further, it is easily found from (A.1) and (A.2), and the equivalent expressions for n_t, that, although the estimators $\hat{s}_t$ and $\hat{n}_t$ are uncorrelated, certain linear combinations of them — namely, the stationary transformations $\delta_s(B)\ \hat{s}_t$ and $\delta_n(B)\ \hat{n}_t$ — are correlated.

It is worth pointing out an interesting feature of the estimators of nonstationary trend and seasonal components. Although both are nonstationary series which, moreover, cannot be cointegrated (since the unit AR roots are different), they display stationary crosscovariances. Thus, the two estimators diverge in time, each one with a nonstationary variance, but their crosscovariances remain constant.

Back to the covariance between the component estimators, model (3.6) shows that different admissible decomposition would affect its MA part, through $\theta_s(B)$, $\theta_n(B)$, V_s and V_n. But before we look at which admissible decomposition minimizes the covariance between the estimators, let us turn our attention to another possibly desirable feature of the estimators.

3.3 The Error in the Component Estimator

The error in the UC estimator depends on the particular admissible decomposition selected. Since the data do not discriminate among admissible decompositions, the selection of a particular one reflects a choice of the analyst. In the absence of a compelling reason to select a particular noise allocation, why not choose the one that provides the most precise estimators? Since the error in $\hat{s}_t$ is equal to that in $\hat{n}_t$, minimizing both estimation errors seems an attractive feature of the selected model.

To see the dependence of the estimation error on the admissible decomposition chosen, we use the following Lemma, which is an application of Theorem 3 in Pierce (1979), for the case $\delta_c(B) = 1$ and $V_a = 1$.

Lemma 3: Let e_t denote the estimation error $e_t = s_t - \hat{s}_t = \hat{n}_t - n_t$. Then e_t can be seen as the output of the ARMA model

$$\theta(B)\, e_t = \theta_s(B)\, \theta_n(B)\, d_t, \tag{3.7}$$

where d_t is a white noise with variance $(V_s\, V_n)$. ∎

From Lemmas 1 and 3, the following results are trivially obtained.

Lemma 4: The ACGF of e_t is equal to the CCGF between $\hat{s}_t$ and $\hat{n}_t$. ∎

Corollary 1: The admissible decomposition with minimum estimation error of the components minimizes also the covariance between the two component estimators. ∎

We turn our attention to the identification of the admissible decomposition that exhibits those desirable properties.

4 Historical Estimation Error and Admissible Decompositions

As mentioned in Section 2, each admissible decomposition is characterized by a particular allocation of the noise to the two components. Let s_t and n_t denote an admissible decomposition of x_t; then $g_x(\omega) = g_s(\omega) + g_n(\omega)$. Let, for $\omega \in [0, \pi]$, $V_u^s = \min g_s(\omega)$, and $V_u^n = \min g_n(\omega)$. The total amount of "additive" noise in x_t that can be distributed between the components is equal to $V_u = V_u^s + V_u^n$. Following an approach similar to Watson (1987), we shall express each admissible decomposition in terms of a parameter α that reflects the particular noise allocation. Denote by s_t^0 and n_t^0 the decomposition with s_t canonical and n_t with maximum noise, and let $g_s^0(\omega)$, $g_n^0(\omega)$, $A_s^0(B,F)$, and $A_n^0(B,F)$ be the associated spectra and ACGFs of the components. These functions, as well as the models for the underlying components, can be derived from the ARIMA model for the observed series. Since any admissible component s_t^α is equal to s_t^0 plus an amount of noise with variance in the interval $[0, V_u]$, any admissible decomposition, s_t^α and n_t^α, can be expressed as

$$g_s^\alpha(\omega) = g_s^0(\omega) + \alpha\, V_u \tag{4.1.a}$$
$$g_n^\alpha(\omega) = g_n^0(\omega) - \alpha\, V_u \tag{4.1.b}$$

with $\alpha \in [0, 1]$. The two canonical decompositions (one with s_t canonical, the other with canonical n_t) can be seen as the two extreme cases $\alpha = 0$ and $\alpha = 1$. The time domain equivalent of (4.1) is given by the relationships

$$A_s^\alpha(B,F) = A_s^0(B,F) + \alpha\, V_u \tag{4.2.a}$$

$$A_n^\alpha(B,F) = A_n^0(B,F) - \alpha\, V_u, \tag{4.2.b}$$

and, for any α, $A_x(B,F) = A_s^\alpha(B,F) + A_n^\alpha(B,F)$. Our aim is to derive an expression that relates the variance of the component estimation error, $V(e_t^\alpha)$, to the parameter α. That variance, we recall, is also the covariance between the two component estimators.

For $0 \le \alpha \le 1$ denote the estimators of the components for a particular α by

$$\hat{s}_t^\alpha = \nu_s^\alpha(B,F)\, x_t \tag{4.3.a}$$

$$\hat{n}_t^\alpha = \nu_n^\alpha(B,F)\, x_t, \tag{4.3.b}$$

where the WK filter is ($k = s, n$):

$$\nu_k^\alpha(B,F) = \sum_{j=0}^{\infty} \nu_{k,j}^\alpha\, (B^j + F^j).$$

Thus $\hat{s}_t^0$ and $\hat{n}_t^0$ correspond to the decomposition with canonical s_t, and $\hat{s}_t^1$ and $\hat{n}_t^1$ to the one with canonical n_t.

<u>Lemma 5</u>: Let $e_t^\alpha = s_t^\alpha - \hat{s}_t^\alpha = \hat{n}_t^\alpha - n_t^\alpha$. Then,

$$V(e_t^\alpha) = V(e_t^0) + (1 - 2\,\nu_{s,0}^0)\, V_u\, \alpha - h_0\, V_u^2\, \alpha^2, \tag{4.4}$$

where e_t^0 is the error in $\hat{s}_t^0$, $\nu_{s,0}^0$ is the central weight of the filter $\nu_s^0(B,F)$, and h_o is given by (1.12). ∎

Lemma 5 expresses the variance of the component estimation error as a second–order polynomial in α, with coefficients that can be obtained from the "observed" ARIMA model. Considering that $V(e_t^0)$ is the variance of model (3.7) and $\nu_{s,0}^0$ is the variance of model (3.5), both for the case of a canonical s_t, and h_0 is the variance of the inverse model (1.9), the three coefficients of (4.4) can be easily computed as the variance of ARMA models with the AR polynomial always equal to $\theta(B)$.

From Lemma 5 it is straightforward to find which admissible decomposition minimizes the variance of the component estimation error:

<u>Lemma 6</u>: For $\alpha \in [0,1]$, $V(e_t^\alpha)$ is minimized

(a) at $\alpha = 0$ when $2\,\nu_{s,0}^0 + V_u\, h_0 \le 1$,

(b) at $\alpha = 1$ when $2\,\nu_{s,0}^0 + V_u\, h_0 \ge 1$. ∎

As a function of α, $V(e_t^\alpha)$ given by (4.4) is a parabola, positive over the interval $[0,1]$, with a finite maximum for, say, α_m. If α_m is contained in the interval $[0,1]$, then either $\alpha = 0$ or $\alpha = 1$ may minimize $V(e_t^\alpha)$; when $\alpha_m > 1$, the minimum will be for $\alpha = 0$, and when $\alpha_m < 0$, it will be for $\alpha = 1$. Since $\alpha_m = (1 - 2\nu_{s,0}^0)/2h_o V_u$, it can be easily checked that the three cases are possible.

Lemma 6 implies that the component estimators with minimum MSE and minimum crosscovariance are always found at one of the two canonical decompositions. Up to now, the two components s_t and n_t have been treated symmetrically. We now break this symmetry and denote by s_t the component with the largest central weight in the associated WK filter that provides the canonical component estimator; these weights are $\nu_{s,0}^0$ and $\nu_{n,o}^1$. Thus, without loss of generality, we assume the following:

Assumption 6a: $\nu_{s,0}^0 \geq \nu_{n,0}^1$. ∎

Now it becomes possible to identify which of the two canonical decompositions has minimum estimation error.

Lemma 7: Among all admissible decompositions, under Assumption 6a, the historical estimator MSE is minimized for the decomposition with canonical n_t. ∎

Lemma 7 provides a simple procedure to determine which canonical decomposition provides minimum component estimation error (and minimum covariance between the two component estimators). For each of the two components compute the central weight of the WK filter that yields the estimator of the component in its canonical form. Then, set as canonical component the one with the smallest weight (i.e., add all noise to the one with the largest weight). Notice that, from the two canonical specifications, the central weights of the WK filters can be simply computed as the variance of the ARMA model (3.5). Three remarks seem worth adding:

(a) Since $\nu_{k,0}^0$ measures the contribution of observation x_t to the component estimator, the precision of the estimator is maximized by assigning all additive noise to the component for which that contribution is largest.

(b) In the important application to seasonal adjustment, if s_t denotes the seasonal component and n_t the adjusted series, it is often the case that $\nu_{s,0}^0 < \nu_{n,0}^0$ and hence the most precise estimates of s_t and n_t are obtained with a canonical seasonal component. In these cases, the "minimum extraction" principle used in the AMB approach to seasonal adjustment provides also the most precise estimators, with minimum crosscovariance.

(c) While one of the two canonical decompositions always provides the most precise estimators, the other may or may not yield estimators with maximum MSE. When $\alpha_m < 0$ or $\alpha_m > 1$, then it maximizes $V(e_t^\alpha)$, and coincides thus with the minimax solution of Watson (1987). For this solution, of course, the covariance between the estimators is also maximized.

   It is worth noticing that the two opposite criteria (choosing the admissible decomposition with maximum or with minimum estimation error variance) stem from a "philosophical" difference. While Watson believes that there is a "true" underlying (unknown) seasonal component model among the set of admissible ones, we believe that reality does not provide for a particular allocation of noise among the two components and that this allocation is, in essence, arbitrary. In so far as unobserved components, such as trend or seasonality, are tools designed by the analyst to address problems, it makes sense to choose the most precise tool among the admissible ones.

(d) Expression (4.4) corresponds to expression (3.9) in Watson (1987). The difference is due to the fact that Watson considers a fixed filter, while the filter, in our case, is the optimal one for every value of α. The fact that the filter depends on α invalidates the derivation in Watson, and expression (4.4) is obtained instead.

5 Preliminary Estimation Error, Revisions, and Admissible Decompositions

Up to now we have considered estimation of the components for an infinite realization of the series. Since the WK filter converges in both directions, as mentioned in Section 3, it can be safely truncated and, for most series lengths, the estimator for the central periods can be seen as the one obtained with the complete filter (the historical or final estimator). While it seems reasonable that, a data–producing agency wishing to produce historical series as precise as possible, minimizes the error in the final estimator, it also seems reasonable that someone involved in short–term monitoring or policy–making would seek to minimize the error in the estimator for the most recent periods, in order to avoid error–induced policy actions (this concern is certainly present in, for example, monetary policy). Given that for the most recent observation the WK filter cannot be applied, a preliminary estimator has to be used instead. We proceed to consider the error in this preliminary estimator.

Assume that only a finite realization of the series is available. Denote this realization by $X_T = [x_1, x_2, \ldots, x_T]$, and by $x_{t/T}$ the forecast of x_t when observations are available up to and including period T. Then, as shown by Cleveland and Tiao (1976), the optimal "preliminary" estimator of s_t is given by

$$\hat{s}_{t/T} = E_T\, s_t = \nu_s(B, F)\, x^e_{t|T}, \tag{5.1}$$

where $\nu(B, F)$ is the WK filter given by (3.4), and $x^e_{t|T}$ is the series extended with forecasts $x_{T+j/T}$ and backcasts $x_{1-j/T}$, $j = 1, 2, \ldots$. As new observations become available, the forecasts are updated or replaced by the new data and, as a consequence, the estimator of s_t will be revised until it becomes the historical estimator, once the filter has converged. Since the above forecasts and backcasts are linear functions of the elements of X_T, expression (5.1) can be rewritten

$$\hat{s}_{t|t+k} = \nu_s(B, F, k)\, x_t, \tag{5.2}$$

where $T = t+k$, $\nu_s(B, F, k)$ denotes the truncated, asymmetric filter, and x_t the elements of X_T. We shall assume that the series is long enough for the weights of (5.2) to have converged in the direction of the past. In the vast majority of practical applications this is not a restrictive assumption, and it allows us to associate the finite–sample effect on the preliminary estimator with the unavailability of future observations. We can then write the error in the preliminary estimator, $d_{t|T} = s_t - \hat{s}_{t/T}$ as

$$d_{t|T} = e_t + r_{t|T},$$

where $e_t = s_t - \hat{s}_t$ is the error in the final estimator $\hat{s}_t$ (analysed in Section 3.3), and $r_{t|T} = \hat{s}_t - \hat{s}_{t/T}$ is the "revision error" in the preliminary estimator. Under Assumptions 1–3, the two errors, e_t and $r_{t|T}$, are independent (see Pierce, 1980), and this will be true for any admissible decomposition. Rewrite expression (A.1) as

$$\begin{aligned} \hat{s}_t = \xi_s(B,F)\, a_t &= \ldots + \xi_{s,-1}\, a_{t-1} + \xi_{s,0}\, a_t + \xi_{s,1}\, a_{t+1} + \ldots + \xi_{s,T-t}\, a_T + \\ &+ \xi_{s,T-t+1}\, a_{T+1} + \ldots = \xi_s(B)^-\, a_T + \xi_s(F)^+\, a_{T+1}. \end{aligned} \tag{5.3}$$

The weights $\xi_{s,j}$ are easily determined from the identity.

$$\phi_s(B)\, \theta(F)\, \xi_s(B,F) = V_s\, \theta_s(B)\, \theta_s(F)\, \phi_n(F). \tag{5.4}$$

Under suitable conditions concerning the starting values (see Bell, 1984), the estimator $\hat{s}_{t|T}$ can be obtained by taking conditional expectations at time T in (5.3), yielding

$$\hat{s}_{t|T} = \xi_s(B)^-\, a_T, \tag{5.5}$$

since $E_T\, a_{T+j} = 0$ for $j \geq 1$. Substracting (5.5) from (5.3), the revision in the concurrent estimator can be expressed as

$$r_{t|T} = \xi_s(F)^+\, a_{T+1} = \sum_{j=T-t+1}^{\infty} \xi_{s,j}\, a_{t+j}, \tag{5.6}$$

which involves only the coefficients of F^j, $j \geq 1$, in (5.4) and hence is a convergent filter that can be truncated after a finite number of terms. Expression (5.6), properly truncated, can then be used to compute the ACGF of $r_{t|T}$; in particular

$$V(r_{t|T}) \simeq \sum_{j=T-t+1}^{M} \xi_{s,j}^2, \tag{5.7}$$

where M is the truncation point.

Up to now, the discussion in this section applies equally to preliminary estimators, for which $T \geq t$, and to forecasts of the component, for which $T < t$. We proceed to consider first preliminary estimation and, for notational convenience, set $T = t + k (k = 0, 1, 2, \ldots)$. For the admissible decomposition associated with α, $x_t = s_t^\alpha + n_t^\alpha$, the components preliminary estimation error and revision error can be expressed as:

$$d^\alpha_{t|t+k} = s^\alpha_t - \hat{s}^\alpha_{t|t+k} = e^\alpha_t + r^\alpha_{t|t+k} \tag{5.8}$$

$$r^\alpha_{t|t+k} = \sum_{j=k+1}^{\infty} \xi^\alpha_{s,j}\, a_{t+j}, \tag{5.9}$$

respectively. In the previous section we looked at the dependence of e_t^α on α. Now we look at the dependence of the revision error, $r^\alpha_{t|t+k}$, and of the total error, $d^\alpha_{t|t+k}$, on α.

From (1.11), (3.4), and (5.4) it is seen that

$$\pi(B)\, \xi_s^\alpha(B,F) = \nu_s^\alpha(B,F), \tag{5.10.a}$$

or, since $\pi(B) = 1/\psi(B)$,

$$\xi_s^\alpha(B,F) = \nu_s^\alpha(B,F)\, \psi(B). \tag{5.10.b}$$

Equating coefficients of B^0 in (5.10.a), it is obtained that

$$\nu_{s,0}^\alpha = \sum_{j=0}^{\infty} \xi_{s,j}^\alpha\, \pi_j, \tag{5.11}$$

where $\nu_{s,0}^\alpha$ is the coefficient of x_t in the estimator (4.3.a). Denote by $\nu_{s,0}^\alpha(k)$ and by $h_0(k)$ the sum of the first $(k+1)$ terms in the r.h.s. of (5.11) and of (1.12), respectively. Thus,

$$\nu_{s,0}^\alpha(k) = \xi_{s,0}^\alpha + \pi_1\, \xi_{s,1}^\alpha + \ldots + \pi_k\, \xi_{s,k}^\alpha, \tag{5.12}$$

$$h_0(k) = 1 + \pi_1^2 + \ldots + \pi_k^2, \tag{5.13}$$

and the following lemma is proved in the Appendix.

Lemma 8: The variance of the revision error in the preliminary estimator $\hat{s}_{t|t+k}^\alpha$, is given by

$$V(r_{t|t+k}^\alpha) = V(r_{t|t+k}^0) + 2\left[\nu_{s,0}^0 - \nu_{s,0}^0(k)\right] V_u\, \alpha + \left[h_0 - h_0(k)\right] V_u^2\, \alpha^2, \tag{5.14}$$

where the superscript 0 denotes the decomposition with s_t canonical. ∎

As a result, the variance of the revision in a preliminary estimator is given by a polynomial in α of degree 2, where the coefficients can be derived from the overall ARIMA model. From Lemma 8, the following results are obtained.

Lemma 9: For $\alpha \in [0,1]$, $V(r_{t|t+k}^\alpha)$ is maximized:

(a) at $\alpha = 0$ when $2\nu_{s,0}^0 + (h_0 - h_0(k))\, V_u \le 2\, \nu_{s,0}^0(k)$;

(b) at $\alpha = 1$ when $2\nu_{s,0}^0 + (h_0 - h_0(k))\, V_u \ge 2\, \nu_{s,0}^0(k)$. ∎

Corollary 2: The variance of the revision error in the preliminary estimator (of s_t and of n_t) is maximized at one of the two canonical decompositions. ∎

Corollary 2 generalizes the result in Maravall (1986), and shows an unpleasant feature of the canonical decompositions: they may imply relatively large revisions in the concurrent estimator of the signal. However, since $V(r_t^\alpha)$ is a convex parabole, it follows that, as was the case for the error in the historical estimator, while one of the two canonical decompositions maximizes the variance of the revision error, it may well be that the other canonical decomposition minimizes that variance. This will happen when α_m, the value of α that minimizes (5.14), falls outside the interval [0,1].

Be that as it may, the main concern is not the revision error *per se*, but the total error in the preliminary estimator of the signal. The dependence of the variance of this error on the particular admissible decomposition selected is shown in the following lemma, obtained by using expressions (4.4) and (5.14) in $V(d^{\alpha}_{t|t+k}) = V(e^{\alpha}_{t}) + V(r^{\alpha}_{t|t+k})$.

Lemma 10: The variance of the error in the preliminary estimator $\hat{s}_{t|t+k}$ is given by the second-degree polynomial in α

$$V(d^{\alpha}_{t|t+k}) = V(d^{0}_{t|t+k}) + (1 - 2\nu^{0}_{s,0}(k))\, V_u\, \alpha - h_o(k)\, V_u^2\, \alpha^2, \tag{5.15}$$

where $d^{0}_{t|t+k}$ is the error that corresponds to the canonical signal. ∎

Lemma 10 allows us to determine which admissible decomposition minimizes the error in the preliminary estimator.

Lemma 11: For $\alpha \in [0,1]$, $V(d^{\alpha}_{t|t+k})$ is minimized

(a) at $\alpha = 0$ when $2\nu^{0}_{s,0}(k) + h_0(k)\, V_u \leq 1$;

(b) at $\alpha = 1$ when $2\nu^{0}_{s,0}(k) + h_0(k)\, V_u \geq 1$. ∎

Corollary 3: The variance of the error in the preliminary estimator of the signal is always minimized at one of the two canonical decompositions. ∎

As a consequence, when the effects of the historical estimation error and of the revision error are aggregated, it still remains true that a canonical specification yields the most precise preliminary estimators of the components. Which one of the two canonical decompositions displays that property can be determined through Lemma 11 or, as was done in Section 4, by breaking the symmetric treatment of the two components. Proceeding in this way, it is possible to express the general result in a very simple way:

For a particular admissible decomposition, rewrite expression (5.2) as

$$\hat{s}^{\alpha}_{t|t+k} = \nu^{\alpha}_{s}(B,F,k)\, x_t, \tag{5.16}$$

so that the decomposition with s_t canonical yields the estimators $\hat{s}^{0}_{t|t+k}$ and $\hat{n}^{0}_{t|t+k}$, while that with n_t canonical yields $\hat{s}^{1}_{t|t+k}$ and $\hat{n}^{1}_{t|t+k}$. It will be convenient to consider the filter that yields the preliminary estimator of u_t given by (A.5), that is,

$$\hat{u}_{t|t+k} = \nu_u(B,F,k)\, x_t, \tag{5.17}$$

where $u_t \sim \text{niid}(0, V_u)$. The parameters $\nu^{0}_{s,0}(k)$ and $h_0(k)$, defined by (5.12) and (5.13), turn out to have a simple interpretation in terms of the filters that provide the preliminary estimators of the components, as shown by the following lemma.

Lemma 12:

(a) $\nu^{0}_{s,0}(k)$ is the weight of B^0 in the filter $\nu^{0}_{s}(B,F,k)$.

(b) $h_0(k)$ is the weight of B^0 in the filter $\nu_u(B,F,k)$. ∎

As before, without loss of generality, denote by s_t the component with the largest weight for x_t in the filter that provides the preliminary estimator of the component in its canonical form, i.e.:

Assumption 6b: $\nu^0_{s,0}(k) \geq \nu^1_{n,0}(k)$. ∎

Lemma 13: Among all admissible decompositions, under Assumption 6b, the most precise preliminary estimators are obtained for the decomposition with canonical n_t. ∎

From Lemma 13, the decomposition with the most precise estimators of the components is straightforward to obtain.

Corollary 4: Let (1.1) and (1.2) represent the admissible decompositions of a given ARIMA model under Assumptions 1-3. To select the decomposition with smallest MSE in a preliminary estimator of s_t and n_t,

a) compute the weight of x_t in the two filters that provide the preliminary estimators of the components specified in their canonical form;
b) choose the canonical decomposition with canonical component the one with the smallest weight. ∎

Since when $k \to \infty$, from (5.12) and (5.13), $\nu^0_{s,0}(k) \to \nu^0_{s,0}$ and $h_0(k) \to h_0$, expression (5.15) becomes then (4.4), in agreement with the fact that, for $k \to \infty$, the preliminary estimator becomes the historical one. A particular case of considerable importance is when $k = 0$. The associated estimator, $\hat{s}_{t|t}$, is denoted the "concurrent" estimator. Obviously, to use the most precise concurrent estimator (i.e., the estimator of the signal for the most recent period) could be a reasonable choice for an agency involved in short-term economic policy and monitoring.

By setting $k = 0$ in (5.12) and (5.13), it is seen that $h_0(0) = 1$, and $\nu^0_{s,0}(0) = \xi^0_{s,0}$. Expression (5.15) becomes

$$V(d^{\alpha}_{t|t}) = V(d^0_{t|t}) + (1 - 2\,\xi^0_{s,0})\,V_u\alpha - V_u^2\,\alpha^2,$$

Assumption 6b becomes: $\xi^0_{s,0} \geq \xi^1_{n,0}$, and the decomposition with most precise estimators is the one that sets n_t canonical, and adds all noise to s_t. As was the case with historical estimation, while one of the two canonical decompositions always minimizes the variance of the error in the preliminary estimator, the other canonical decomposition may or may not maximize that variance. It will maximize the variance when α_m, the value of α that maximizes the function (5.15), falls outside the interval $[0, 1]$. From (5.15) it is easily seen that α will fall outside the unit interval when $\nu^0_{s,0}(k)$ is outside the interval $[.5 - h_0(k)\,V_u, .5]$; in this case the two canonical decompositions provide the most and the least precise estimators.

Although historical and preliminary estimators have minimum MSE when one of the two canonical specifications is employed, the canonical specification may well not be the same for different estimators. Thus, for example, there are models, as we shall see in the next section, for which the historical seasonally adjusted series is best estimated with a canonical seasonal component, while the concurrent seasonally adjusted series is best estimated with a canonical trend. The switching of solutions is due to the fact that Assumption 6b may imply that, for different values of k, different components may have the largest $\nu_0^0(k)$ weight. Focussing attention on historical and concurrent estimation, from Lemmas 6 and 11, the following corollary is immediately obtained.

Corollary 5: Under Assumption 6a, when $\xi_{s0}^0 < (1 - V_u)/2$, the historical estimation error is minimized with a canonical n_t and the concurrent estimation error is minimized with a canonical s_t. Otherwise, n_t canonical minimizes both types of errors.

Under Assumption 6b (with $k = 0$), replacing ξ_{s0}^0 with ν_{s0}^0, and V_u with $h_0 V_u$ in the above inequality, the same result holds. ∎

The possible switching of solutions is an inconvenient feature since, in practice, it could mean that agencies producing historical series and agencies involved in short–term policy would use different seasonally adjusted series. Perhaps the most sensible procedure would be, in the case of switching solutions, to publish the most precise historical estimator, and use the most precise concurrent estimator for internal short–term policy making. In any event, as will be seen in the next section, the switching of solutions tends to occur when the difference between the two solutions is small, and hence the inconvenience is minor.

6 Forecasts

Since any admissible component can be expressed as the sum of the canonical component plus an orthogonal white-noise component (with variance αV_u), the forecast of the component will be that of the canonical one plus the forecast of orthogonal white noise. Since the latter will always be zero, it follows that, although different admissible decompositions will provide different historical and preliminary estimators, they will all provide the same forecasts. The standard errors of these forecasts, however, will differ: obviously, they will become larger as αV_u increases. Trivially, thus, the decomposition that minimizes the standard error of the component forecast is that with the component itself canonical. Contrary to the case of estimation errors in current or past signals, the forecasting errors of s_t^α and n_t^α are not the same. The minimum variance forecast error of s_t^α is reached at the canonical decomposition with $\alpha = 0$, while that of n_t^α at the canonical decomposition with $\alpha = 1$. There is not an admissible decomposition that simultaneously minimizes the forecasting error variance of s_t and n_t. Still, if forecasts are the estimators of interest, the selection of an admissible decomposition is not a very relevant issue, since all decompositions yield identical forecasts.

7 Examples

7.1 Trend–plus–Cycle Model

We begin with the same example used to illustrate identification in Section 2. The mode is that of equation (2.2) with $\theta(B) = (1 + .364B - .025B^2)$, and accepts a "trend–plus-cycle" decomposition, where the admissible decompositions are given by components of the type (2.3). The identity (1.8) becomes:

$$(1 + .364B - .025B^2)\, a_t = (1 + .7B)(1 + \theta_s B)\, a_{st} + (1 - B)(1 + \theta_n B)\, a_{nt} \qquad (7.1)$$

Since (7.1) is an identity among three MA(2) processes, the associated system of co-variance equations consists of 3 equations (one for the variance, and one for each of the lag–1 and lag–2 covariances). The unknowns are the 4 parameters θ_s, θ_n, V_s, and V_n, and hence (2.3) is not identified.

As seen before, an easy way to identify the component models is by adding the zero-coefficient restriction $\theta_s = 0$, which yields of course the decomposition (2.1), with $V_s = 5V_u = .621$ (model (2.2) is standardized by setting $V_a = 1$). From this initial decomposition, it is found that $g_s(\omega) = V_s/2(1-\cos\omega)$, so that for $\omega \in [0, \pi]$, $\min g_s(\omega) = g_s(\pi) = V_s/4 = .155$. Similarly, $g_n(\omega) = V_n(1.04 + .4\cos\omega)/(1.49 + 1.4\cos\omega)$, and hence $\min g_n(\omega) = g_n(0) = .062$. Since the amount of additive noise that can be exchanged between the components is the sum of these two minima, $V_u = .217$.

Starting from the decomposition (2.1), if we substract from $g_s(\omega)$ its minimum .155, the resulting spectra can be factorized to obtain the model for the canonical signal (for a simple algorithm to factorize a spectrum see Maravall and Mathis, 1994.) This model is found to be

$$\nabla s_t^0 = (1 + B)\, a_{st}^0, \qquad V_s^0 = .155. \qquad (7.2.a)$$

Since the noise removed from s_t is added to n_t, factorizing the spectrum $(g_n(\omega) + .155)$ yields the model for the component n_t^0, associated with the canonical s_t^0; namely.

$$(1 + .7B)\, n_t^0 = (1 + .443B)\, a_{nt}^0, \qquad V_n^0 = .301. \qquad (7.2.b)$$

From models (2.2) and (7.2), expressions (3.7), (3.5), and (1.9) can be used to compute the variance of the estimation error. The variance $V(e_t^0)$, the central weight of the WK filter for $\hat{s}_t^0$, $\nu_{s,0}^0$, and the coefficient h_0 of Lemma 5 are the variances of the processes

$$\begin{aligned}
\theta(B)\, z_t &= (1 + B)(1 - .443B)\, b_t, && V_b = V_s^0 V_n^0 = .047,\\
\theta(B)\, z_t &= (1 + B)(1 + .7B)\, b_t, && V_b = V_s^0 = .155,\\
\theta(B)\, z_t &= (1 + .7B)(1 - B)\, b_t, && V_b = V_a = 1,
\end{aligned}$$

respectively, where $\theta(B) = (1 + .364B - .025B^2)$ in all cases. This yields $V(e_t^0) = .101$, $\nu_{s,0}^0 = .441$, $h_0 = 1.653$, and, using (4.4), for any admissible decomposition

$$V(e_t^\alpha) = .101 + .026\alpha - .078\alpha^2. \qquad (7.3)$$

The historical estimation error variance is seen to be minimized for $\alpha = 1$, that is, for the decomposition with canonical n_t, in which case $V(e_t^1) = .049$. The maximum value of $V(e_t^\alpha)$ is reached for $\alpha_m = .164$, an interior point of the interval [0, 1]; therefore, the other canonical, decomposition (7.2), is not, in this case, a minimax solution. That the decomposition with minimum estimation error is the one with canonical n_t can also be found through Lemma 7: The decomposition with canonical n_t is found by removing $\min g_n(\omega) = .062$ from $g_n(\omega)$, and adding it to $g_s(\omega)$ in the initial decomposition (2.1). Factorizing the resulting spectra yields the models

$$\nabla s_t^1 = (1 - .084)\, a_{st}^1, \qquad V_s^1 = .739,$$
$$(1 + .7B)\, n_t^1 = (1 - B)\, a_{nt}^1, \qquad V_n^1 = .018.$$

Proceeding as before, $\nu_{n,0}^1$ is the variance of the model

$$\theta(B)\, z_t = (1 - B)^2\, b_t, \qquad V_b = V_n^1 = .018,$$

equal to .200. Thus, since $\nu_{s,0}^0 = .441 > \nu_{n,0}^1 = .200$, Assumption 6a holds and Lemma 7 can be directly applied. For this example, thus, the MSE of the historical estimators of the two components are minimized when the cycle is made canonical. (Notice that, if x_t is a series observed every 6 months, the component n_t represents a seasonal component. The most precise estimator of the seasonally adjusted series would then be obtained by removing a canonical seasonal component.)

Concerning preliminary estimation, we focuss on the concurrent estimator and its one–period revision. In order to obtain the error variances for any admissible decomposition, from (5.15), we need the parameters $V(d_{t|t+k}^0)$, $\nu_{s,0}^0(k)$, and $h_0(k)$, for $k = 0, 1$. The first parameter $V(d_{t|t+k}^0)$ is equal to the sum of $V(e_t^0)$, already computed, plus $V(r_{t|t+k}^0)$, which can be computed through (5.7). For this we need the coefficients in F^j, $j = 0, 1, \ldots, M$ of the filter $\xi_s^0(B, F)$, given by (5.4). For this example,

$$\xi_s^0(B, F) = V_s^0\, \frac{(1 + B)\,(1 + F)\,(1 + .7F)}{(1 - B)\,(1 + .364F - .025F^2)} = V_s^0\, \eta(B, F). \tag{7.4}$$

In order to express the filter $\eta(B, F)$ as the sum of a filter in B and a filter in F, we first write the numerator and denominator of $\eta(B,F)$ as $(1 + B)\,(.7 + 1.7B + B^2)\, F^2$ and $(1 - B)\,(-.025 + .364B + B^2)\, F^2$, respectively, and then obtain the partial fractions decomposition:

$$\eta(B, F) = \frac{c_0}{1 - B} + \frac{c_1 + c_2 B + c_3 B^2}{-.025 + .364B + B^2}. \tag{7.5}$$

The coefficients c_0, c_1, c_2, and c_3 are determined by removing denominators in (7.5), and equating coefficients of B^0, B, B^2, and B^3 in the left– and right–hand–side of the resulting identity. This yields a linear system of equations with solution $c_0 = 5.078$, $c_1 = .827$, $c_2 = 1.378$, and $c_3 = -1$. The filter $\eta(B, F)$ can then be expressed as $\eta(B, F) = \eta^-(B) + \eta^+(F)$, where $\eta^-(B) = 5.078\,(1 - B)^{-1}$, and $\eta^+(F) = (-1 + 1.378F + .827F^2)\,(1 + .364F - .025F^2)^{-1}$. Multiplying by V_s^0, it is found that

$$\begin{aligned}
\xi_{sj}^0 &= .788, && j < 0, && \xi_{s0}^0 = .633, \\
\xi_{s1}^0 &= .270, && \xi_{s2}^0 = .026, && \xi_{s3}^0 = -.003, \\
\xi_{s4}^0 &= .002, && \xi_{s5}^0 = -.001, && \xi_{sj}^0 \simeq 0, && j > 5.
\end{aligned}$$

Table 1: TREND–PLUS–CYCLE EXAMPLE: Estimation Error Variance

	Concurrent Estimator	One-period Revision	Final Estimator
canonical seasonal component ($\alpha = 0$)	.175	.103	.101
canonical seasonally adjusted series ($\alpha = 1$)	.070	.055	.049

Expression (5.7) yields $V(r^0_{t|t}) = .074$, and hence $V(d^0_{t|t}) = V(e^0_t) + V(r^0_{t|t}) = .175$. For the one–period revision of the concurrent estimator, since $V(d^0_t) = V(d^0_{t|t+1}) + (\xi^0_{s,1})^2$, it follows that $V(d^0_{t|t+1}) = .103$. The coefficients $h_0(k)$ are found through (5.13), with $\pi(B) = (1 + .713B)\nabla/\theta(B)$. In particular $\pi_0 = 1$, $\pi_1 = -.664$, and hence $h_0(0) = 1$, $h(1) = 1.441$. Finally, from (5.12), $\nu^0_{s,0}(0) = .633$, and $\nu^0_{s,0}(1) = .453$. Replacing the coefficients of α in (5.15) with their computed values, the estimation error of the concurrent estimator and of its one–period revision, for any admissible decomposition, are equal to

$$V(d^\alpha_{t|t}) = .175 - .057\alpha - .047\alpha^2, \tag{7.6}$$
$$V(d^\alpha_{t|t+1}) = .103 + .020\alpha - .068\alpha^2. \tag{7.7}$$

Expression (5.14) provides also the variance of the revision error; for the concurrent estimator it is found equal to

$$V(r^\alpha_{t|t}) = .074 - .083\alpha + .031\alpha^2. \tag{7.8}$$

The four variances (7.3), (7.6), (7.7), and (7.8) are represented in Figure 3. For this example, consideration of different estimators does not produce any switching of solutions, and the specification with canonical $n_t (\alpha = 1)$ always minimizes the estimation error variance. (It is straightforward to find that $\xi^1_{n0} = .150 < \xi^0_{s,0} = .633$, and hence the conditions of Assumptions 6a and 6b are both met.) The variances of the concurrent, one–period revision, and final estimation errors are given in Table 1. The use of a canonical n_t component instead of a canonical s_t cuts in less than half the variance of the error, a nonnegligible gain in precision.

7.2 Quarterly ARIMA Model

We consider the model

$$(1 - B^4)\, x_t = (1 - .5B)\, a_t, \tag{7.9}$$

which is the same example used for illustration by Kohn and Ansley (1986) and Gómez and Maravall (1994). The AR part of (7.9) can be rewritten $(1 - B^4) = \nabla U$, where $U = 1+B+B^2+B^3$, and hence the model can be decomposed into a seasonal component, s_t, and a seasonally adjusted series, n_t, having models of the type

$$\begin{aligned} U s_t & \quad \theta_s(B)\, a_{st} \\ \nabla n_t & \quad \theta_n(B)\, a_{nt}, \end{aligned}$$

where, under Assumption 4b, $\theta_s(B)$ and $\theta_n(B)$ are, in general, of order 3 and 1, respectively. The identity (1.8) is now given by

$$(1 - .5B)\, a_t = (1 - B)\, \theta_s(B)\, a_{st} + U\, \theta_n(B)\, a_{nt} \tag{7.10}$$

and there will be, in general, 5 covariance equations associated with this identity. Since there are 6 unknowns $(\theta_{s1}, \theta_{s2}, \theta_{s3}, \theta_n, V_s \text{ and } V_n)$, the model is not identified. Proceeding as before, we start with an initial decomposition identified with the use of zero–coefficient restrictions. Restricting to 2 the order of $\theta_s(B)$ and to 0 that of $\theta_n(B)$, the system of covariance equations has now 4 equations and 4 unknowns $(\theta_{s1}, \theta_{s2}, V_s, V_n)$. The system, however, is highly nonlinear and a more efficient way to proceed is the following.

Setting $\theta_s(B) = (1 + \theta_{s1}B + \theta_{s2}B^2)$ and $\theta_n(B) = 1$, the Fourier transform of the identity between the ACGF of the left– and right–hand–side of (7.10) yields

$$\begin{aligned} 1.25 - \cos\omega \; &= \; (g_0 + g_1 \cos\omega + g_2 \cos 2\omega)\,(2 - 2\cos\omega) + (4 + 6\cos\omega + \\ &\quad + 4\cos 2\omega + 2\cos 3\omega)\, V_n, \end{aligned} \tag{7.11}$$

where $g_o = (1 + \theta_{s1}^2 + \theta_{s2}^2)\, V_s$, $g_1 = \theta_{s1}(1 + \theta_{s2})\, V_s$, and $g_2 = \theta_{s2}\, V_s$. Using the identity $2\cos(j\omega)\, \cos\omega = \cos(j-1)\,\omega + \cos(j+1)\,\omega$, operating in (7.11), and equating coefficients in $\cos(j\omega)$, $j = 0, 1, 2, 3$, a linear system of equations is obtained, with solution $g_0 = .656$, $g_1 = .125$, $g_2 = .031$, and $V_n = .016$. Therefore, the initial decomposition is given by

$$g_s(\omega) = \frac{.656 + .125\cos\omega + .031\cos 2\omega}{4 + 6\cos\omega + 4\cos 2\omega + 2\cos 3\omega}, \tag{7.12.a}$$

$$g_n(\omega) = .016/(2 - 2\cos\omega). \tag{7.12.b}$$

From these spectra it is found that, for $\omega \in [0, 1]$, $\min g_s(\omega) = g_s(0) = .051$, and $\min g_n(\omega) = g_n(\pi) = .004$. Therefore, $V_u = g_s(0) + g_n(\pi) = .055$.

To obtain the canonical decomposition for $\alpha = 0$ (i.e., the decomposition with canonical seasonal), one simply needs to substract $g_s(0)$ from (7.12.a). Factorizing the spectrum obtained, the model for the canonical s_t component is found to be given by

$$U\, s_t^0 = (1 - .501B - .342B^2 - .156B^3)\, a_{st}^0, \qquad V_s^0 = .325. \tag{7.13.a}$$

Adding, in turn, $g_s(0)$ to (5.6.b) and factorizing the resulting spectrum yields the model for n_t^0:

$$\nabla n_t^0 = (1 - .578B)\, a_{nt}^0, \qquad V_n^0 = .088. \tag{7.13.b}$$

We can now compute $V(e_t^0)$, $\nu_{s,0}^0$, and h_0 of Lemma 5 as the variances of the models

$$(1-.5B)\,z_t = (1-501B-.342B^2-.156B^3)\,(1-.578B)\,b_t, V_b = V_s^0\,V_n^0 = .029,$$
$$(1-.5B)\,z_t = (1-501B-.342B^2-.156B^3)\,(1-B)\,b_t, \quad V_b = V_s^0 = .325,$$
$$(1-.5B)\,z_t = (1-B^4)\,b_t, \quad V_b = V_a = 1,$$

which yields $V(e_t^0) = .042$, $\nu_{s0}^0 = .701$, and $h_0 = 2.5$. Since $2\nu_{s0}^0 + V_u h_0 = 1.54 > 1$, according to Lemma 6 the decomposition with minimum estimation error variance is that with a canonical n_t component ($\alpha = 1$). This is easily confirmed by the expression for $V(e_t^\alpha)$, from (4.4) equal to

$$V(e_t^\alpha) = .042 - .022\alpha - .008\alpha^2, \tag{7.14}$$

in the interval $\alpha \in [0,1]$. The minimum is reached for $V(e_t^1) = .013$. Notice that, in this case, the maximum of $V(e_t^\alpha)$ is reached at $\alpha_m < 0$, and hence $(s_t^0,\, n_t^0)$ represents the decomposition with the largest error variance in the component estimator; i.e., the minimax solution.

The model for the canonical n_t component is found by removing from (7.12.b) the constant $\min g_n(\omega) = g_n(\pi) = .004$ and factorizing the resulting spectrum; the model is found to be $\nabla n_t^1 = (1+B)\,a_{nt}^1$, $V_n^1 = .004$. According to (3.5), ν_{n0}^1 is equal to the variance of the model $(1-.5B)\,z_t = (1+B)\,U\,b_t$, $V_b = .004$, so that $\nu_{n0}^1 = .162$. Since $\nu_{s0}^0 > \nu_{n0}^1$, Assumption 6a is satisfied and Lemma 7 confirms that the decomposition with n_t canonical provides the most precise component estimators. Notice that, while in the first example, these estimators are obtained with a canonical seasonal (or cyclical) component, in the second example they are obtained with a canonical trend. (It can be seen that the result still holds if $\theta = .5$ in (7.9) is replaced by any invertible value of θ.)

In order to obtain the variances of the preliminary estimation error, $V(d_{t|t+k}^\alpha)$, and of the revision error, $V(r_{t|t+k}^\alpha)$, we also need the parameters $V(r_{t|t+k}^0)$, $V(d_{t|t+k}^0)$, $\nu_{s,0}^0(k)$, and $h_0(k)$ of Lemmas 8 and 10. As in the previous example, we consider the concurrent estimator and its one-period revision. The variance $V(d_{t|t+k}^0)$ for $k = 0, 1$ are obtained from the sum $V(e_t^0) + V(r_{t|t+k}^0)$, where the first term has already been computed, and the second term is found using (5.7), once the coefficients in $F^j (j > 0)$ of $\xi_s^0(B,F)$ have been obtained. These ξ-coefficients can be obtained as follows. As in the first example, write $\xi_s^0(B,F) = V_s^0\,\eta(B,F)$, with

$$\eta(B,F) = \left[\frac{\theta_s^0(B)}{U}\right]\left[\frac{\theta_s^0(F)(1-F)}{\theta(F)}\right],$$

where $\theta_s^0(B)$ and $\theta(B)$ denote the MA polynomials in (7.13.a) and (7.9), respectively. Then $\eta(B,F)$ can be rewritten as

$$\eta(B,F) = \frac{F^4}{F}\left[\frac{\theta_s(B)\,\bar\theta_s(B)\,(1-B)}{U\,\bar\theta(B)}\right], \tag{7.15}$$

where $\bar\theta(B)$ denotes $\theta(B^{-1})$ as a function of B (i.e. if $\theta(B) = 1-.5B$, $\bar\theta(B) = -.5+B$). Using a partial fraction expansion, the expression in brackets in (7.15) can be decomposed as in

$$\frac{c(B)}{U} + \frac{d(B)}{\theta(B)},$$

where $c(B)$ and $d(B)$ are of degree 2 and 4, respectively. Therefore,

$$\eta(B,F) = F^3 \frac{c(B)}{U} + \frac{\bar{d}(F)}{\theta(F)} = F^3 c_1(B) + d_1(F).$$

Shifting forward by 3 periods the coefficients of $c_1(B)$, adding them to the coefficients of $d_1(F)$, and multiplying by V_s^0 (given by (7.13.a)), the coefficients of $\xi_s^0(B,F)$ are obtained. For our purposes, only the weights in F^j, $j \geq 0$, are of interest, namely $\xi_{s0}^0 = .824$, $\xi_{s1}^0 = -.135$, $\xi_{s2}^0 = -.099$, $\xi_{s3}^0 = -.065$, $\xi_{s4}^0 = .018$, $\xi_{s5}^0 = .009$, $\xi_{s6}^0 = .005$, $\xi_{s7}^0 = .002$, $\xi_{s8}^0 = .001$, $\xi_{si}^0 \simeq 0$ $(i > 8)$. Using expression (5.7), $V(r_{t|t}^0) = .033$, and hence $V(d_{t|t}^0) = V(e_t^0) + V(r_{t+t}^0) = .075$. Further, $V(d_{t|t+1}^0) = V(d_{t|t}^0) - (\xi_{s,1}^0)^2 = .060$. Finally, the weights π_0 and π_1 are obtained from $\pi(B) = (1-B^4)(1-.5B)^{-1}$, which yields $\pi_0 = 1$ and $\pi_1 = .5$. From (5.12) and (5.13), the parameters $\nu_{s,0}^0(k)$ and $h_0(k)$ can now be computed, and Lemmas 8 and 10 yield

$$\begin{aligned}
V(r_{t|t}^\alpha) &= .033 - .014\alpha + .005\alpha^2 \\
V(d_{t|t}^\alpha) &= .075 - .036\alpha - .003\alpha^2 \\
V(d_{t|t+1}^\alpha) &= .060 - .028\alpha - .004\alpha^2.
\end{aligned}$$

Figure 4 plots these variances in the admissible range $\alpha \in [0,1]$, together with (7.14). It is seen how the canonical decomposition with n_t canonical ($\alpha = 1$) minimizes all estimation errors, while the decomposition with canonical seasonal component maximizes them. (Again, this result is valid for any invertible value of θ in (7.9).) Table 2 presents the variances of the errors in the concurrent, 1–period revision, and final estimators of the components. As in the previous example, there is a large gain in the precision of the component estimators when moving from the canonical decomposition with $\alpha = 0$ to the one with $\alpha = 1$. Since the two canonical decompositions represent the maximum and minimum values of the estimation error variance, they are bounds for the estimation error variance associated with any other admissible decomposition. Finally, compared to the first example, the revision between the concurrent and final estimator now lasts longer: the first–period revision accounts for roughly 40% of the total revision.

7.3 The "Airline Model"

We consider a class of models, appropriate for monthly or quarterly series, that display trend and seasonality. The model is given by the multiplicative ARIMA expression

$$\nabla \nabla_\tau x_t = (1 + \theta_1 B)(1 + \theta_\tau B^\tau)\, a_t, \tag{7.16}$$

where τ is the number of observations per year and, as before, $V_a = 1$. Following the work of Box and Jenkins (1970), model (7.16) is often referred to as the "Airline Model".

Table 2: QUARTERLY ARIMA EXAMPLE: Estimation Error Variance

	Concurrent Estimator	One-period Revision	Final Estimator
canonical seasonal component ($\alpha = 0$)	.075	.060	.042
canonical seasonally adjusted series ($\alpha = 1$)	.037	.028	.013

On the one hand, it is often encountered in practice; on the other hand, it provides a convenient reference example, since the parameters θ_1 and θ_τ are directly related to the stability of the trend and of the seasonal component. In particular, a value of the parameter θ_1 (or θ_τ) close to -1 indicates the presence of a stable trend (or seasonal) component. For $-1 < \theta_1 < 1$ and $-1 < \theta_\tau < \theta^*$, where θ^* is a small positive value (see Figure 6), the model accepts a decomposition of the type (A.5); see Hillmer and Tiao (1982). If the two components decomposition is considered, as in (1.1), with s_t denoting the seasonal component and n_t the seasonally adjusted series, then, for an admissible decomposition, the components follow models of the type

$$U s_t^\alpha = \theta_s^\alpha(B)\, a_{st}^\alpha; \qquad \nabla^2 n_t^\alpha = \theta_n^\alpha(B)\, a_{nt}^\alpha,$$

where $\theta_s^\alpha(B)$ and $\theta_n^\alpha(B)$ are, in general, polynomials in B of order $\tau - 1$ and 2, respectively.

We have seen earlier that the component estimators with minimum MSE are always obtained with one of the two canonical specifications. Tables 3 and 4 present the final and concurrent estimation error variance associated with the two canonical decompositions, for $\tau = 12$, and for different values of θ_1 and θ_{12} within the admissible region. For both types of errors, the variance is large for models whose spectra are dominated by a very stochastic trend (values of θ_1 close to 1). On the other hand, the estimation error variance is small when the model contains relatively stable components.

An interesting result from Tables 3 and 4 is that, when the error variance is large, the difference between the two canonical decompositions is relatively small; in that case, which canonical decomposition (and more generally, which admissible decomposition) is chosen has little effect on the precision of the estimator. On the contrary, when the error variance is small, the difference between the two decompositions becomes more pronounced.

Comparing Tables 3 and 4, it is further seen that the variance of the final estimation error accounts for (roughly) between 1/3 and 1/2 of the variance of the concurrent estimation error; the revision error is, thus, typically larger than the final estimation error.

Table 3: AIRLINE MODEL: Variance of Error in Final Estimator

Theta(1)	Model Spec.	Theta(12) = 0	Theta(12) = -.25	Theta(12) = -.5	Theta(12) = -.75
.75	Canonical s_t	.410	.504	.436	.259
	Canonical n_t	.407	.504	.439	.267
.50	Canonical s_t	.308	.377	.327	.195
	Canonical n_t	.300	.376	.337	.220
.25	Canonical s_t	.226	.274	.239	.144
	Canonical n_t	.210	.271	.255	.190
0	Canonical s_t	.164	.197	.173	.106
	Canonical n_t	.138	.186	.191	.168
-.25	Canonical s_t	.121	.143	.129	.081
	Canonical n_t	.082	.119	.139	.146
-.50	Canonical s_t	.096	.113	.106	.070
	Canonical n_t	.042	.070	.095	.118
-.75	Canonical s_t	.077	.118	.116	.076
	Canonical n_t	.019	.036	.054	.074

s_t: seasonal component
n_t: nonseasonal component

Table 4: AIRLINE MODEL: Variance of Error in Concurrent Estimator

Theta(1)	Model Spec.	Theta(12) = 0	Theta(12) = -.25	Theta(12) = -.5	Theta(12) = -.75
.75	Canonical s_t	1.257	1.151	.905	.521
	Canonical n_t	1.261	1.157	.913	.532
.50	Canonical s_t	.956	.873	.685	.393
	Canonical n_t	.964	.888	.710	.433
.25	Canonical s_t	.699	.641	.505	.292
	Canonical n_t	.710	.665	.551	.369
0	Canonical s_t	.491	.458	.367	.215
	Canonical n_t	.498	.483	.426	.327
-.25	Canonical s_t	.333	.323	.269	.164
	Canonical n_t	.326	.336	.324	.292
-.50	Canonical s_t	.228	.239	.214	.139
	Canonical n_t	.193	.217	.234	.244
-.75	Canonical s_t	.149	.205	.207	.143
	Canonical n_t	.097	.120	.141	.161

s_t: seasonal component
n_t: nonseasonal component

Table 5: AIRLINE MODEL: Estimation Error Variance

	Concurrent Estimator	12–period Revision	Final Estimator
canonical seasonal component ($\alpha = 0$)	.263	.153	.125
canonical seasonally adjusted series ($\alpha = 1$)	.293	.124	.116

Using, as an example, $\theta_1 = -.34$ and $\theta_{12} = -.42$, the parameters of expressions (4.4), (5.14), and (5.15) for the decomposition with a canonical seasonal component, can be derived from the overall ARIMA model in a manner similar to that illustrated in the two previous examples. For an admissible decomposition, the variances of the errors can be expressed as

$$
\begin{aligned}
V(r_t^\alpha) &= .138 - .018\alpha + .057\alpha^2 \\
V(d_t^\alpha) &= .263 + .081\alpha - .051\alpha^2 \\
V(d_{t|t+12}^\alpha) &= .153 + .065\alpha - .094\alpha^2 \\
V(e_t^\alpha) &= .125 + .099\alpha - .108\alpha^2,
\end{aligned}
$$

and they are represented in Figure 5. This example illustrates a case of "switching solutions": while the final estimation error is minimized with the decomposition with canonical seasonally adjusted series, the concurrent estimation error is minimized with the decomposition with a canonical seasonal component. Still, as seen in Table 5, the difference between the errors associated with the two canonical decompositions is relatively small, in particular for the final estimation error case.

For the monthly and quarterly Airline Model, Figure 6 displays the lines that separate the regions of the admissible parameter space where a canonical seasonal minimizes the final and concurrent estimation error, from that where the minimum is achieved with a canonical seasonally adjusted series. The region where a canonical seasonal component provides the most precise estimators is larger for the concurrent estimation error, and the area between the two lines represents the region of switching solutions. What is clearly seen in Figure 6 is that stable trends imply the use of a canonical seasonally adjusted series (i.e., of a canonical trend), while stable seasonals imply the use of a canonical seasonal component. This was to be expected from Assumption 6b and Lemma 10, since more stable components will have smaller central weights in the corresponding WK filter.

Appendix

Proof of Lemma 1: Combining (3.4), (1.6), and (1.7), it is possible to express the estimator $\hat{s}_t$ in terms of the innovations a_t of the model for the observed series. After simplification, it is found that

$$\phi_s(B)\ \hat{s}_t = \theta_s(B)\ \alpha_s(F)\ a_t, \tag{A.1}$$

where $\alpha_s(F)$ is the (convergent) forward filter

$$\alpha_s(F) = V_s\ \frac{\theta_s(F)\ \phi_n(F)}{\theta(F)}. \tag{A.2}$$

An equivalent expression is found for $\hat{n}_t$ by simply interchanging the subindices s and n. Combining the two expressions and cancelling common factors, it is obtained that

$$C(B,F) = (V_s\ V_n)\ \frac{\theta_s(B)\ \theta_n(B)\ \theta_s(F)\ \theta_n(F)}{\theta(B)\ \theta(F)}, \tag{A.3}$$

which is the ACGF of model (3.6). ■

Proof of Lemma 5: From Lemma 4, ACGF (e_t^α) = CCGF $(s_t^\alpha,\ n_t^\alpha)$. Since the latter can be expressed as $(A_s^\alpha(B,F)\ A_n^\alpha(B,F))/A_x(B,F)$, considering (4.2),

$$\begin{aligned}\text{ACGF}\ (e_t^\alpha) &= \left[A_s^0(B,F) + \alpha\ V_u\right]\ \left[A_n^0(B,F) - \alpha\ V_u\right]\ [A_x(B,F)]^{-1} = A_s^0(B,F) \\ &A_n^0(B,F)/A_x(B,F) + \left[1 - 2A_s^0(B,F)/A_x(B,F)\right]\ V_u\ \alpha - [1/A_x(B,F)]\ V_u^2\ \alpha^2.\end{aligned} \tag{A.4}$$

Equating constant terms in the identity (A.4) directly yields (4.4). ■

Proof of Lemma 6: Expression (4.4) implies that $V(e_t^\alpha)$ is a concave function of α. It follows that, within the interval $0 \le \alpha \le 1$, the minimum of $V(e_t^\alpha)$ will always be at one of the two boundaries. Since $V(e_t^1) - V(e_t^0) = V_u\ (1 - 2\nu_{s,0}^0)$ - $V_u^2\ h_0$, under condition (a), $V(e_t^1) \ge V(e_t^0)$ and $\alpha = 0$ will provide the minimum; trivially, $\alpha = 1$ provides the minimum otherwise.

When $2\nu_{s,0}^0 + V_u\,h_o = 1$, then $V(e_t^0) = V(e_t^1)$, and both canonical solutions provide the same estimation MSE, and provide thus two minima for $V(e_t^\alpha)$, within the admissible range for α. ■

Proof of Lemma 7: The series x_t can always be decomposed as in

$$x_t = s_t^0 + n_t^1 + u_t, \tag{A.5}$$

where s_t^0 and n_t^1 are the two canonical components, and u_t is white noise with variance V_u. The WK filter for u_t is given by

$$\nu_u(B,F) = V_u\ \frac{\phi(B)\ \phi(F)}{\theta(B)\ \theta(F)} = V_u\ \pi(B)\ \pi(F), \tag{A.6}$$

equal thus to the ACGF of the inverse model (1.10), scaled by V_u. It follows that $V_u\,h_0$ is the central coefficient of the WK filter for u_t. Therefore,

$$\nu_{s,0}^1 = \nu_{s,0}^0 + V_u h_0 \tag{A.7}$$

is the central weight of the WK filter associated with the decomposition that assigns all white noise to s_t (i.e., with the decomposition with n_t canonical). From Assumption 6a, $\nu^0_{s,o} \geq \nu^1_{n,0}$ or, adding $\nu^0_{s,0} + V_u h_0$ to both sides of the inequality,

$$2\nu^0_{s,0} + V_u h_0 \geq \nu^1_{n,0} + \nu^1_{s,0}, \tag{A.8}$$

where use has been made of (A.7). From $x_t = \hat{n}^1_t + \hat{s}^1_t$, the r.h.s. of (A.8) must equal one, hence we are in case (b) of Lemma 6. (When Assumption (6a), and hence (A.8), holds as an equality, then the two canonical decompositions provide two identical minima of $V(e^\alpha_t)$.) ∎

<u>Proof of Lemma 8</u>: From (3.2), (3.3), and (4.2.a),

$$\begin{aligned} \nu^\alpha_s(B,F) &= A^\alpha_s(B,F)/A_x(B,F) = \\ &= [A^0_s(B,F) + \alpha V_u]/A_x(B,F) = \\ &= \nu^0_s(B,F) + \alpha V_u h(B,F), \end{aligned}$$

where use has been made of $h(B,F) = 1/A_x(B,F)$. For the coefficient of F^j,

$$\nu^\alpha_{s,j} = \nu^0_{s,j} + \alpha V_u h_j. \tag{A.9}$$

Substracting (5.1) from (3.2),

$$r^\alpha_{t|t+k} = \hat{s}^\alpha_t - \hat{s}^\alpha_{t|t+k} = \sum_{j=k+1}^{\infty} \nu^\alpha_{s,j}(x_{t+j} - x_{t+j|t+k}),$$

where use has been made of the fact that $x_{t+j|t+k} = x_{t+j}$ when $j \leq k$. Or, using (A.9),

$$r^\alpha_{t|t+k} = \sum_{j=k+1}^{\infty} (\nu^0_{s,j} + \alpha h_j V_u)(x_{t+j} - x_{t+j|t+k}). \tag{A.10}$$

The $(i-k)$–period–ahead forecast error $\varepsilon(i-k) = x_{t+i} - x_{t+i|t+k}$ can be expressed as

$$\varepsilon(i-k) = \sum_{j=0}^{i-k-1} \psi_j a_{t+i-j}, \qquad (\psi_0 = 1), \tag{A.11}$$

and inserting (A.11) in (A.10),

$$\begin{aligned} r^\alpha_{t|t+k} = & \sum_{i=k+1}^{\infty} \Big[(\nu^0_{s,i} + \psi_1 \nu^0_{s,i+1} + \ldots) a_{t+i} + \\ & + \alpha V_u(\pi_i + \psi_1 \pi_{i+1} + \ldots) a_{t+i}\Big], \end{aligned} \tag{A.12}$$

Define

$$\ell_i = \nu^0_{s,i} + \psi_1 \nu^0_{s,i+1} + \psi_2 \nu^0_{s,i+2} + \ldots, \tag{A.13}$$

$$m_i = \pi_i + \psi_1 \pi_{i+1} + \psi_2 \pi_{i+2} + \ldots, \tag{A.14}$$

$$\ell_t = \sum_{i=k+1}^{\infty} \ell_i a_{t+i}, \tag{A.15}$$

$$m_t = \sum_{i=k+1}^{\infty} m_i a_{t+i}, \tag{A.16}$$

Then, we can write

$$V(r^\alpha_{t|t+k}) = V(\ell_t) + 2\alpha V_u \operatorname{Cov}(\ell_t, m_t) + \alpha^2 V_u^2 V(m_t). \tag{A.17}$$

Setting $\alpha = 0$, $V(\ell_t)$ is the variance of the revision in the canonical specification of the component, that is

$$V(\ell_t) = V(r^0_{t|t+k}). \tag{A.18}$$

From (A.16), $V(m_t) = \sum_{i=k+1}^{\infty} m_i^2$, where, according to (A.14), m_i is the coefficient of F^i in the polynomial $h(B,F)\,\psi(B)$. Given that $\psi(B) = 1/\pi(B)$, from (1.10), $m_i = \pi_i$. Thus

$$V(m_t) = h_0 - h_0(k). \tag{A.19}$$

Finally, Cov $(\ell_t, m_t) = \sum_{i=k+1}^{\infty} \ell_i\, m_i = \sum_{i=k+1}^{\infty} \pi_i\, \ell_i$. From (A.13) and (5.10.b), it is seen that ℓ_i is the coefficient of F^i in $\xi^0_s(B,F)$, or Cov $(\ell_t, m_t) = \sum_{i=k+1}^{\infty} \pi_i\, \xi^0_{s,i}$. Using (5.11) and (5.12), it follows that

$$\text{Cov}\,(\ell_t, m_t) = \nu^0_{s,0} - \nu^0_{s,0}(k). \tag{A.20}$$

Plugging (A.18), (A.19), and (A.20) in (A.17),

$$V(r^\alpha_{t|t+k}) = V(r^0_{t|t+k}) + 2\left[\nu^0_{s,0} - \nu^0_{s,0}(k)\right] V_u\, \alpha + [h_0 - h_0(k)]\, V_u^2\, \alpha^2.$$

■

<u>**Proof of Lemma 9**</u>: $V(r^\alpha_{t|t+k})$ is a parabole with positive constant term. Since, by construction, the coefficient of α^2 is positive, over the range $\alpha \in [0,1]$, $V(r^\alpha_t)$ is a positive, convex, function and will, as a consequence, display a maximum at one of the boundary values $\alpha = 0$ or $\alpha = 1$. By comparing $V(r^0_t)$ and $V(r^1_t)$, conditions (a) and (b) of the Lemma are immediately obtained. ■

<u>**Proof of Lemma 11**</u>: As a function of α, since $h_0(k) > 0$, $V(d^\alpha_{t|t+k})$ is a concave parabole, and within the interval $0 \le \alpha \le 1$, it will display a minimum at one of the two boundaries. By comparing the values of (5.15) for $\alpha = 0$ and $\alpha = 1$, conditions (a) and (b) are obtained. ■

<u>**Proof of Lemma 12**</u>: Inserting (3.1) in (5.2),

$$\hat{s}_{t|t+k} = \nu_s(B,F,k)\,\psi(B)\,a_t, \tag{A.21}$$

where the superscript α has been deleted for notational simplicity. Taking conditional expectations at time $T = t+k$, expression (5.3) yields

$$\hat{s}_{t|t+k} = \xi_s(B,F,k)a_t, \tag{A.22}$$

where $\xi_s(B,F,k)$ is the filter $\xi_s(B,F)$ truncated at F^k, and use has been made of the property $E_{t+k}\, a_T = 0$ when $T > t+k$. Comparing (A.21) and (A.22), it is seen that

$$\nu_s(B,F,k) = \xi_s(B,F,k)\pi(B). \tag{A.23}$$

Equating the coefficients of B^0 at both sides of the identity (A.23), if w_0 denotes that of the l.h.s.,

$$w_0 = \sum_{i=0}^{k} \xi_{s,i}\, \pi_i. \tag{A.24}$$

For the canonical specification of s_t, (5.12) and (A.24) imply $w_0 = \nu^0_{s,0}(k)$. Part (b) is proved in an identical manner, by noticing that (5.17) and (A.6) imply

$$\hat{u}_{t|t+k} = a_t + \pi_1\, a_{t+1} + \ldots + \pi_k\, a_{t+k} = \pi(F,k)\, a_t,$$

and hence $\xi_u(B,F,k) = \pi(F,k)$. ■

<u>**Proof of Lemma 13**</u>: Using the decomposition (A.5), the lemma is proved in the same way as Lemma 7, replacing the $\nu-$ by the $\nu(k)-$coefficients, and h_0 by $h_0(k)$. Then, condition (b) of Lemma 11 is seen to be satisfied. ■

Fig. 1 : SERIES

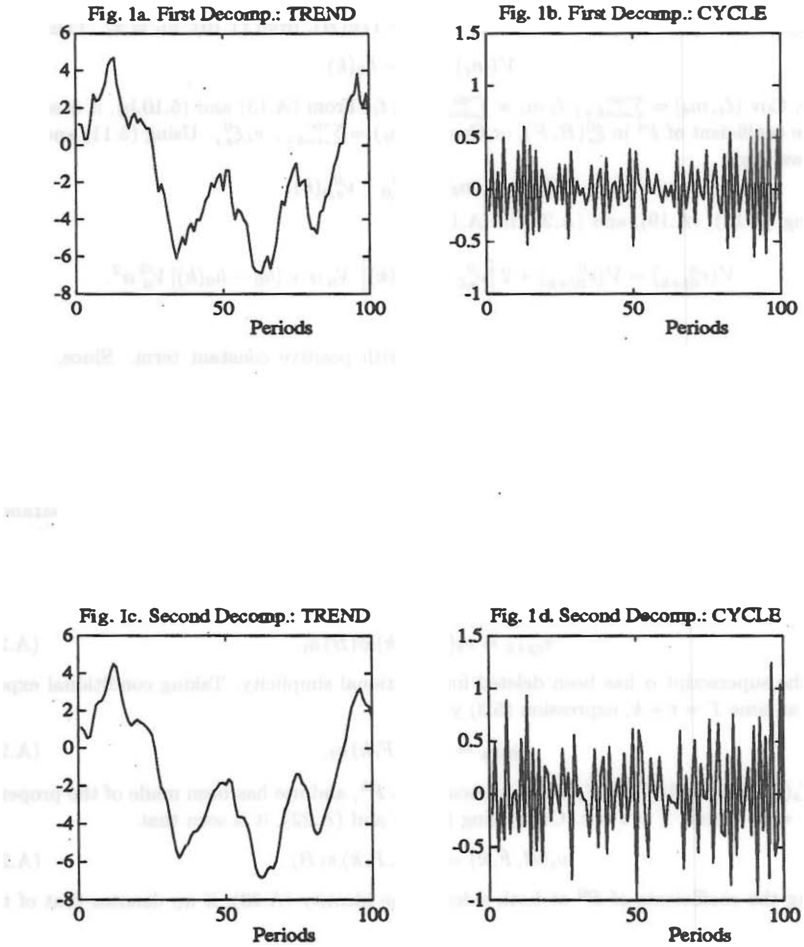


Fig. 1a. First Decomp.: TREND
6
4
2
0
-2
-4
-6
-8
0
50
100
Periods
Fig. 1b. First Decomp.: CYCLE
1.5
1
0.5
0
-0.5
-1
0
50
100
Periods
Fig. 1c. Second Decomp.: TREND
6
4
2
0
-2
-4
-6
-8
0
50
100
Periods
Fig. 1d. Second Decomp.: CYCLE
1.5
1
0.5
0
-0.5
-1
0
50
100
Periods

Fig. 2 : SPECTRAL DECOMPOSITION

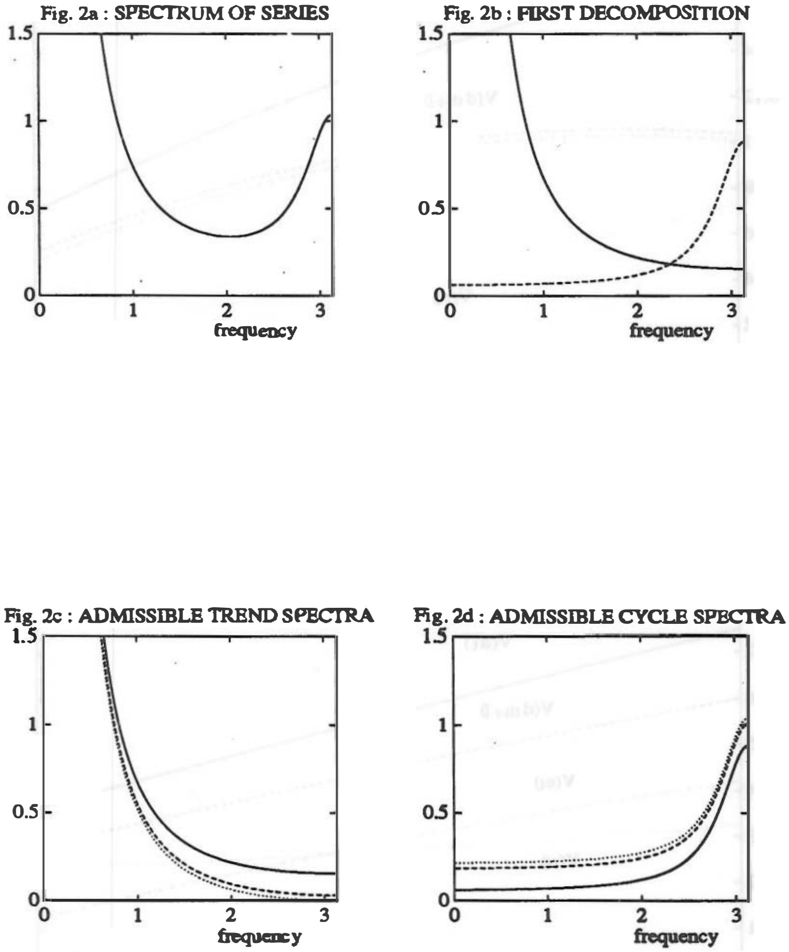


Fig. 3 : Variance of est. errors : TREND-CYCLE EXAMPLE

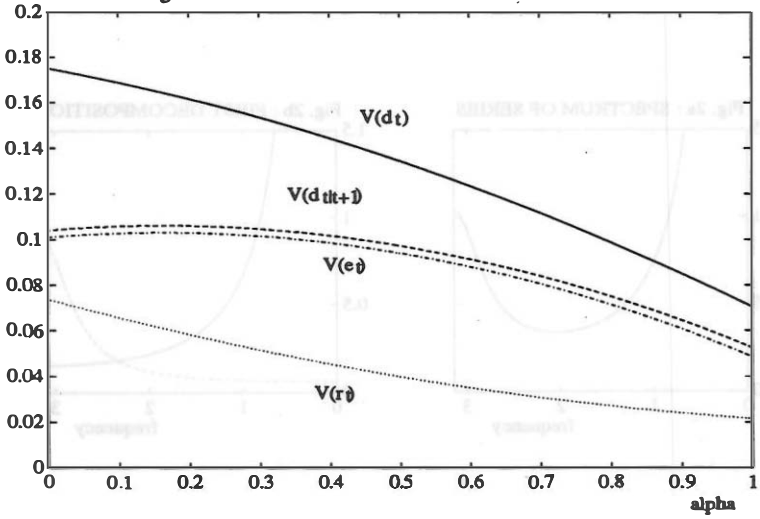


Fig. 4 : Variance of est. errors : QUARTERLY ARIMA EXAMPLE

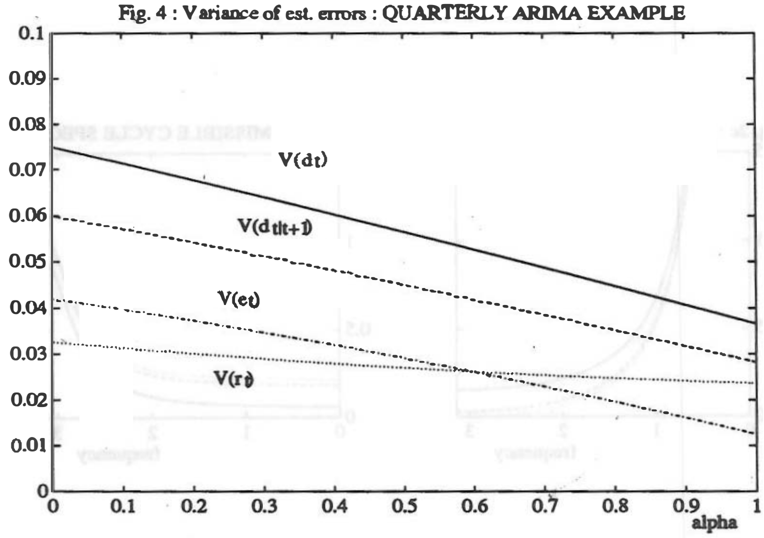


Fig. 5 : Variance of est. errors : AIRLINE MODEL EXAMPLE

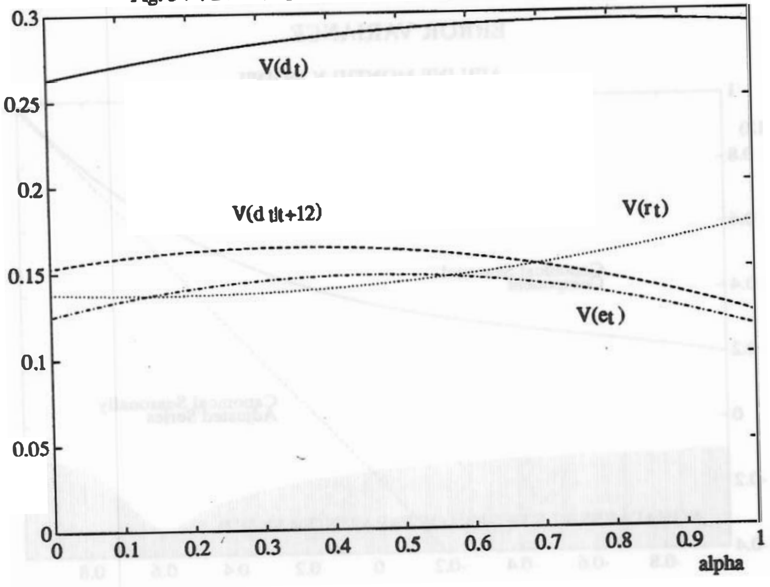
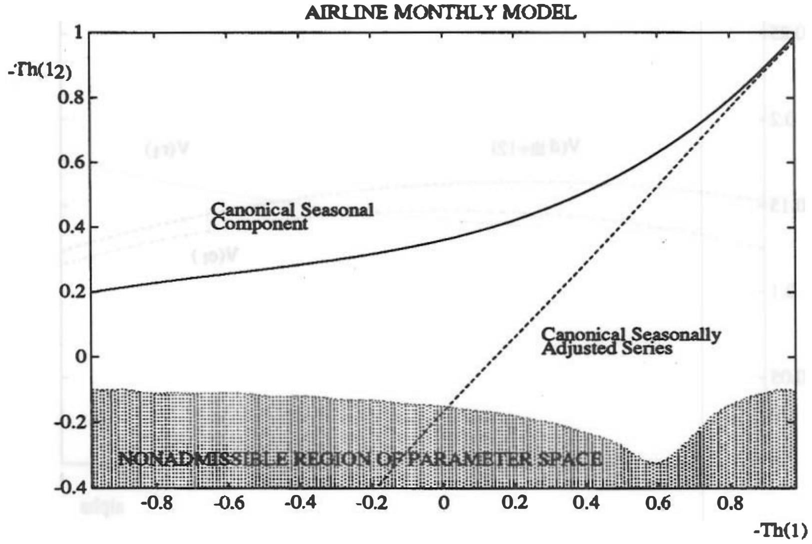
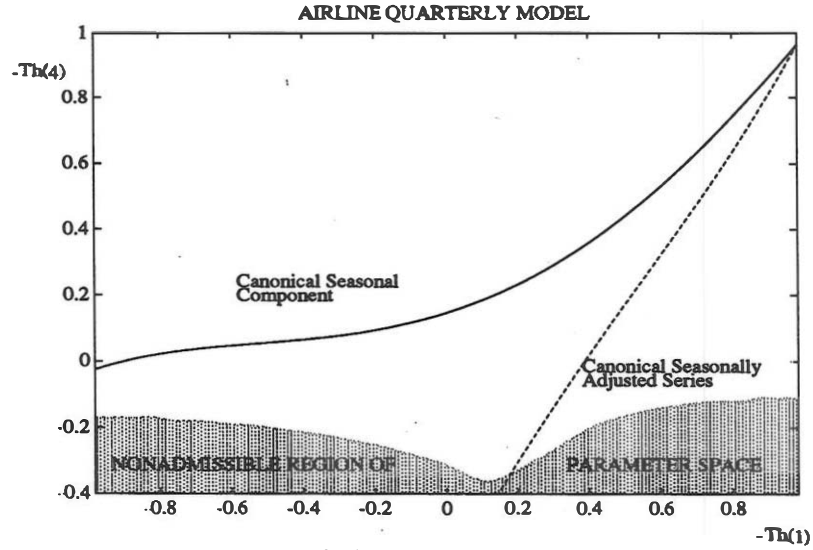


Fig. 6 : CANONICAL SOLUTION THAT MINIMIZES THE ESTIMATION ERROR VARIANCE





— Border line for final estimation error variance

- - Border line for concurrent estimation error variance

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