ESTIMATING INFLATION EXPECTATIONS USING FRENCH GOVERNMENT INFLATION-INDEXED BONDS

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ESTIMATING INFLATION EXPECTATIONS USING FRENCH GOVERNMENT INFLATION-INDEXED BONDS (*)

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Abstract

Inflation-indexed bonds are fixed-income securities whose nominal cash flows are adjusted to an inflation index. In countries where these securities exist, inflation expectations are sometimes estimated as the spread between the nominal yield on a conventional bond and the real yield on an indexed bond with a similar maturity and issued in the same currency and by the same issuer. However, this indicator, known as the break-even rate, may estimate inflation expectations with some biases. In this paper, we discuss, and quantify where possible, the size of such biases. Then, focusing on the 10-year French indexed bond, we compute an alternative indicator, called the inflation compensation measure, which corrects some of these biases and find very few differences between both indicators in our sample period. Finally, the comparison with other indicators of long-term inflation expectations shows that measures based on indexed-bond prices are more time-varying than non-financial indicators, but less variable than other financial indicators.

1. Introduction

Inflation-indexed bonds (indexed bonds hereafter) are fixed-income securities whose nominal cash flows are adjusted to inflation. While nominal payments are known ex-ante in a conventional bond, and real payments depend on inflation; the opposite occurs in the case of indexed bonds¹.

Indexed-bond markets have traditionally emerged in countries experiencing very high rates of inflation. In these cases, the issuance of indexed debt by the government has commonly been aimed at developing long-term capital markets and improving the credibility of anti-inflation policies. This has been the case, for example, in Israel, Brazil, Mexico or Argentina. More recently, indexed-bond markets have also emerged in a number of industrialised countries, with moderate inflation rates and with price stability-oriented monetary policies. These countries include the United Kingdom, Australia, Canada, Sweden, the United States, France and New Zealand, where indexed-bond markets have sought to reduce debt costs, to complete financial markets and to reinforce the credibility of non-inflationary policies.

Financing through indexed, as opposed to conventional, bonds may reduce the cost of debt through different channels. Firstly, if investors demand an inflation-risk premium on conventional bonds, the issuer of indexed bonds will save this premium in exchange for bearing the inflation risk. If the issuer is less risk-averse than the investors, the resulting cost-risk combination must be superior. Secondly, issuing indexed bonds may further reduce the expected cost of debt if, as appeared to be the case in United Kingdom in 1981, investors have higher inflation expectations than those of the issuer. Finally, as Barro (1995) noted, another advantage of issuing theses securities is the greater smoothness of their real payments in comparison with those of conventional debt.

From the investors' point of view, the main reason for demanding indexed bonds is to hedge against inflation risk. For this reason, the main holders of these financial assets are conservative long-term investors such as pension funds and insurance companies. Additionally, indexed securities allow for greater portfolio diversification and active management of inflation expectations. For example, investors with higher (lower) inflation expectations than the market will overweigh (underweigh) indexed bonds in their portfolios.

For the economic authorities, indexed bonds are also useful due to their information content. In particular, the prices of these bonds are sometimes used to derive estimates of real interest rates and market inflation expectations. These non-observable variables are relevant elements in the assessment of the macroeconomic outlook as determinants of

¹ In practice, however, some characteristics of indexed bonds imply that the real cash-flow of indexed bonds may also depend on the evolution of inflation, albeit to a lesser extent than in the case of conventional bonds.

economic agents' saving and expenditure decisions. In the case of market inflation-expectations, they are also useful for the monetary authorities as an indicator of the credibility of price stability. Against this background, one advantage of using indexed-bond prices for the estimation of inflation expectations is that they readily provide daily information. This makes them very attractive for regular monetary policy assessment and for timely evaluation of the credibility that the market accords to price stability, in comparison with alternative methods such as those based on opinion surveys or econometric analysis.

In countries where indexed bonds exist, the most widely used measure of inflation expectations derived from bond markets is the spread between the nominal yield on a conventional bond and the real yield on an indexed bond with a similar maturity, the same issuer and denominated in the same currency. However, this indicator, which is habitually referred to as the *break-even inflation rate*, may measure inflation expectations with an error due to the effect of, among other things, imperfect indexation, taxes, inflation risk and liquidity premia.

Alternative and more sophisticated methods to estimate inflation expectations using prices of indexed bonds have appeared in the literature. For example, Deacon and Derry (1994) proposed the estimation of an *Inflation Term Structure* for the British indexed-bond market. More recently, Sack (2001) proposed an alternative indicator, which he called *Inflation Compensation Measure*, for the US indexed-bond market.

Against this background, in this paper we discuss, and quantify where possible, the size of the biases included in the break-even rate for the estimation of inflation expectations. Then, focusing on the French indexed-bond market, we evaluate the applicability of alternative methodologies for estimating inflation expectations. Finally, we compare alternative indicators of long-term inflation expectations. We focus on French indexed bonds since they are the only such instruments issued by governments in the euro area, and their information content is regularly exploited by the European Central Bank in its regular assessment of inflation expectations for the euro area.

The rest of the paper is structured as follows. Section 2 describes the functioning of the French inflation-indexed bond market. Section 3 discusses the biases of the break-even rate for the estimation of inflation expectations. Section 4 evaluates alternative measures of inflation expectations using the French 10-year indexed-bond prices. Section 5 compares different indicators of inflation expectations for the French economy and, finally, the last section draws the main conclusions.

2. The structure of inflation-indexed bonds

2.1. General issues

Indexed bonds are fixed-income securities whose nominal cash flows are adjusted by the performance of an inflation index. While nominal cash flows of conventional bonds are known ex-ante, they are uncertain for indexed bonds. By contrast, real cash flows are only certain for indexed bonds, provided they are perfectly indexed to inflation and inflation compensation gains are not taxed.

The real yield (r) on an indexed bond is computed in the same way as the nominal yield on a conventional bond, but using real flows instead of nominal ones (nominal payments are not known ex-ante). More precisely, the real yield is obtained implicitly from the following expression:

$$P_{t} = \sum_{i=1}^{N} \frac{c}{(1+r)^{i}} + \frac{100}{(1+r)^{N}}$$

where P_t is the price of the indexed bond, c is the real coupon rate, and N is the term to maturity (years).

The most widespread class of indexed bonds are *capital-indexed bonds*² employed, for instance, in Australia, Canada, New Zealand, Sweden, the United Kingdom, the United States and France. These bonds pay a periodic coupon, which is the result of applying a fixed real coupon to the inflation-adjusted principal. And, at maturity, the inflation-adjusted principal is paid.

The reference price index employed for the adjustment of the bond's principal usually coincides with a consumer price index. This type of index is particularly appropriate since it provides inflation coverage for a wide group of investors and it is published regularly, with a short lag and minor revisions. Additionally, a highly reputable institution independent of the issuer is, in most cases, the provider of the price index.

The adjustment of the principal of the bond requires a daily reference of the price index, which obviously is not available contemporaneously. For this reason, the daily reference index is computed using past values of the price index. The time lag between the updating date and the date of the price index employed is known as the *inflation lag* of the indexed bond. Many countries like Canada, the United States, Sweden or France apply an inflation

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 $^{^2}$ See Deacon and Derry (1997) for a description of other alternatives, such as interest-indexed bonds, current pay bonds, indexed-annuity bonds, indexed zero-coupon bonds.

lag of 3 months. But there are cases, such as the British Indexed-Linked Gilts, for which the lag is longer (8 months in the British case) due to the rules applied to accrued interest.

Finally, the taxation on indexed bonds may be levied on nominal or real returns. In the first case, the inflation compensation gain is taxed, and hence the protection against inflation is reduced. If, alternatively, only real yields are taxed, as in the British case, then after-tax real cash flows are certain, increasing the attractiveness of these bonds for inflation risk-averse investors subject to taxes.

2.2. The inflation-indexed French government bonds: OATis

The French Treasury is currently the only sovereign issuer of indexed bonds in the euro area. It started issuing a 10-year bond (OATi 3% 07/2009) in September 1998, and placed a second line of 30-year bonds (OATi 3.40% 07/2029) in September 1999. Since then, these *capital-indexed bonds* have been tapped regularly and in March 2001 their outstanding amount reached €9.5 and €3.6 billion, respectively. These amounts are quite small compared with that of the 10-year conventional bond used for the calculation of the breakeven rate (OAT 4% 04/2009), which was €20.2 billion on the same date. Nevertheless, the differences have been narrowing over recent months due to the buy-backs of the conventional bond carried out by the French Treasury and new issues of the indexed bonds (see Figure 1).

The daily price reference used to adjust the nominal cash flows is based on the non-seasonally adjusted French consumer price index (CPI), excluding tobacco, which is published by the French Institute for National Statistics every month. The annual payment of the coupon is computed as follows:

 $Coupon_t = Real\ Coupon\ Rate\ x\ Nominal\ x\ Index\ Ratio_t$

$$Index \ Ratio_t = \frac{Daily \ price \ reference_t}{Base \ index} .$$

The nominal value of the bond is ≤ 1 and the coupon is 3% for the 10-year issue and 4% for the 30-year one. The *Base index* is the value of the price index on the accrual date (25 July 1998, in the case of the 10-year issue), and the *Daily price reference* on day t of month m is calculated by linear interpolation of two monthly price indices:

$$Daily\ price\ reference_{t} = CPI_{m-3} + \frac{ndb}{ND_{m}}(CPI_{m-2} - CPI_{m-3}) \ ,$$

where CPI_i is the consumer price index of month i, nbd is the number of days since the start of the month up to day d, and ND_i is the number of days in month i. Note that CPI_{m-2} is always published before the end of month m-1 and is therefore available to calculate the $Daily\ price\ reference$ for each day of month m.

The calculation of accrued interest is similar to that of conventional bonds:

Accrued interest_t=Real Coupon Rate x Index Ratio_t x Nominal x
$$\frac{days \ accrued}{days \ in \ the \ coupon \ period}$$

At maturity, bond holders receive the inflation-adjusted principal, which is computed as the product of the nominal value and the index ratio provided this ratio is higher than one, otherwise (i.e. in the event of deflation) the par value is guaranteed.

The tax treatment of French indexed bonds is very similar to that of conventional French-government bonds. French-resident retail investors pay taxes on both coupon interest income and capital gains, including those derived from the inflation compensation, which are paid when they are realised. These investors are also entitled to opt for a final and definitive one-off flat-rate withholding tax of 25%. If they choose not to do so, income from these bonds must be declared together with their taxable income. Institutional investors are also taxed on interests received but the redemption premium generated by indexation of the principal may be taxed partly before divestment or maturity, even though the premium is not yet received. And finally, non-resident investors are exempt from the one-off flat-rate withholding tax.

In the secondary market, primary dealers have market-maker responsibilities in respect of this new security to ensure the transparency and liquidity of the market. In the future, OATis could possibly be eligible for both stripping and repo operations.

3. Measuring inflation expectations using the break-even inflation rate

3.1. The break-even inflation rate

The break-even inflation rate (break-even rate, hereafter) is generally defined as the spread between the nominal yield on a long-term conventional bond and the real yield on an indexed bond issued with a similar maturity, the same issuer and the same currency of denomination. In the French case, the 10-year break-even rate is currently computed using the conventional and indexed bonds maturing in April and July 2009, respectively. As Figure 2 shows, the yield on the 10-year French indexed bond has trended in a much more stable

way than that on the 10-year conventional bond.³ As a consequence, movements in the yield on the conventional bond are mostly transmitted to the break-even rate. During the period considered in Figure 2, this rate ranged from 0.7% to 2%, with a mean value of 1.5%. The yield on the indexed bond ranged from 2.7% to 3.9%, with a mean value of 3.4%.

The break-even rate is the rate of inflation that would approximately equalise the total return on an indexed bond with that on a conventional bond. Under some assumptions this indicator is an unbiased estimator of average inflation expectations over the remaining life of the bonds. Section 3.2 discusses these assumptions and analyses the impact of relaxing them.

3.2. Biases

To discuss the biases associated with the use of the break-even rate as a measure of inflation expectations, we first introduce an initial scenario, called scenario 1, with a number of simplifying assumptions. Later, we introduce new scenarios relaxing the initial assumptions. In each scenario we derive an analytical expression for the bias and, in some cases, we try to give estimates of this bias or simulate it for different combinations of parameters.

Scenario 1: A simple (but unrealistic) world

In this initial scenario we introduce the following six simplifying assumptions: i) the inflation compensation of the indexed bond coincides with the contemporaneous realised inflation (there is no inflation lag), ii) returns on bonds are tax-free, iii) investors are risk-neutral, iv) both bonds have the same degree of market liquidity, v) both bonds are zero-coupon securities with an identical maturity date, and vi) inflation compensation is symmetric (same treatment of inflation and deflation).

Under assumptions i-vi the real yield on the indexed bond will be equal to the real rate of interest demanded by investors for the time to maturity of the bond. Given that investors are risk-neutral, the conventional bond will only include a compensation for the expected inflation in addition to the real rate remuneration. That is to say, the following equation must hold:

$$(1+i)=(1+\mathbf{p})(1+\mathbf{r}) \tag{1}$$

³ Throughout this paper information on conventional bond prices and yields is taken from Bloomberg, whereas information on the indexed bond is taken from Reuters due to the existence of missing data at Bloomberg for this security.

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where i is the yield on the nominal bond, p is the expected inflation rate and r is the real rate of interest. This is the simple version of the Fisher identity that holds assuming risk-neutrality.⁴

After some algebra, equation (1) can be rewritten as

$$i = r + p + pr \tag{2}$$

Taking into account that in this case the real yield on the indexed bond (*r*) is equal to the real rate of interest, the break-even inflation rate will be:

$$i - r = \mathbf{p} + \mathbf{p}\mathbf{r} \tag{3}$$

In other words, in this simple world the break-even rate is an upward-biased estimator of inflation expectations provided that inflation expectations are positive. The bias, which will be referred to as the *compound bias*, is equal to pr. However, for small values of p and r the bias will be very small and, as a consequence, the break-even rate will be a good approximation of the expected inflation rate. For instance, if the real rate of interest is 4% and the expected inflation rate is 2% the bias will only be 8 basis points (b.p.).

It is worth noting that, due to the compound effect, changes in the break-even rate will also overstate changes in inflation expectations by a factor of one plus the real rate of interest (see expression (3)).

In any case, it is straightforward to see that in this simple world an unbiased estimator of the expected inflation rate can be recovered using the expression (i-r)/(1+r).

In the absence of indexed bonds, changes in long-term inflation expectations are sometimes approximated by changes in yields on long-term conventional bonds. A comparison between equations (2) and (3) shows that, under the assumptions of scenario 1, the break-even rate captures more accurately the changes in inflation expectations only if the real rate of interest is time-varying. The higher the variability of the real rate of interest, the better the relative performance of the break-even rate in comparison with the yield on the nominal bond to measure changes in inflation expectations.

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 $^{^4}$ Note that in equation (1) there is no mention of the time to maturity. Hence variables i, p and r reflect the cumulative rates between now and the maturity. Alternatively, equation (1) can be applied to variables expressed on an annual basis by simply taking the time to maturity power to both sides. In this case p should be interpreted as the annual rate of the expected cumulative inflation between now and the redemption date of the bonds. The interpretation of p as annual inflation expectations is affected by the Jensen inequality.

Scenario 2: Introducing an inflation lag

In this scenario we relax assumption i). In particular, we assume that indexed-bond holders are compensated using a lagged measure of inflation instead of contemporaneous inflation. Under this assumption, the following simple Fisher identity must hold in the indexed-bond market:

$$(1+r)(1+\mathbf{p}_1) = (1+\mathbf{p})(1+\mathbf{r}) \tag{4}$$

where p_i stands for the expected lagged inflation. It is worth noting that the yield on the indexed bond will cease to be equal to the real rate of interest provided that the lagged and contemporaneous expected inflation are different. In particular, the real yield on the indexed bond will be higher (lower) than the real rate of interest if the lagged expected inflation is lower (higher) than the contemporaneous expected inflation. This is because investors would require a positive (negative) premium to compensate them for the expected under-(over-) compensation for inflation.

In the nominal-bond market, the Fisher identity (1) will be unchanged. Solving equation (4) for r and using (2), the following expression is given for the break-even rate:

$$i - r = \mathbf{p} + \mathbf{p}\mathbf{r} + \frac{\mathbf{p}_{l} - \mathbf{p}}{1 + \mathbf{p}_{l}}(1 + \mathbf{r})$$
(5)

In this case the break-even rate includes an additional bias reflecting the relative discrepancy between the lagged and the contemporaneous expected inflation rates. The sign of the bias will depend on the term structure of inflation expectations. In particular, the sign will be positive (negative) if inflation between t-t and t is expected to be higher (lower) than inflation between t-t and t, where t indicates the current period and t is the redemption date of the bond. In practice, the absolute value of the bias will depend directly on the relative length of the lag in comparison with the time to maturity of the bond. In the French case the lag length is three months, which seems small compared to the 10-year maturity of the bond. With the aim of evaluating empirically the significance of this bias, we have computed the first factor of the last term on the right-hand side of equation (5) for the French indexed bond. Instead of inflation expectations we have used average inflation over a 10-year horizon with data from 1960 to 2000. An average absolute value of 2 b.p. and a maximum absolute value of 7 b.p. is obtained, suggesting that the bias produced by the inflation lag is very small in the French case.

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9

⁵ This result can be reached ignoring the problems caused by the Jensen inequality.

Scenario 3: Introducing taxes

In this scenario we relax assumption ii) and keep the other assumptions of scenario 1 unchanged. In particular, we introduce two different taxes: one is applied to the interest income and the other to the inflation compensation gain. Let t_1 and t_2 be, respectively, the rates of these taxes for the marginal investor. In this case, the Fisher identities for the conventional and indexed-bond markets turn out to be:

$$(1+i(1-t_1)) = (1+\mathbf{p})(1+\mathbf{r})$$
(6)

$$(1+r(1-t_1))(1+\mathbf{p})-t_2\mathbf{p} = (1+\mathbf{p})(1+\mathbf{r})$$
(7)

Note that now the left hand side of the Fisher identities reflect the after-tax expected nominal gross return. Similarly, ρ should be interpreted as the after-tax real rate of interest. Solving equation (6) for i, we get:

$$i = \frac{\mathbf{r}}{1 - t_1} + (1 + \frac{\mathbf{r} + t_1}{1 - t_1})\mathbf{p}$$
(8)

Solving (7) for *r* and using (8) we obtain the following expression for the break-even rate:

$$i - r = \mathbf{p} + \frac{\mathbf{pr}}{1 - t_1} + \frac{(t_1 - t_2 + \mathbf{p}t_1)\mathbf{p}}{(1 - t_1)(1 + \mathbf{p})}$$
(9)

The second and third terms on the right-hand side of expression (9) capture the bias produced when measuring inflation expectations using the break-even rate in the presence of taxes. The first term of them is the compound bias, which has been discussed previously. It is positive and higher than in the case of no taxes provided inflation expectations are positive. The last term captures the error caused by the existence of taxes and will be referred to as the tax bias. This bias will be positive (negative) if t_2 is lower (higher) than $t_1(1+p)$ and the inflation expectations are positive. In practice, it will normally be positive since the marginal tax rate on inflation compensation gains is not higher than that of the interest income. Therefore, the introduction of taxes tends to increase the upward bias of the break-even rate in measuring the *level* of inflation expectations even when the marginal tax rates are the same $(t_1=t_2=t)$.

Note that changes in the break even rate overestimate *changes* in inflation expectations due to tax effects provided inflation expectations are positive and the marginal tax rate on inflation compensation gains is not higher than that of the interest income (see expression (9)). However, a comparison between (8) and (9) shows that this bias will generally be

higher when using changes in the conventional bond yield. In other words, the existence of taxes improves the relative performance of the break-even rate to capture *changes* in inflation expectations in comparison with the yield on the nominal bond provided that compensation gains are taxed.

Table 1 shows the bias in using the break-even rate to estimate the level of inflation expectations for different combinations of parameters t_1 , t_2 and p. The marginal tax rate on interest income, t_1 , is set between 0% and 40%, the marginal tax rate on inflation compensation gains, t_2 , is set to be a proportion of the previous tax rate (between 0% and 100%), and inflation expectations range from 0% to 5%. Biases are computed using expression (9) assuming that the real rate of interest is 3.5%. In the absence of taxes the bias (compound bias) appears to be very small (between 0 and 18 b.p.). The introduction of taxes increases the bias up to a maximum of 363 b.p. when the marginal tax rate is 40%, inflation compensation gains are not taxed and inflation expectations are 5%. Obviously this is an extreme assumption, but it is useful for illustrating the high potential effects of taxes when estimating inflation expectations using the break-even rate. When the marginal rate on inflation compensation gains is set at 50% of that on the interest income, the maximum bias diminishes to 204 b.p. Finally, when both tax rates are equal the maximum bias is only 45 b.p.

Table 1 also shows, for the same combinations of parameters, the bias in the sensitivity of the estimated changes in inflation expectations when using both the changes of the breakeven rate and the changes in the yield on the nominal bond. When inflation compensation gains are not taxed, the sensitivity reaches a maximum value of 1.73 times for both the yield on the nominal bond and the break-even rate, i.e. an increase in inflation expectations of 100 points is reflected in an increase of 173 points in the estimated change in inflation expectations. This bias is reduced in the case of the break-even rate when the inflation compensation gains are taxed, whereas it remains unchanged in the case of the yield on the nominal bond. For instance, when both tax rates are set at the same level, the sensitivity reaches a maximum value of 1.09 times.

The relative extensiveness length of the ranges of the estimated biases in Table 1 suggests the importance of the estimation of marginal tax rates to assess accurately the biases of the break-even rate in measuring inflation expectations.

Unfortunately, marginal tax rates are not normally observed due to the existence of investors with different tax rates. Therefore, it will be difficult in practice to control for the tax bias. In the French case, given that compensation gains and interest income are equally taxed, we might think that the impact of taxes on the bias of the break-even rate would be limited. In particular, according to the results of Table 1, for a marginal tax rate of 40% (which is approximately the maximum applied to French investors) the tax bias would be

lower than 7 b.p. for expected inflation below 2%⁶. However, the marginal tax rates will not necessarily be equal even if, at individual level, both sources of income are equally taxed. Additionally, it is possible that some non-resident investors may be subject to a two-rate tax system. As a consequence, the tax bias might be much higher than that estimated previously.

Scenario 4: Introducing risk aversion

In this scenario we relax assumption iii) and keep the other assumptions of scenario 1 unchanged. In this case, investors in the conventional bond would demand a compensation for bearing inflation risk and the Fisher identity in this market would be:

$$(1+i) = (1+\mathbf{p})(1+\mathbf{r}+\mathbf{g}_n)$$
(10)

where g_i stands for the inflation risk premium. Conversely, the yield on the indexed bond does not include this compensation since it offers a real return without uncertainty and, as a consequence, the yield on this bond will continue to be the real rate of interest.

Solving (10) for i we have:

$$i = \mathbf{r} + \mathbf{g}_n + \mathbf{p}(1 + \mathbf{r} + \mathbf{g}_n) \tag{11}$$

Taking into account equation (11) and r=r, the following expression for the break-even rate is derived:

$$i - r = \boldsymbol{p} + \boldsymbol{g}_n + (\boldsymbol{r} + \boldsymbol{g}_n)\boldsymbol{p} \tag{12}$$

Equation (12) shows that the introduction of risk-averse investors tends to increase the upward bias of the break-even rate as a measure of the level of inflation expectations in comparison with scenario 1. The sensitivity of changes to inflation expectations also increases in comparison with scenario 1.

The empirical evidence on inflation risk premia suggests that they are not negligible although they are relatively stable. For instance, Alonso and Ayuso (1996) report inflation risk premia in the Spanish bond market between 20 b.p. and 40 b.p. for a 5-year horizon.

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⁶ As Table 1 shows, when the tax rates are both at 40% and the expected inflation is 2%, the bias is 14 b.p., but up to 7 b.p. correspond to the compound bias.

Scenario 5: Introducing different degrees of liquidity

In this scenario we relax assumption iv) and keep the other assumptions. In particular, we assume that the indexed bond has a lower degree of liquidity compared to the conventional bond. This is a realistic assumption since a relatively high proportion of indexed bond holders tend to keep these assets until maturity, which is reflected in relatively low trading volumes. For instance, in the French case the turnover of the 10-year indexed bond was, in 1999, around 100 times lower than that of the nominal bond used to compute the breakeven rate.

In the indexed-bond market, investors demand compensation for bearing higher liquidity risk (i.e. the risk of not finding a counterparty or having to incur higher transaction costs in the event of selling before maturity). The Fisher identity for the indexed bond is:

$$(1+r)(1+\mathbf{p}) = (1+\mathbf{p})(1+\mathbf{r}+\mathbf{g}_i)$$
(13)

where g stands for the liquidity premium. Solving (13) for r and using equation (2) we obtain the following expression for the break-even rate:

$$i - r = \mathbf{p} - \mathbf{g}_i + r\mathbf{p} \tag{14}$$

Equation (14) shows that the existence of a liquidity premium in the indexed-bond market tends to understate the level of expected inflation derived from the break-even rate. By contrast, this premium does not affect the sensitivity of the break-even rate to changes in inflation expectations.

Under the assumptions of this scenario, the relative performance in changes of the breakeven rate in comparison with changes in the nominal bond yield to capture *changes* in inflation expectations depends on the relative volatility of the real rate of interest in comparison with the volatility of the liquidity premium. In particular, the measure derived from the break-even rate will only be a better proxy for changes in inflation expectations if the real rate of interest is more volatile than the liquidity premium.

To approximate the importance of liquidity premia in the French market we have compared the implied interest rates quoted on the bond market with those quoted on the STRIP market, the latter having a lower degree of liquidity. We approximate the liquidity premium as the difference between the implied interest rates in both markets. To do this, we estimate zero-coupon yield curves in both markets using the Nelson and Siegel model with daily frequency between September 1998 and May 2001. Figure 3 shows the average difference

between both curves together with the 5% and 95% percentile values. At the short end of the curve (up to 2.5 years), the average difference appears to be positive up to a maximum of 5 b.p. As Clermont-Tonnerre (1993) documented, this result is not consistent with the existence of a liquidity premium in the STRIPs market and may be attributed to a relative excess of demand at the short end of the STRIPs market due to the behaviour of guaranteed mutual funds. For horizons longer than 2.5 years the average difference between the curves turns out to be negative, as expected. However, the value of the average difference is almost negligible (around 1 b.p.) and the 95% percentile is always positive. These results suggest the existence of very low liquidity premia in the STRIPs market. Additionally, the existence of other effects such as imbalances between supply and demand in the STRIPs market may also distort the estimation of these premia.

Scenario 6: Relaxing simultaneously a number of assumptions

In this scenario we relax at the same time assumptions i-iv in order to analyse the interactions between the different assumptions. For instance, the existence of both taxes on inflation gains and an inflation lag means that the indexed bond is no longer an inflation risk-free asset; and, in a context where investors are risk-averse, indexed-bond holders would demand compensation for bearing this risk.

The Fisher identities for the conventional and the indexed bonds are, respectively:

$$(1+i(1-t_1)) = (1+\mathbf{p})(1+\mathbf{r}+\mathbf{g}_n)$$
(15)

$$(1+r(1-t_1))(1+\mathbf{p}_1)-t_2\mathbf{p}_1=(1+\mathbf{p})(1+\mathbf{r}+\mathbf{g}_1)$$
(16)

Note that in this case g includes both a liquidity and an inflation risk premium. The latter will generally be smaller than the inflation risk premium demanded on conventional bonds since the inflation risk for the indexed bond will also be smaller.

Solving (15) for i, we have:

$$i = \frac{\mathbf{r} + \mathbf{g}_n}{1 - t_1} + (1 + \frac{\mathbf{r} + \mathbf{g}_n + t_1}{1 - t_1})\mathbf{p}$$
(17)

and solving (16) for r and using (17), we have the following expression for the break-even rate:

$$i - r = \boldsymbol{p} + \frac{\boldsymbol{g}_n}{1 - t_1} - \frac{1 + \boldsymbol{p}}{1 + \boldsymbol{p}_1} \frac{\boldsymbol{g}_i}{1 - t_1} + \frac{(\boldsymbol{r} + \boldsymbol{g}_n)\boldsymbol{p}}{1 - t_1} + \frac{\boldsymbol{p}t_1 - \boldsymbol{p}_1 t_2 + \boldsymbol{p} \boldsymbol{p}_1 t_1}{(1 - t_1)(1 + \boldsymbol{p}_1)} + \frac{\boldsymbol{p}_1 - \boldsymbol{p}}{1 + \boldsymbol{p}_1} (1 + \frac{\boldsymbol{r}}{1 - t_1})$$
(18)

In expression (18) we see that when assumptions i-iv are relaxed, the bias of the breakeven rate in measuring the *level* of inflation expectations can be broken down into five components. These components are: a) the inflation-risk premium of the nominal bond, b) the (liquidity and inflation-risk premia) of the indexed bond, c) the compound error, d) the tax effect and e) the inflation lag effect. It is interesting to see that the marginal tax rate on interest income affects all five components. In particular, the higher this tax rate the higher the absolute value of any of these components. Also, the inflation lag not only appears in the last term but also affects the second and fourth terms.

Scenario 7: Introducing coupon bonds

In this scenario we consider a more realistic case in which conventional and indexed bonds are coupon bonds whose maturity dates could be different (i.e. we relax assumption v and keep the other assumptions of scenario 1).

As is well known, the yield on a conventional coupon bond does not coincide with the interest rate at the horizon of its term to maturity, unless the nominal term structure is flat. Something similar applies in the case of indexed bonds for real rates of interest –i.e. the real yield does not coincide with the real rate of interest at the horizon of its term to maturity, unless the real term structure is flat. Therefore, the break-even rate is implicitly assuming that both the nominal and real term structures are flat - and therefore that the term structure of the expected inflation is also flat. This assumption will generally not be satisfied and the break-even rate will estimate inflation expectations with a bias that we will refer to as the coupon bias.

In order to analyse the sign and size of the coupon bias it is useful to approximate the (nominal and real) yield to maturity as the interest rate at the horizon of the duration of the bond. Applying this approximation to the conventional and indexed bonds, and using the Fisher identity, we can rewrite the break-even rate as:

$$i - r \approx \boldsymbol{p}_{T} + \boldsymbol{p}_{di} r_{di} + (\boldsymbol{p}_{di} - \boldsymbol{p}_{T}) + (i_{dc} - i_{di})$$
 (19)

where p_m and r_m are the inflation expectations and (zero-coupon) real interest rate at a m-horizon, respectively, for $m=\{T,dc,di\}$; T is the horizon for which inflation expectations are measured, dc is the duration of the conventional bond and di is the duration of the indexed bond⁷. The last three terms of expression (19) approximate the bias incurred when computing the break-even rate with coupon bonds. The first one, $(p_{di} r_{ic})$, is the result of the compound effect. And the other two, $(p_{di} - p_T) + (i_{dc} - i_{di})$, reflect the coupon bias. The first term of the coupon bias $(p_{di} - p_T)$ depends on the slope of the term structure of the expected

⁷ The duration of the indexed bond is computed in the conventional way using the real cash flows.

inflation. More specifically, it is negative (positive) with an upward (downward) slope since dc is lower than T, and its absolute value increases with the absolute value of the slope. This term will only be zero if the inflation term structure is assumed to be flat -i.e. parallel nominal and real term structures.

The second term of the coupon bias $(i_{dc}-i_{di})$ depends on both the duration of the bonds and the slope of the nominal term structure. This term will be zero if the nominal term structure is flat, as the break-even rate implicitly assumes, or if, alternatively, the duration of both bonds is the same. If the duration of the indexed bond is higher than that of the conventional bond, this term will be positive (negative) for a downward (upward) sloping nominal term structure, whereas the opposite sign will appear if the duration of the indexed bond is lower than that of the conventional bond.

Hence, if the nominal and real term structures are flat, as the break-even rate implicitly assumes, there is no coupon bias. If these term structures are parallel, but not flat, - i.e. if the inflation term structure is flat - the coupon bias will depend on the duration of the indexed and conventional bond and on the slope of the nominal term structure, and it can be approximated by $(i_{dc}-i_{di})$. When the inflation term is not flat, an additional term will appear in the coupon bias that can be approximated by $(p_{di}-p_T)$.

To estimate the importance of the coupon bias we have carried out a simulation exercise assuming different combinations of nominal and real interest rate term structures. Note that the term structure of expected inflation is implicitly defined given the nominal and real term structures. In particular, the expected inflation rate for a m-horizon will be $p_m = (i_m - r_m)/(1 + r_m)$, where i_m is the nominal zero-coupon yield at a m-horizon. More specifically, six scenarios for nominal interest rates and three for real rates are considered. In all of them the term structure of interest rates is assumed to follow the Nelson and Siegel model, i.e.

$$y_m = \boldsymbol{b}_0 + (\boldsymbol{b}_1 + \boldsymbol{b}_2) \frac{\boldsymbol{t}}{m} \left(1 - \exp\left(-\frac{m}{\boldsymbol{t}}\right) \right) - \boldsymbol{b}_2 \exp\left(-\frac{m}{\boldsymbol{t}}\right)$$
 (20)

where y_m is the m-horizon instantaneous interest rate (nominal or real). The scenarios considered impose $\mathbf{b}_2=0$ (i.e. yield curves are monotonous), $\mathbf{t}=4^8$ and combine different values for parameters \mathbf{b}_o , which captures the long-term interest rate (asymptotic value), and \mathbf{b}_l , which captures the (opposite of the) slope of the term structure. In all real interest rate scenarios the long-term interest rate is set at 3.5%, a value which is close to the average observed yield on the French indexed bond since its launch. Scenario 1 of nominal interest rates approximately reflects the average observed level of the estimated zero-coupon yield

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⁸ This corresponds approximately to the average estimated value for this parameter for the nominal interest rate term structure in the French market between September 1998 and May 2001.

curve. For each combination of nominal and real interest rate structures the coupon bias is computed as the difference between the break-even rate adjusted for the compound effect⁹ and the implied inflation expectation at a 10-year horizon. The break-even rate is computed using the characteristics of the French 10-year conventional and indexed bonds in July 1999 (i.e. 10 years before maturity).

As Table 2 reports, the sign and size of the coupon bias mainly depends on the implied slope of the term structure of the expected inflation. This reflects the fact that both bonds have a very similar duration: around 8.1 years and 8.7 years for the conventional and indexed bond, respectively. The bias only reaches relative high values (-32 b.p. and 38 b.p.) when the slope of the term structure of expected inflation is supposed to be high in absolute terms also (9 pp. and –9 pp., respectively). Conversely, when the expected inflation is flat (same slope in the nominal and real interest rate curves) the bias is very low. Finally, when we take the nominal rates that reflect the recent period (scenario 1) the bias ranges from –4 b.p. (when we assume a flat inflation term structure) to –17 b.p.

Scenario 8: Introducing asymmetry in inflation compensation

Sometimes, as is the case in France, the par value of indexed bonds is guaranteed at maturity, introducing asymmetry in the compensation for inflation (i.e. assumption vi does not hold). The existence of such asymmetry means that the bond includes a put option. If this option is priced the bond will have a higher price (lower yield) than that of an otherwise identical bond with symmetric compensation. As a consequence of this effect, the breakeven rate will overestimate inflation expectations. The size of this bias will be higher the greater the likelihood of deflation over the horizon of the residual life of the bond. In the French case, the lack of historical negative figures for inflation over horizons of 10 years suggests that the likelihood of deflation is very low, and, as a consequence, the size of the bias caused by the asymmetry of compensation for inflation should also be very low.

Summary of results

To sum up, in this section we have shown that the break-even rate may measure inflation expectations with an error. Table 3 summarises the main biases that this indicator could have. In particular, the sign of the bias and an estimation of its size, in the case of the 10-year French bond, are shown.

⁹ This correction is made in order to isolate the coupon bias from the compound bias.

4. Improving the information content of the break-even rate

In the previous section we have discussed different sources of errors associated with the use of the break-even rate as a measure of market inflation expectations. We have also seen that the size and sign of the joint impact of these biases is difficult to establish. The literature on indexed bonds has provided some methodologies to overcome some of these problems, especially regarding the inflation-lag and coupon biases. In this section, we discuss the applicability and added value of these alternative measures of inflation expectations for the 10-year French indexed bond.

One simple approach to eliminate the impact of the inflation lag in the estimation of expected inflation has consisted of introducing inflation expectations over the lagging period in the calculation of the real yield. An iterative procedure is then used aimed at achieving consistency between the inflation expectations over the lagging period and the break-even rate. More recently, Evans (1998) has proposed an alternative framework to deal with the inflation lag based on a no-arbitrage relation between nominal and indexed bonds. Both methods have mainly been employed for the British indexed bonds due to their relatively long inflation lag (eight months). For the French case, as we saw in Section 3, the impact of the inflation lag is negligible.

The break-even rate is normally computed using bonds with different maturities, and as was argued in Section 3, different cash-flow structures, both causing coupon bias. One alternative considered in the literature has involved comparing bonds with a similar duration, instead of a similar maturity. But this alternative has been rejected by some authors because the nature of the interest rate risk differs for nominal and indexed bonds (see Bootle (1991)). Another, more refined, approach for dealing with coupon bias has consisted of the estimation of an implied forward inflation rate curve, commonly known as Inflation Term Structure or ITS. The ITS is obtained as the difference between the forward rates of nominal and real yield curves. One of the advantages of this procedure lies in the fact that it uses information of the whole market, avoiding the bond-specific distortions of the breakeven rate. Deacon and Derry (1994) proposed this methodology for the British market, fitting a real and nominal yield curve with a parametric model and employing an iterative process to deal with the inflation lag¹⁰. The Bank of England employed it in some Inflation Reports (see, for example, Bank of England (1994)). More recently, Anderson and Sleath (2001) have also followed the same approach but using the framework proposed by Evans (1998) to control the inflation lag and a spline-based technique to fit the yield curves.

¹⁰ The real yield curve is re-estimated until consistency between the assumed ITS and the estimated ITS is achieved. The initial assumption is a flat inflation rate.

Unfortunately, this methodology cannot be applied to the French market, given the limited supply of indexed bonds.

Alternatively, Sack (2000), focusing on the U.S. 10-year indexed bond (TIPS 11), proposed an alternative measure, which he called *inflation compensation measure*. This takes into account the structure of payments of the bonds but does not require fitting a real yield curve. The *inflation compensation measure* is defined as the constant rate, p, needed to adjust the principal of the indexed bond so that the present value of the nominal coupon and principal payments are equal to the price of the indexed bond. More formally,

$$P_{t} = \sum_{i=1}^{N} c(1+\boldsymbol{p})^{i} d_{t}(i) + 100(1+\boldsymbol{p})^{N} d_{t}(N)$$
(21)

where N is the term to maturity (in years) of the bond, c is the annual fixed real coupon rate, and $d_t(i)$ is the discount function associated with the nominal term structure of interest rates.

It can be proved that the *inflation compensation measure* is an unbiased estimator of market inflation expectations under the following assumptions: i) flat inflation expectations, ii) no tax effects, iii) zero inflation-risk premium 12 , iv) indexed bonds have the same degree of liquidity as bonds employed for fitting the nominal yield curve. To see this, note first that if inflation expectations are constant over the life of the indexed bond (assumption i) at a rate p, and ignoring the inflation lag effect, the right-hand side of expression (21) gives the price of a portfolio of bonds that matches the expected cash flows of the indexed bond. And under assumptions iii) and iv) the price of this portfolio should be the same as that of the indexed bond in order to avoid arbitrage opportunities.

Then, the *inflation compensation measure* improves the break-even rate in some aspects. Firstly, it controls part of the coupon bias since it relaxes the assumption included in the break-even rate of flat nominal term structure. However, as in the case of the break-even rate, this measure also assumes a flat inflation term structure – or parallel nominal and real term structures – and therefore it does not completely eliminate the coupon effect. Looking back to the discussion of the coupon bias in Section 3, we know that the part of the coupon bias corrected with this measure will depend on the differences in duration of the conventional and indexed bonds, which are very small in the French case. Therefore, the improvement on this matter will be small.

Secondly, the inflation compensation measure avoids any bias caused by idiosyncratic factors of the nominal bond used to compute the break-even rate. Thirdly, if the nominal yield curve employed to discount the expected cash flows has similar liquidity to that of

¹¹ Treasury inflation-protected securities.

¹² Note that under assumptions i) and iii) the inflation lag does not introduce any additional bias.

indexed bonds, the *inflation compensation measure* also controls the liquidity bias. In this regard, Sack (2000) fitted the nominal yield curve using the prices of Treasury STRIPS assuming that these securities and indexed bonds have a similar degree of liquidity. And, finally, by definition the *inflation compensation measure* does not include a compound bias.

We derive the *inflation compensation measure* for the French market, applying expression (21) to the 10-year French indexed bond and using the zero-coupon yield curve fitted with French conventional bonds¹³ as discount function. This inflation compensation measure, p_{conv}^{oati} , proves to be very similar to the break-even rate, with the differences always below 10 b.p. (see Figure 4). Hence, in this sample, the joint impact of the biases corrected by the *inflation compensation measure* is not very important.

In order to quantify these biases separately we break down the difference between the break-even rate and the inflation compensation measure p_{conv}^{oati} into three components:

$$Breakeven - \mathbf{p}_{conv}^{oati} = Compound \ Bias + Idiosyncratic \ Effect + Coupon \ Bias$$

The *Compound Effect* is calculated as the difference between the break-even rate and the rate adjusted for the compound effect (r(i-r)/(1+r)). The *Idiosyncratic Effect* captures the specific distortions caused by the particular 10-year nominal bond employed to compute the break-even rate. In order to approximate this component we compute a compensation measure $(\boldsymbol{p}_{conv}^{oat})$ applying (21) to the 10-year nominal bond, in the same way as we did for the indexed bond. Note that in this case the compensation measure captures the yield error of the bond associated with fitting a zero-coupon yield curve. Finally, the part of the *Coupon Bias* corrected by the inflation compensation measure is computed as a residual, assuming that potential premia due to liquidity, tax effects and inflation risk, included in both measures, cancel one another out.

The *Compound Effect* proves to be small and stable and, in this sample, it implies an overestimation of inflation expectations in the break-even rate of 4 b.p. on average (see Figure 5). The part of *Coupon Bias* that is corrected by the inflation compensation measure proves to be very stable around -4 b.p. The course and size of this effect is consistent to the results in Section 3 given that the differences in the duration of the indexed and conventional bonds are very small. In Sack (2000), this bias also accounts for less than 5 b.p., but in his case it is strongly influenced by changes in the references used to compute the break-even rate ¹⁴. Finally, the *Idiosyncratic Effect* is also small but more volatile, ranging from –6 to 4 b.p. The development of this component seems to be related to the benchmark

¹³ We fit a Nelson-Siegel model for the estimation of the yield curve and minimise price errors weighted by duration.

¹⁴ The on-the-run nominal security, employed for the break-even rate in the American case, is auctioned quarterly (with some reopenings), while new indexed bonds are issued annually.

status of the nominal bond used in the break-even rate. For example, from November 1998 to June 1999, the bond acted as a benchmark in the French market, and, hence, its yield included a negative liquidity premium in relation to the remaining conventional bonds. This introduced a negative bias into the break-even rate that implied an underestimation of expected inflation. From November 1999, the *Idiosyncratic Effect* was mainly positive due to the loose of the liquidity of the conventional bond.

The compensation measure p_{conv}^{oati} probably still incorporates a liquidity premium because, unlike in Sack (2000), the expected payments have been discounted using the zero-coupon yield curve of conventional bonds. However, we have seen in Section 3 that the impact of liquidity on the estimation of inflation expectations with the break-even rate seems to be small when we evaluate liquidity differences through the STRIPs yield curve. To confirm this, we compute the compensation measure using the STRIPs yield curve, as in Sack (2000), $m{p}_{strips}^{oati}$. The difference between these alternative compensation measures, $m{p}_{conv}^{oati}$ and $m{p}_{strips}^{oati}$ will be a good approximation of the liquidity bias included in the inflation compensation measure $m{p}_{conv}^{oati}$ if STRIPs and conventional bond markets differ solely in their degree of liquidity, and if the liquidity of STRIPs is similar to that of indexed bonds. As Figure 7 shows, the liquidity bias is, as expected, very small, and in some intervals of the sample its sign is even negative, indicating a potential negative liquidity premium. These results may be related to the existence of very small liquidity premia or, alternatively, to the fact that prices of STRIPs reflect additional factors, other than liquidity, such as clientele effects, related to the specific nature of these securities (as was mentioned in Section 3). Additionally, it cannot be ruled out that the liquidity premium included in the price of the indexed bond is higher than that estimated here if the degree of liquidity of these bonds were lower than that of the STRIPs.

Summarising, the limited number of indexed bonds in the French market restricts the scope to improve the break-even rate as a measure of inflation expectations. However, we can calculate an inflation compensation measure, as Sack (2000) did for the U.S. market, that overcomes some biases of the break-even rate, namely the biases induced by the different cash-flow structures of the bonds (coupon bias) and the idiosyncratic effects of the conventional bond (idiosyncratic bias). In the French case, the inflation compensation measure proves to be very similar to the break-even rate. Even though the inflation compensation measure improves the break-even rate, it may still incorporate an inflation-risk premium embedded in conventional bond prices. Additionally, the existence of biases due to the liquidity of indexed bonds and tax effects cannot be ruled out.

5. Comparing some inflation expectation measures

Figure 5 depicts the annual rate change of the French CPI (excluding tobacco), together with three indicators of long-run inflation expectations: the inflation compensation measure, the forecast from a univariate ARIMA model, and the *consensus forecast*, a measure from survey data published by Consensus Economics. The time horizon of all of them is approximately 10 years.

Comparison of these indicators reveals interesting differences in both the level and the changes. According to the *consensus forecast*, long-run inflation expectations have remained very stable. During 1999 and 2000 this indicator decreased slightly from 1.7% to 1.5%, and in the first half of 2001 it revised inflation expectations moderately upwards to 1.6%.

The inflation expectations derived from the univariate model are also very stable and their fluctuations incorporate recent past movements in current inflation. In this way the increase in inflation during 2000 is translated into a slight upward revision of inflation expectations from approximately 1% to 1.3%.

Finally, the inflation compensation measure, computed as an average of its daily values, reflects more volatile inflation expectations. During 1999 this indicator followed an upward trend (from levels around 1% to 1.6%) coinciding with the increase in observed inflation ¹⁵. During 2000, this indicator remained relatively stable in spite of the increase in the current inflation rate, and its levels were much closer to those derived from the consensus forecast.

Therefore, during 1999 and 2000, the *consensus forecast* considers the increase in observed inflation as transitory (because it perhaps reflects transitory increases in energy prices), while the other indicators find some persistent factors and reviewed long-term expected inflation upwards. During 2000 the inflation compensation measure and the consensus forecast indicate very similar levels of inflation expectations. Finally, at the beginning of 2001, all indicators coincide in correcting the expected long-term inflation upwards.

The greater variability of the inflation compensation measure might reflect, to some extent, the fact that participants in financial markets revise their inflation expectations more thoroughly than is inferred from the other indicators. This explanation is clear in the case of the univariate model since the information set of the investors is much wider than the past values of the CPI. Nevertheless, it cannot be ruled out that the relatively greater volatility of the inflation compensation measure was, to some extent, induced by some of the biases

¹⁵ During the first part of the sample a case could be argued for the existence of a learning process and price discovery that might have distorted the inflation expectations derived from the new French indexed-bond market.

discussed in Section 3, like for example that associated with the inflation-risk premium, liquidity differences or tax issues.

If, alternatively, we focus on estimating changes in inflation expectations, rather than their levels, we can compare the inflation compensation measure with other indicators based on the information provided by financial markets. Under some assumptions about the behaviour of real interest rates, Mishkin (1990) proposed using the slope of the nominal term structure as an indicator of changes in inflation expectations. When the slope of the yield curve steepens (flattens) it is an indication that the inflation rate will rise (fall) in the future. Alternatively, given that long-term real interest rates are supposed to be relatively stable, the fluctuations in the yield on the 10-year nominal bond are also interpreted as changes in inflation expectations.

Figure 8 tracks changes in long-term inflation expectations based on these three financial indicators¹⁶: monthly changes in the slope of the term structure (slope indicator), monthly changes in the yield on the 10-year nominal benchmark bond (yield indicator) and monthly changes in the inflation compensation measure. The daily average is used in all cases. The yield indicator is highly correlated with the inflation compensation measure, although the size of the movements in inflation expectations are generally smaller in the case of the inflation compensation measure, but the sign tends to coincide. There are two possible explanations for these differences. On the one hand, they may reflect fluctuations in real interest rates, which are incorporated into the *yield indicator*. On the other, as we saw in Section 3, due to tax effects, the overestimation associated with the *yield indicator* is higher than that of the inflation compensation measure. In the case of the slope indicator, it usually coincides in sign with the other indicators, but there are some periods in which the message is different. For example, during May 2000 the slope indicator predicted a strong reduction in inflation expectations (while the other indicators predicted an increase). One explanation for this contradictory message is the influence of expected movements in official interest rates. During this period, the expected increase in official interest rates might have flattened the slope of the yield curve. At the end of 2000 something similar occurs: expectations of a cut in official interest rates may have increased the slope of the yield curve, reflecting, as a consequence, a strong increase in expected inflation.

Figure 9 depicts changes in inflation expectations during the last six months according to previous financial measures. From this perspective the evolution of these indicators is smoother and it also reflects aforementioned differences. During 1999 all indicators predict increases in inflation expectations, in year 2000, only the *slope indicator* shows quite strong decreases in inflation expectations while according to the other indicators there are no

¹⁶ For each month we calculate the average of the daily values of the slope of the term structure, the 10-year nominal yield and the inflation compensation measure, respectively. Then the indicator of changes in inflation expectations is calculated as monthly differences.

changes. As already mentioned, this behaviour of the *slope indicator* may be induced because this period was characterised by reductions in official interest rates.

In summary, the comparison of financial indicators of changes in inflation expectations shows that the compensation measure is relatively smooth, and it is also useful for measuring the level of inflation expectations. Although it may incorporate some biases it seems to overestimate to a lesser extent changes in inflation expectations than the yield measure, and it is not affected either by changes in real interest rates or by the expected movements in official interest rates.

6. Conclusions

Long-term inflation expectations are relevant variables for monetary authorities, especially for those with price stability goals. The break-even rate is a widely used measure of inflation expectations that uses prices of indexed bonds. In this paper, we have seen that this indicator is an unbiased estimator of inflation expectations under very restrictive assumptions, which, in practice, are not fulfilled. The joint impact of potential biases is difficult to establish, although the analysis made in this paper sheds some light on their possible sign and size.

The literature on indexed bonds provides alternative methods to deal with the biases in the break-even rate, but unfortunately the limited supply of indexed bonds in the French market does not allow use of the most refined approaches. However, Sack (2000) proposed an alternative indicator, which he called inflation compensation measure, that employs richer information and avoids some of the biases included in the break-even rate such as (part of) the coupon bias, the compound bias and the idiosyncratic effects of the 10-year conventional bond. Under some conditions, this measure also avoids liquidity bias. Thus, this alternative estimator improves the break-even rate as an estimator of long-term inflation expectations.

In the French case, the inflation compensation measure proves to be very similar to the break-even rate, suggesting that, in this sample, the joint impact of the biases controlled by this alternative indicator is small.

The comparison of alternative measures of long-term inflation expectations, such as survey statistics, or those derived from an univariate model, shows significant differences. For example, the inflation compensation measure is more time-varying. This may be attributed to the fact that investors revise inflation expectations more thoroughly or, on the other hand, it could also partially reflect the potential biases of this measure. However, among financial indicators, the inflation compensation measure seems to be more stable, and although it

may incorporate some biases, it seems to reduce the impact of movements on real interest rate, tax effects or market-expected movements in official interest rates that may distort other financial indicators.

Thus, the analysis performed in this paper suggests that indexed bonds provide valuable information on inflation expectations for monetary authorities. Nevertheless, caution is required when interpreting the estimated inflation expectations since, at least potentially, they may incorporate some biases. If these biases are quite stable, it would be better to estimate changes in, instead of the levels of, inflation expectations. In any event, the potential variability of these biases in the very short term suggests that it would be better not to interpret changes in these indicators over very short periods of time.

TABLE 1

Estimation of the tax bias (a)

	Bias in the level (percentage points)			Sensitivity to changes in the inflation rate			
				Break-even rate			Yield on
	π=0 %	π= 2 %	π=5 %	π= 0 %	π=2%	π= 5 %	conventional bond
Case I: t ₂ =0							
$t_1=0 \%$ $t_1=10 \%$ $t_1=30 \%$ $t_1=40 \%$	0 0 0 0	0.07 0.30 0.96 1.45	0.18 0.75 2.39 3.63	1.04 1.15 1.48 1.73	1.04 1.15 1.48 1.73	1.04 1.15 1.48 1.73	1.04 1.15 1.48 1.73
Case II : t ₂ =0.5 t ₁							
t ₁ =10 % t ₁ =30 % t ₁ =40 %	0 0 0	0.19 0.54 0.80	0.49 1.37 2.04	1.09 1.26 1.39	1.10 1.27 1.40	1.10 1.27 1.41	1.15 1.48 1.73
Case III: t ₂ =t ₁							
$t_1=10 \%$ $t_1=30 \%$ $t_1=40 \%$	0 0 0	0.08 0.12 0.14	0.22 0.35 0.45	1.04 1.05 1.06	1.04 1.06 1.07	1.04 1.07 1.09	1.15 1.48 1.73

⁽a) These estimations include the bias induced by the compound effects. t_1 and t_2 are, respectively, the marginal tax rates on interest income and capital gains, π is the expected inflation rate. The real rate of interest is set at 3.5%

TABLE 2

Estimation of the duration bias included in the break-even rate (a)

basis points

Term structure of real interest rates assumption Long-term real interest rate β_0 = 3.5%

			Slope (- β_1) (perc.points)			
	Long-term nominal interest rate	Slope $(-\beta_1)$ (perc.points)				
			0	3	-3	
Term structure of nominal interest rate	5 %	3 0 -3	-10.3 0.0 11.3	-4.3 6.2 17.8	-16.8 -6.7 4.5	
	10 %	6 0 -6	-25.1 0.0 30.8	-19.0 6.6 37.8	-31.6 -7.0 23.3	

⁽a) The term structure is assumed to follow the Nelson & Siegel model with β_2 =0 and τ =4. The estimates are made using the characteristics of the 10-year conventional and indexed French bonds in July 1999.

TABLE 3

Summary of the biases of the break-even rate

Source of bias	Expression	Sign (a)	Size(b.p.)	Assumptions (b)
Compound effect	πρ	+	5	Expected inflation: 1.5 % Real interest rate: 3.5 %
Inflation lag	$(\pi_{l} - \pi)(1 + \rho) / (1 + \pi_{l})$	+/-	+/- 2	Mean deviation of the CPI with its three month lag considering an horizon of 10 years.
Taxes (c)	$(t_1 - t_2 + \pi t_1) \pi / (1 - t_1)(1 + \pi)$	+	7	Expected inflation: 2 % Tax rate: 40 %
Inflation risk aversion	$(1 + \pi) \gamma_n$	+	N.A.	
Liquidity	-γι	-	-1	STRIPs and indexed bonds share the same liquidity
Coupon effect	$(\pi_{di} - \pi_{T}) + (i_{dc} - i_{di})$	+/-	-4 a -10	Observed nominal interest rates Slope of real term structure between 0 and 3 p.p.
Asymmetry in the compensation		+	small	Low probability of deflation

⁽a) Inflation expectations are assumed to be positive. + (-) indicates that the break-even rate overestimates (underestimates) inflation expectations.

⁽b) Assumptions used to estimate the size of the bias.

⁽c) The positive sign is derived assuming that the marginal tax rate for capital gains is less or equal to the marginal tax rate on interest income. The estimated size of the bias assumes that these rates are equal.

FIGURE 1 **Outstanding volume** EUR billion OAT 4% 25 April 2009 25 OATi 3% 25 July 2009 20 15 10 5 0 Dec-98 Dec-99 Dec-00 Mar-01 Source: French Treasury.

FIGURE 2

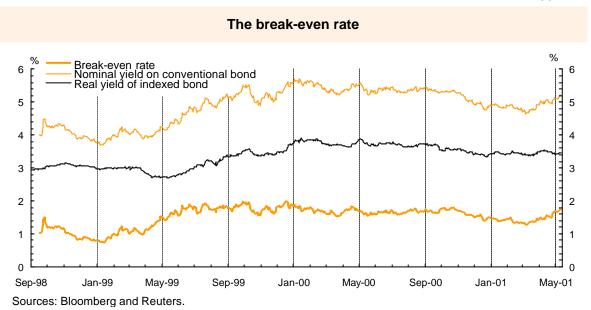


FIGURE 3

Difference in the estimated yield curves between conventional bonds and STRIPs

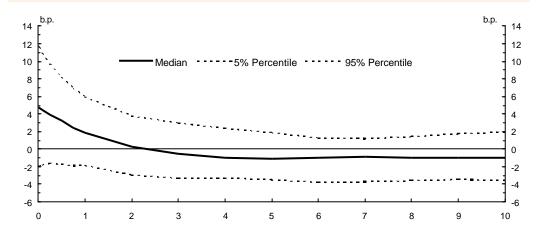


FIGURE 4

Long-term inflation expectations derived from indexed-bond prices

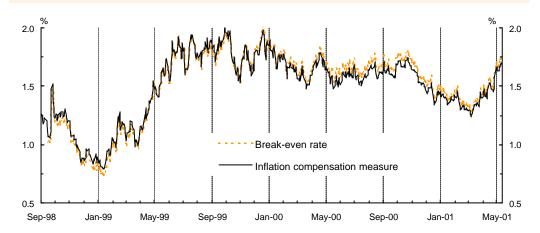
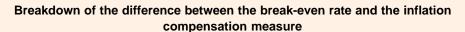


FIGURE 5



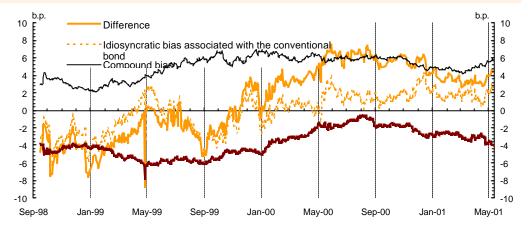


FIGURE 6

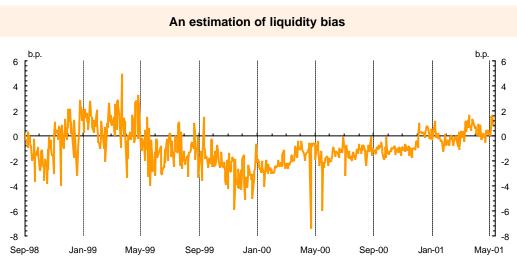


FIGURE 7

Long-term inflation expectations and actual inflation

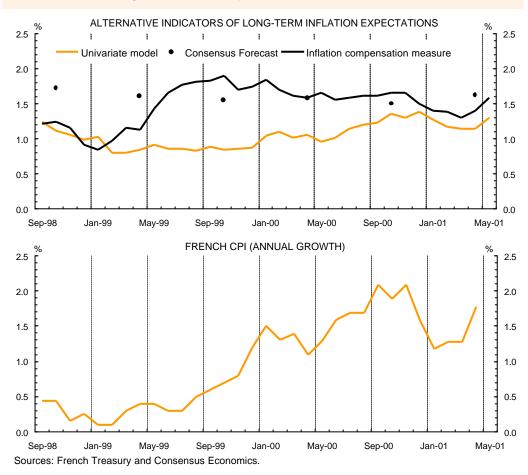
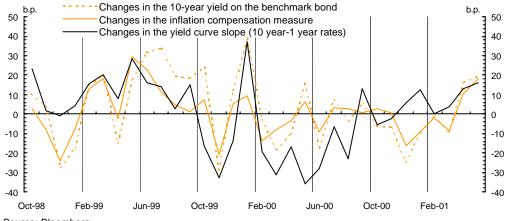


FIGURE 8

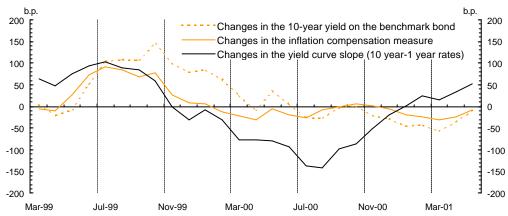
Financial indicators of monthly changes in long-term inflation expectations



Source: Bloomberg.

FIGURE 9

Financial indicators of half-year changes in long-term inflation expectations



Source: Bloomberg.

REFERENCES

Alonso, F., Ayuso, J. (1996), Primas de Riesgo por Inflación en España', Revista de Economía Aplicada vol IV, n.12, pp. 5-19.

Anderson N., Sleath J. (2001), 'New Estimates of the UK Real and Nominal Yield Curves', Bank of England, working paper no 126.

Bank of England (1994), Inflation Report. November.

Barro (1995), 'Optimal Debt Management', NBER Discussion Paper No 5327.

Bootte, R. (1991), 'Index-linked Gilts', Cambridge Woodhead-Faulkner (2nd. Edition)

Breedon F.J., Chadha J.S. (1997), 'The Information Content of the Inflation Term Structure', Bank of England, Working Paper Series No. 75.

Clermont-Tonnerre, A. (1993), 'Bond stripping, the French experience', The Journal of International Securities Markets, IFR, vol 7, Fall.

Deacon M., Derry. A. (1994), 'Deriving Estimates of Inflation Expectations from the Prices of U.K. Government Bonds'. Bank of England, Working Paper Series No23

Deacon M., Derry. A. (1998), Inflation-indexed Securities, Prentice-Hall, Hemel Hempstead.

Evans M.D.D. (1998), 'Real Rates, Expected Inflation, and Inflation Risk Premia', The Journal of Finance, Vol LIII, No. 1, February.

France Trésor, (2001) 2000 Annual Review.

Mishkin, F (1990) 'The Information in the Longer Maturity Term Structure about Future Inflation', Quarterly Journal of Economics, 105, pp. 815-28.

Nelson, C. R., and Siegel, A. F. (1987), 'Parsimonious Modelling of Yield Curves for U.S. Treasury Bills', Journal of Business, vol 60, N. 4, pp. 473-489.

Sack, B. (2000), 'Deriving Inflation Expectations from Nominal and Inflation-indexed Treasury Yields', Finance and Economics Discussion Series. Federal Reserve Board, 2000-33.