AN APLICATION OF TRAMO-SEATS: AUTOMATIC PROCEDURE AND SECTORAL AGGREGATION
The Japanese Foreing Trade Series

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Banco de España — Servicio de Estudios
Documento de Trabajo n.º 0207

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February 2002

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Abstract

Programs TRAMO and SEATS, that contain an ARIMA-model-based methodology, are applied for seasonal adjustment and trend-cycle estimation of the exports, imports, and balance of trade Japanese series. The programs are used in an automatic mode, and the results are found satisfactory. It is shown how the SEATS output can be used to discriminate among competing models. Finally, using the balance of trade series, direct and indirect estimation are analyzed and discussed.

Keywords: Applied Time Series Analysis; regression - ARIMA models; Seasonal Adjustment; Trend-cycle estimation; Direct / Indirect Adjustment.

1. INTRODUCTION

In the early eighties, the work of Bell, Box, Burman, Cleveland, Hillmer, Pierce and Tiao set up the basis of an alternative methodology for seasonal adjustment of time series [see, for example, Burman (1980) or Hillmer and Tiao (1982)]. In essence, the methodology consists of minimum mean square error (MMSE) estimation of unobserved components ("signal extraction") hidden in an observed time series, for which an ARIMA model has been identified. This methodology has been termed the "ARIMA-model-based" (AMB) approach, and an important precedent is the work contained in Nerlove, Grether and Carvalho (1979). Typically, the components (or signals) are the seasonal, trend-cycle, and irregular components, the latter two comprising the seasonally adjusted (SA) series. The three components are assumed mutually orthogonal, and follow linear stochastic processes, usually non-stationary for the case of the trend-cycle and seasonal component, with niid ("white noise") innovations. The models for the components accept ARIMA-type parametric expressions and are derived in such a way that they aggregate into the ARIMA model identified for the observed series [see, for example, Maravall (1995)]. Estimators of the components are computed via the so-called Wiener-Kolmogorov (WK) filter, as applied to nonstationary series (see Bell, 1984).

It is often the case that, before it can be assumed the output of an ARIMA model, the series needs prior treatment. Important preadjustments are outlier correction, the removal of Calendar, intervention variable, and other possible regression effects, and interpolation of missing values; see, for example, Chang, Tiao, and Chen (1988), Box and Tiao (1975), Chen and Liu (1993), Hillmer, Bell, and Tiao (1983), Gómez and Maravall (1994), and Gómez, Maravall and Peña (1999). Awareness of the preadjustment problem has been steadily growing, and extends beyond model-based signal extraction methods [see, for example, Findley et al (1998)].

The AMB methodology had some appealing features. On the one hand, compliance with the ARIMA model of the observed series would seem a good protection against spuriousness of results or model misspecification. On the other hand, the parametric model-based framework would facilitate analysis and inference [see, for example, Pierce (1979, 1980), Bell and Hillmer (1984), Hillmer (1985), Maravall (1987) and Maravall and Planas (1999)]. Yet, despite the smart and efficient Burman and Wilson algorithm for finite sample implementation of the WK filter [see Burman (1980)], real-world application of the procedure proved elusive, in particular, for large-scale applications; it seemed to require heavy dosis of time-series analysts and computing resources, which were related, of course, to the lack of a reliable and efficient automatic (or quasi-automatic) procedure. As a consequence, the AMB methodology remained latent for some years. The appearance of the programs TRAMO and SEATS [Gómez and Maravall (1996)] has somewhat changed the situation, and the AMB methodology is presently being used or tested intensively at many agencies, institutions, and companies throughout the world.

The next section summarizes programs TRAMO and SEATS. Next, their use is illustrated with an application to Japanese foreign trade series. The paper concludes with some comments on model selection and direct versus indirect adjustment.

2. BRIEF DESCRIPTION OF PROGRAMS TRAMO AND SEATS.

TRAMO ("Time series Regression with ARIMA noise, Missing values, and Outliers") is a program for estimation and forecasting of regression models with errors that follow in general nonstationary ARIMA processes, when there may be missing observations in the series, as well as contamination by outliers and other special (deterministic) effects. An important group of the latter is the Calendar effect, composed of the Trading Day (TD) effect, caused by the different distribution of week-days in different months, Easter effect (EE), due to the changing date of Easter, Leap Year (LY) effect, and holidays effect.

If B denotes the lag operator, such that $B\,x(t) = x(t-1)$, and f the number of observations per year, given the observations $y = (y(t_1), y(t_2), \ldots, y(t_m))$ where $0 < t_1 < \ldots < t_m$, TRAMO fits the general model

$$y(t) = \sum_{i=1}^{n_{out}} \omega_i \lambda_i(B)\, d_i(t) + \sum_{i=1}^{n_c} \alpha_i\, cal_i(t) + \sum_{i=1}^{n_{reg}} \beta_i\, reg_i(t) + x(t) \qquad , \qquad (2.1)$$

where $d_i(t)$ is a dummy variable that indicates the position of the i-th outlier, $\lambda_i(B)$ is a polynomial in B reflecting the outlier dynamic pattern, cal_i denotes a calendar-type variable, reg_i a regression or intervention variable, and x is the ARIMA error. The parameter ω_i is the instant i-th outlier effect, α_i and β_i are the coefficients of the calendar and regression-intervention variables, respectively, and n_{out}, n_c and n_{reg} denote the total number of variables entering each summation term in (2.1). In compact notation, (2.1) can be rewritten as

$$y(t) = z'(t)\, b + x(t) \qquad , \qquad (2.2)$$

where b is the vector with the ω, α and β coefficients, and *z' (t)* denotes a matrix with columns the variables

$$[\ cal_1\ (t), \ldots, cal_{n_c}\ (t),\ \lambda_1\ (B)\ d_1\ (t), \ldots, \lambda_{n_{out}}\ (B)\ d_{n_{out}}\ (t), reg_1\ (t), \ldots, reg_{n_{reg}}\ (t)\ ].$$

The first term of the addition in (2.2) represents the effects that should be removed in order to transform the observed series into a series that can be assumed to follow an ARIMA model; it contains thus the preadjustment component.

In compact form, the ARIMA model for *x (t)* can be written as

$$\phi(B)\ \delta(B)\ x(t) = \theta(B)\ a(t) \qquad , \qquad (2.3)$$

where *a (t)* denotes the $N\ (0, V_a)$ white-noise innovation, and $\phi(B)$, $\delta(B)$, and $\theta(B)$ are finite polynomials in *B*. The first one contains the stationary autoregressive (AR) roots, $\delta(B)$ contains the nonstationary AR roots, and $\theta(B)$ is an invertible moving average (MA) polynomial. Often they assume the multiplicative form

$$\delta(B) = \nabla^d\ \nabla_f^{d_s}$$
$$\phi(B) = \left(1 + \phi_1 B + \ldots + \phi_p B^p\right)\left(1 + \Phi_1 B^f + \ldots + \Phi_{p_s} B^{p_s f}\right)$$
$$\theta(B) = \left(1 + \theta_1 B + \ldots + \theta_q B^q\right)\left(1 + \Theta_1 B^f + \ldots + \Theta_{q_s} B^{q_s f}\right)$$

where $\nabla = 1 - B$ and $\nabla_f = 1 - B^f$ are the regular and seasonal difference operators. We shall refer to a model consisting of (2.2) and (2.3) as a regression(reg)-ARIMA model.

When used automatically, TRAMO tests for the log/level transformation, for the possible presence of calendar-type effects, detects and corrects for three types of outliers [namely, additive outliers (AO), transitory changes (TC), and level shifts (LS)], identifies and estimates by maximum likelihood the reg-ARIMA model, interpolates missing values, and computes forecasts of the series. It also yields estimates and forecasts of the preadjustment component *z' (t) b* and of the series *x (t)* in (2.2), that is, the series that can be assumed to be the output of a linear stochastic process. This "linearized" series is equal thus to the interpolated and preadjusted series.

Program SEATS ("Signal Extraction in ARIMA Time Series") estimates unobserved components in series that follow ARIMA models using the AMB methodology, and originated from the 1982 version of a program that Burman was developing for the Bank of England. In SEATS, the unobserved components are the trend-cycle, seasonal, transitory, and irregular components. Broadly, the trend-cycle captures the peak around zero present in the series (pseudo)spectrum, the seasonal component captures the spectral peaks around the seasonal frequencies, the irregular component picks up white-noise variation, and the transitory component captures highly transitory variation different from white noise. From the ARIMA model for the series, SEATS derives the models for the components, which often display the following structure:

For the trend-cycle component (p),

$$\nabla^D p(t) = w_p(t) \quad , \qquad D = d + d_s \quad ,$$

where $w_p(t)$ is a stationary ARMA process.

For the seasonal component (s),

$$S\, s(t) = w_s(t) \quad ,$$

where $S = 1 + B + ... + B^{f-1}$ denotes the annual aggregation operator, and $w_s(t)$ is a stationary ARMA process.

The transitory component (c) is a stationary ARMA process, and the irregular component (u) is white noise.

The processes $w_p(t)$, $w_s(t)$, c (t), and u (t) are assumed Normally distributed and mutually uncorrelated. Aggregation of the models for p, s, c, and u yields the ARIMA model for the series x (t).

The model for the SA series (n) is obtained from the aggregation of the models for p (t), c (t), and u (t). Its basic structure is also of the type

$$\nabla^D n(t) = w_n(t) \quad ,$$

with $w_n(t)$ a stationary ARMA process.

The component estimator and forecast are obtained by means of the WK filter as the MMSE estimators of the signal given the observed series, and, under the normality assumption, are equal to the corresponding conditional expectation. The WK filter is a two-sided, centered, symmetric, and convergent filter; within the AMB framework, it can be given a simple analytical representation. Consider the decomposition of the series x (t), that follows the ARIMA model

$$\phi(B)\ \ x(t) = \theta(B)\ \ a(t) \quad , \quad a(t) \sim wn\,(0, V_a) \ ,$$

where $\phi(B)$ also contains the possible unit roots, into “signal plus non-signal” components as in x (t) = s (t) + n (t). Let the model for the signal be

$$\phi_s(B)\ \ s(t) = \theta_s(B)\ \ a_s(t) \quad , \quad a_s(t) \sim wn\,(0, V_s) \ ,$$

where $\phi_s(B)$ also contains possible unit roots. Denote by $\phi_n(B)$ the polynomial in B that contains the roots of $\phi(B)$ that are not in $\phi_s(B)$. Then, if $F = B^{-1}$ denotes the forward operator (such that F x (t) = x (t+1)), for a doubly infinite series, the WK filter to estimate the signal is given by

$$\nu_s(B,F) = \frac{V_s}{V_a}\ \frac{\theta_s(B)\ \ \phi_n(B)}{\theta(B)}\ \frac{\theta_s(F)\ \ \phi_n(F)}{\theta(F)} \quad ,$$

or, equivalently, by the ACF of the stationary ARMA model

$$\theta(B)\ \ z(t) = \left[\theta_s(B)\ \ \phi_n(B)\right] a_z(t) \quad , \quad a_z(t) \sim wn\,(0, V_s / V_a) \, .$$

The estimator of the signal is obtained through

$$\hat{s}(t) = \nu_s(B,F)\ \ x(t) \quad .$$

In practice, one deals with a finite series, say, [x(1), x(2), … , x(T)]. Because the WK filter converges, for long-enough series, the estimator of the signal for the mid-years of the sample can be considered to be equal to the final estimator (that is, the one that would be obtained with the doubly infinite series). More generally, given the series [x(1), … , x(T)], the MMSE estimators and forecasts of the components (or signals) are obtained applying the WK filter to the series extended at both ends with forecasts and backcasts. The Burman-Wilson algorithm permits us to obtain the effect of the doubly infinite filter with just a small number of forecasts and backcasts. The model-based framework is exploited by SEATS to provide standard errors (SE) of the estimators and forecasts (the SE are exact if the ARIMA model is correct). Being obtained with a two-sided filter, the component estimators at the end of the series are preliminary, and will be subject to future revisions. The model-based framework is also exploited to analyze revisions (size, speed of convergence, etc.) and to provide further elements of interest to short-term monitoring.

TRAMO and SEATS are structured so as to be used together. TRAMO preadjusts the series, and SEATS decomposes the linearized series into its stochastic components. The complete final component is equal to the stochastic one, plus the deterministic effect associated with that component, that has been removed in the preadjustment by TRAMO (for example, an AO outlier will be added to the irregular component, a LS outlier will be added to the trend-cycle, EE will go to the seasonal component, and so on). TRAMO, SEATS, and program TSW, a Windows version that integrates both programs, are freely available at http://www.bde.es, together with documentation.

3. AN APPLICATION TO THE JAPANESE FOREIGN TRADE SERIES.

The Japanese exports, imports, and balance of trade monthly series are used to illustrate the (mostly) automatic functioning of TRAMO-SEATS, as enforced in program TSW. The series span the period September 1989 – August 2001 (144 observations) and are displayed in figures 1.1, 1.2 and 1.3. I shall adjust the series one by one, in a "blind univariate" manner, ignoring in each case the results obtained for the others.

3.1 Automatic Procedure

The automatic procedure of TRAMO-SEATS requires the prior decision of whether or not a test for the presence of calendar effects should be included, and if so which specification for the TD should be used. The different options are controlled by the parameter RSA (see the TSW Reference Manual). The most general case corresponds to the value

* RSA = 8, in which case, the following tests are performed:
  - log / level specification,
  - Easter effect,
  - Leap Year effect,
  - Trading Day effect using a 6-variable specification (one for each day of week).

Then, automatic model identification (AMI), joint with automatic outlier detection and correction (AODC), is performed. In the latter, three types of outliers are considered: AO, TC, and LS outliers.

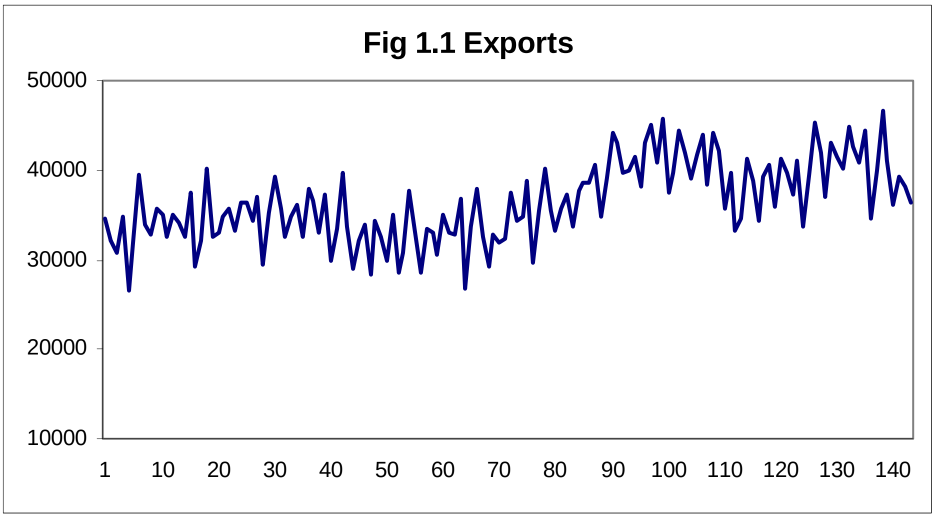


Fig 1.1 Exports
50000
40000
30000
20000
10000
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140

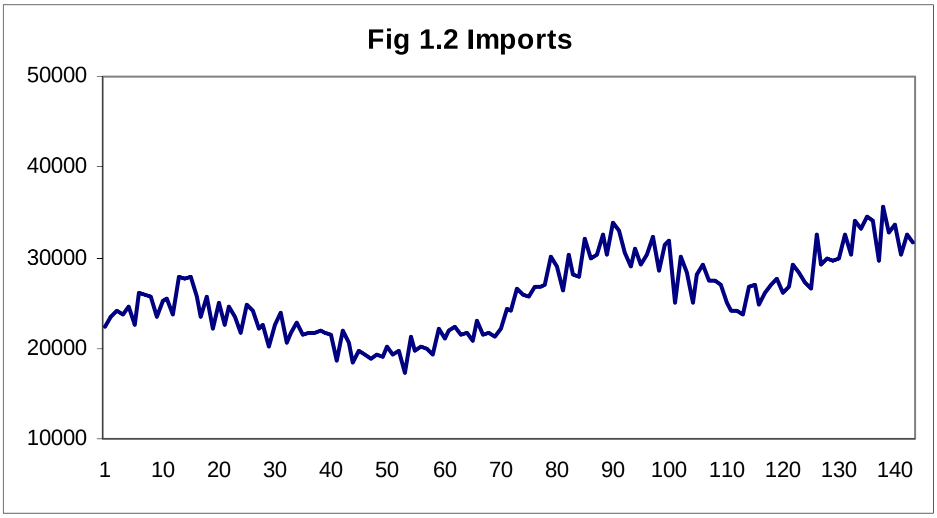


Fig 1.2 Imports
50000
40000
30000
20000
10000
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140

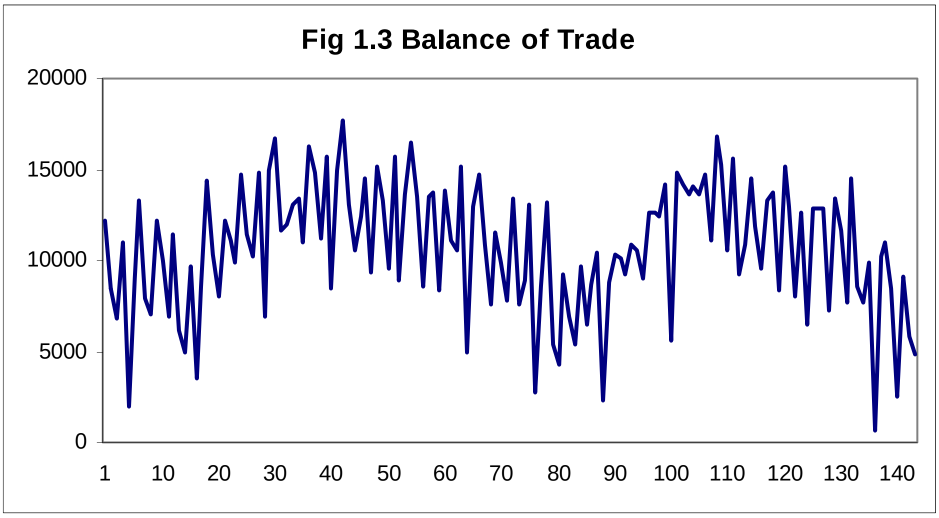


Fig 1.3 Balance of Trade
20000
15000
10000
5000
0
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140

* RSA = 6, as 8, without the LY effect test
* RSA = 5, as 8, with the TD specification changed to a one-variable specification (working / non-working day).
* RSA = 4, as 5, without the LY-effect test.
* RSA = 3, as 8, without tests for EE, TD, and LY effects.
* RSA = 1, as 3, without AMI . The default ("Airline model") is always used. This model is given by

$$\nabla \ \nabla_{12} \ x(t) = (1+\theta_1 B) \ (1+\theta_{12} B^{12}) \ a(t) \quad ,$$

and provides an excellent "benchmark" model, and a good protection in cases of unstable AMI results. [For the empirical relevance of this model, see Fischer and Planas (2000)].

3.2 Exports Series (E)

Starting with the most general case RSA = 8, the model obtained is

$$E\ (t) = OUT_e\ (t) + CAL_e\ (t) + x_e\ (t) \quad , \qquad (3.1)$$

where the first term in the right-hand-side (rhs) of (3.1) is the total outlier effect, which is the sum of three outliers, as in

$$OUT_e\ (t) = 2716\ d_1(t) + 2626\ d_2\ (t) - 3667\ \frac{1}{1-B}\ d_3\ (t)$$

(t-values): (3.4) (3.3) (-3.8)

with $d_1(53)$ =1 (1/94), $d_2(66)$ =1 (2/95), and $d_3(111)$ =1 (11/98), and zero otherwise. The first two are AO outliers, the third is a LS one. The second term in the r.h.s. of (3.1) is the calendar effect, given by

$$CAL_e\ (t) = -609\ TD_1(t) + 29\ TD_2\ (t) + 366\ TD_3\ (t) -$$

(t-values): (-3.7) (.2) (2.3)

$$-\ 82\ TD_4\ (t) + 420\ TD_5\ (t) + 242\ TD_6\ (t) +$$

(-.5) (2.6) (1.5)

$$+\ 1609\ LY(t)$$

(3.1)

where TD $_i$, $i = 1,\ldots, 6$, represents the 6-variable specification, and LY the Leap Year variable. Finally, the linearized series $x_e(t)$ in (3.1) follows the ARIMA model

$$(1+.323\ B)\ \nabla\ \nabla_{12}\ x_e(t) = (1-.742\ B^{12})\ a_e(t)\ ,$$

(t-values): (3.8) (-8.5)

with $\sigma_a^e = 1040$. (On average, the series is forecasted one-month-ahead with a standard error between 2 and 3 % of the series level.)

Summary diagnostics are presented in the first row of Table1 (all tables are given at the end of the paper). The residuals can be comfortably accepted as zero-mean, uncorrelated, Normally distributed, with zero skewness and kurtosis equal to 3; they do not contain residual seasonality, nor nonlinearity (of the ARCH-type), and their signs are randomly distributed. Figure 2.1 displays the residuals; Figure 2.2, the residual ACF; Figure 2.3, the 2-year-ahead forecast function with the associated 95% confidence intervals, and Figure 2.4, the linearized series and the preadjustment component.

SEATS decomposes the linearized series and its ARIMA model into components, which also follow ARIMA-type models, namely,

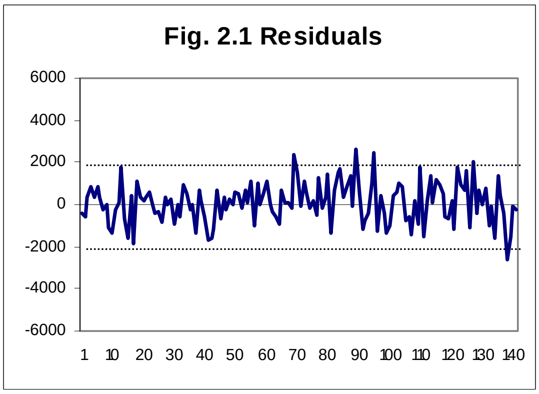
$$\nabla^2 p(t) = (1+.025\ B - .975\ B^2)\ a_p(t) = \\ = (1-.975\ B)(1+B)\ a_p(t)$$

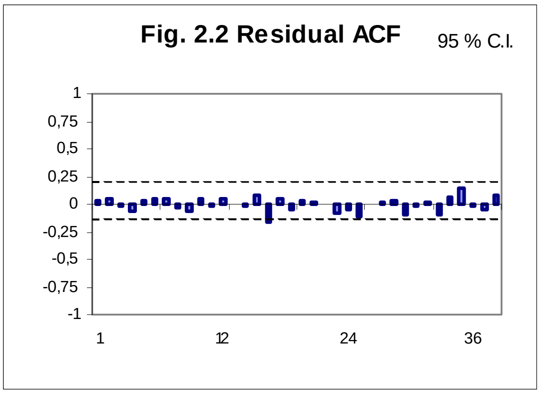
$$S\ s(t) = (1+1.359\ B + 1.756\ B^2 + 1.729\ B^3 + 1.571\ B^4 + 1.356\ B^5 + \\ + 1.072\ B^6 + .737\ B^7 + .479\ B^8 + .289\ B^9 - .023\ B^{10} - .275\ B^{11})\ a_s(t)$$

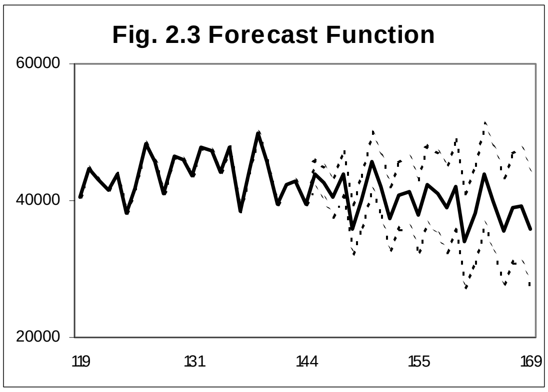
$$(1+.323\ B)\ c(t) = (1+B)\ a_c(t)$$

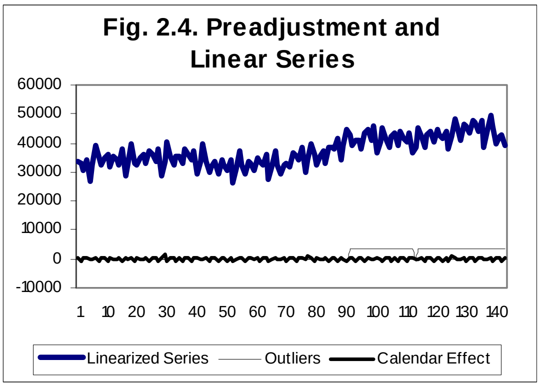
$$(1+.323\ B)\nabla^2 n(t) = (1-.978\ B - .001\ B^2 + .002\ B^3)\ a_n(t)$$

$$u(t) = \text{w.n.}$$



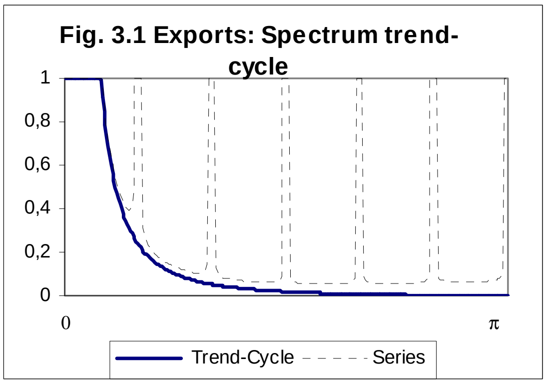


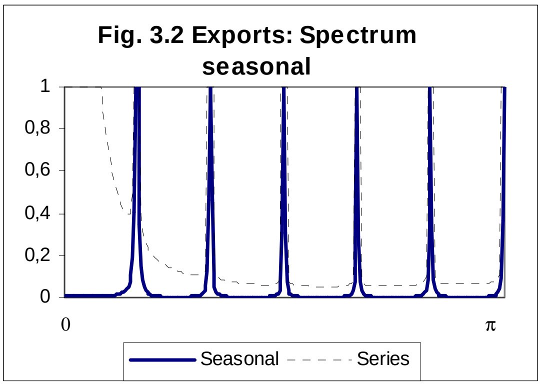


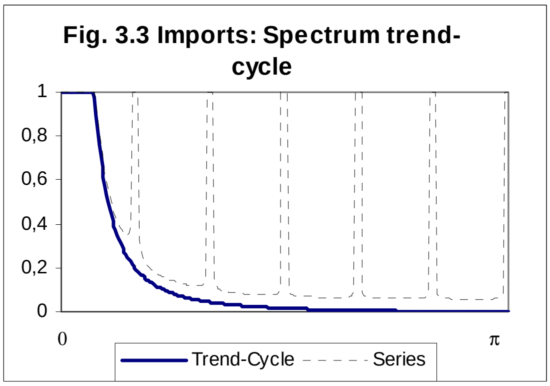


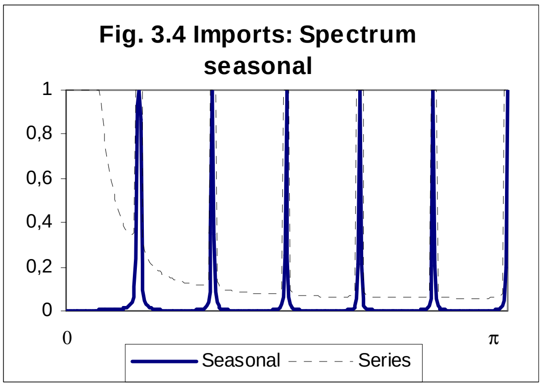
The trend-cycle follows thus an IMA (2,2) process, and factorization of the MA polynomial reveals the factor (1+B), associated with a spectral zero at the π-radians frequency. Figure 3.1 shows the monotonically decreasing trend-cycle spectrum, and the zero is implied by the so-called “canonical property”, used for identification of the trend-cycle and seasonal components in the AMB decomposition [see, for example, Maravall (1995)].

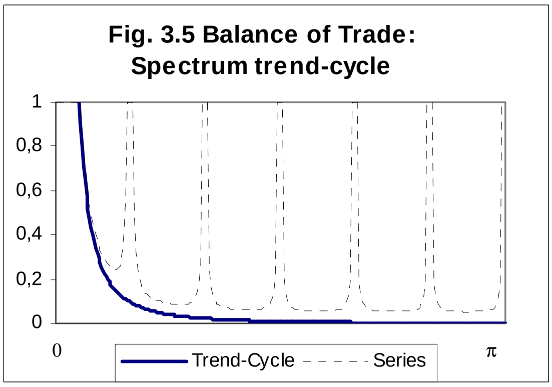
The seasonal component is a nonstationary ARMA (11,11) process, with the AR polynomial given by the annual aggregation operator ($S = 1 + B + \ldots + B^{11}$); its spectrum is given in Figure 3.2, and the spectral zero is located between the last two harmonics. The transitory component picks up the AR factor (1+.323 B), which would otherwise contaminate the trend-cycle with undesirable short-term variation, and follows a stationary ARMA (1,1) model, with the spectral zero for the π-frequency (this transitory component is also included in the SA series). The irregular component is simply white noise. The distinction between a transitory and an irregular component is due to the fact that isolating a white-noise irregular facilitates testing (see Maravall, 1987). Their behavior, however, is very similar, and for the rest of the discussion, both components will be added. The resulting component follows an ARMA (1,1) model, and its variance is the sum of the variances of c (t) and u (t).

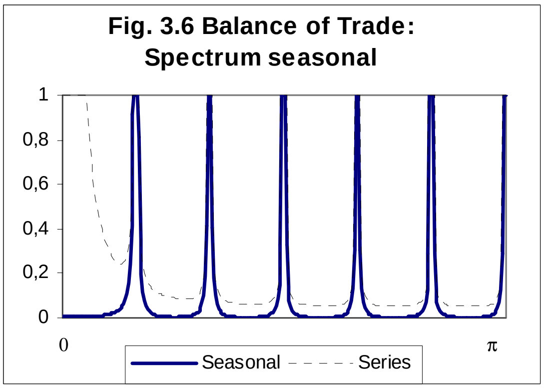












The squared gains of the two WK filters for the historical estimators of the SA series and trend-cycle are given in figures 4.1 and 4.2, and the estimators of the different series components in figures 5.1 – 5.6. Figure 5.1 reveals the relative importance of the seasonal variations. The SA series, nevertheless, contains some noise and, after its removal the trend-cycle (Figure 5.2) still exhibits important short-term variation. The seasonal component (Figure 5.3) is considerably stable, and the irregular and transitory components (Figures 5.4 and 5.6) are seen to contain highly erratic and transitory noise. Figures 6.1, 6.2 and 6.3 present the forecasts of the original series and trend-cycle, and the forecasts of the seasonal and calendar components.

Fig. 4.1 Squared gain of trend-cycle filter

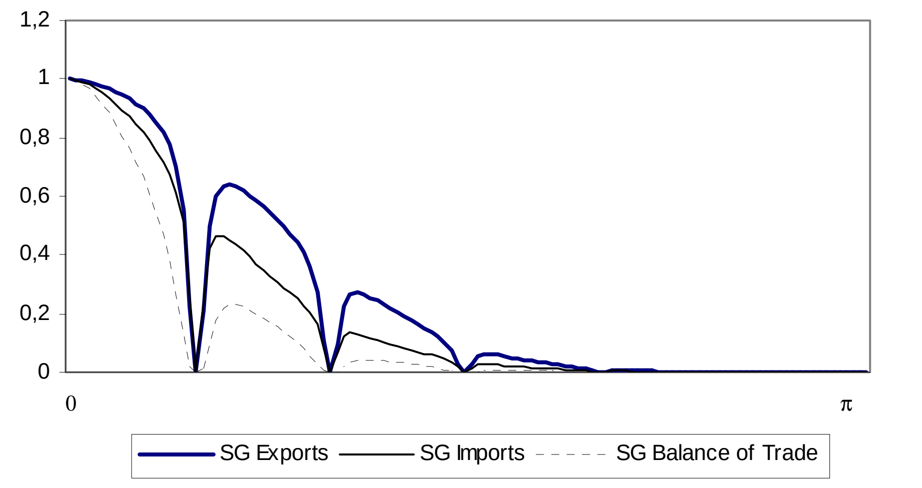


Fig. 4.2 Squared gain of SA series filter

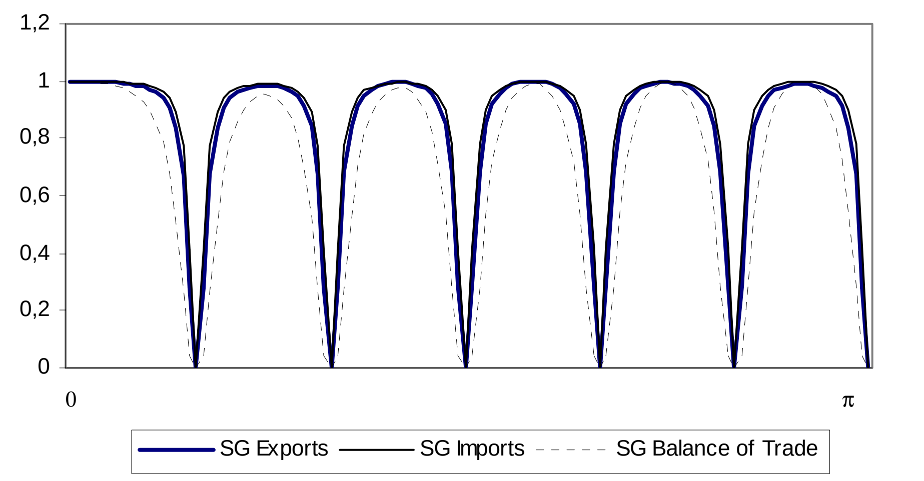


Fig. 5.1 Exports: SA series

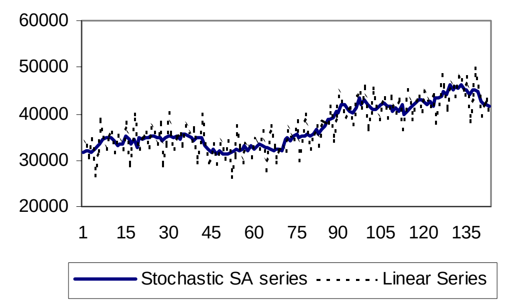


Fig. 5.2 Exports: Trend-Cycle

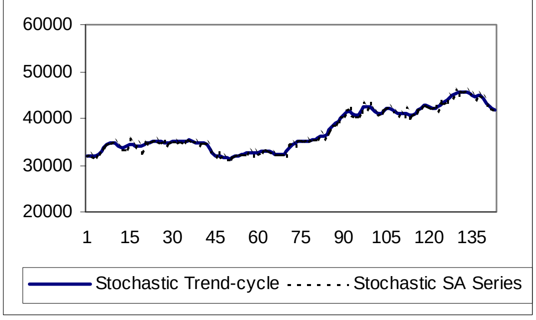


Fig. 5.3 Exports: Seasonal Component

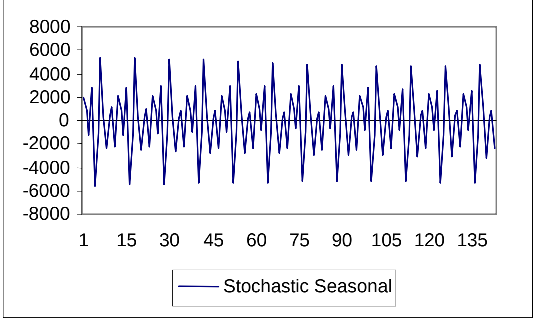


Fig. 5.4 Exports: Irregular Component

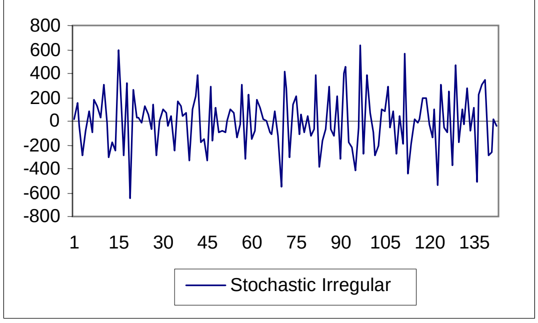


Fig. 5.5 Exports: Calendar Effect

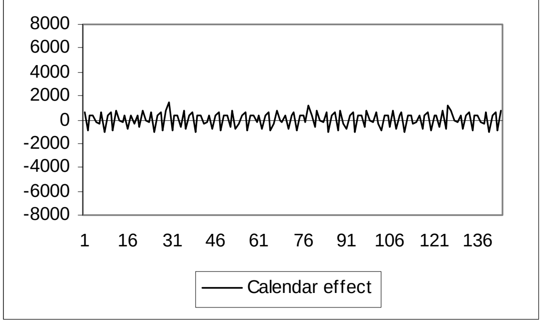


Fig. 5.6 Exports: Transitory component

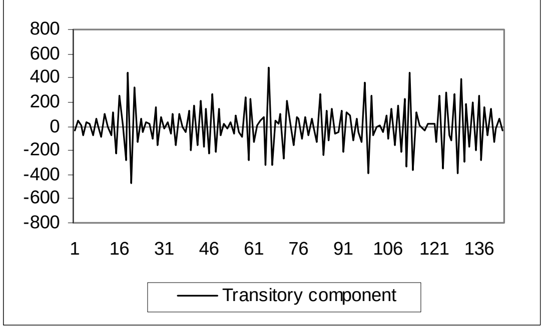


Fig. 6.1 Exports: Forecast of series and of trend

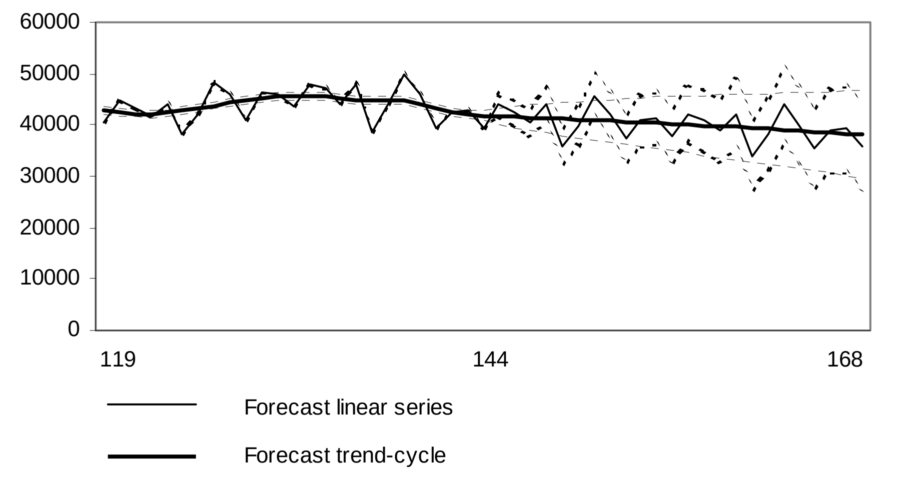


Fig. 6.2 Exports: Forecast of Seasonal

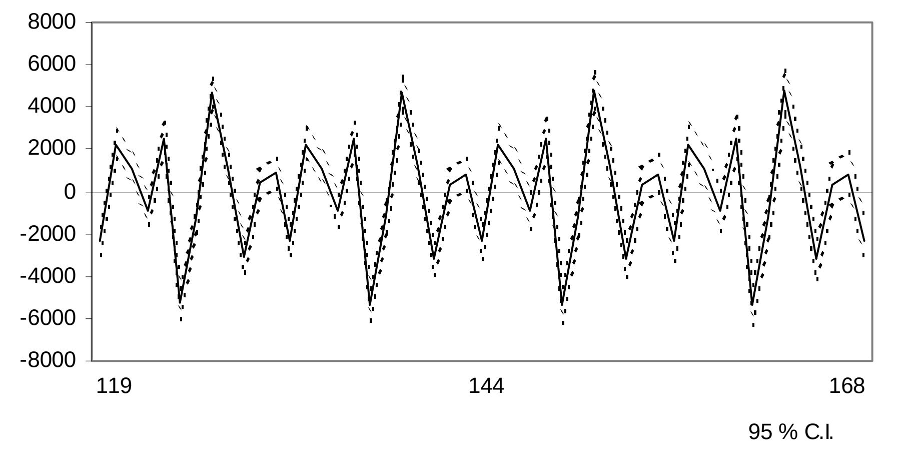
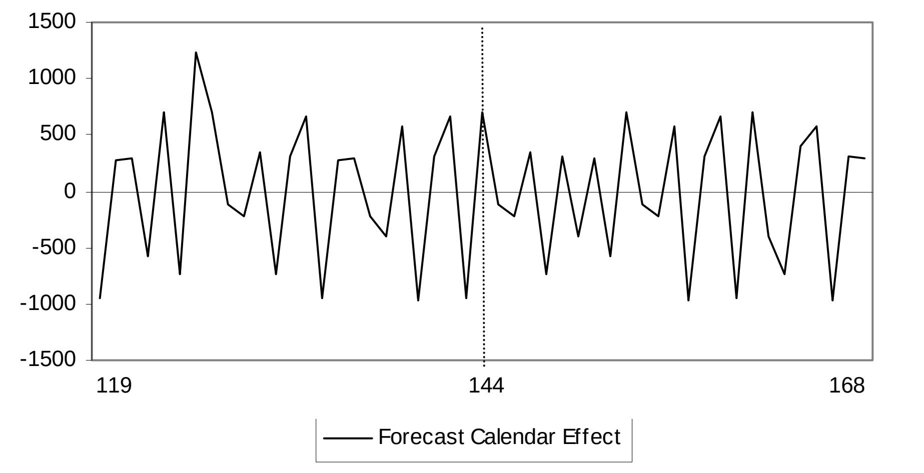


Fig. 6.3 Exports: Forecast of Calendar Effect



Some properties of the decomposition achieved are presented in the first row of tables 2 to 5. Table 2 shows that the seasonal component is relatively stable, while the trend-cycle is subject to a larger stochastic shock every period. As seen in Table 3, the estimation error of the concurrent SA series estimator is smaller than that of the trend-cycle, and the revision the estimator will suffer is also smaller. On the other hand, Table 4 shows that the SA series estimator will converge much slower to the final estimator. Table 5 indicates that the series contains highly significant seasonality, which shows up not only for historical estimation, but also in preliminary estimation and forecasting. Finally, Figures 7.1 and 7.2 exhibit the original series y (t), the final seasonally adjusted series (with the stochastic seasonal estimator in SEATS and the calendar effect estimated by TRAMO removed from the observed series), and the final trend-cycle component, which includes the SEATS stochastic trend-cycle and the LS outlier estimated by TRAMO.

Fig. 7.1 Exports: Final SA Series

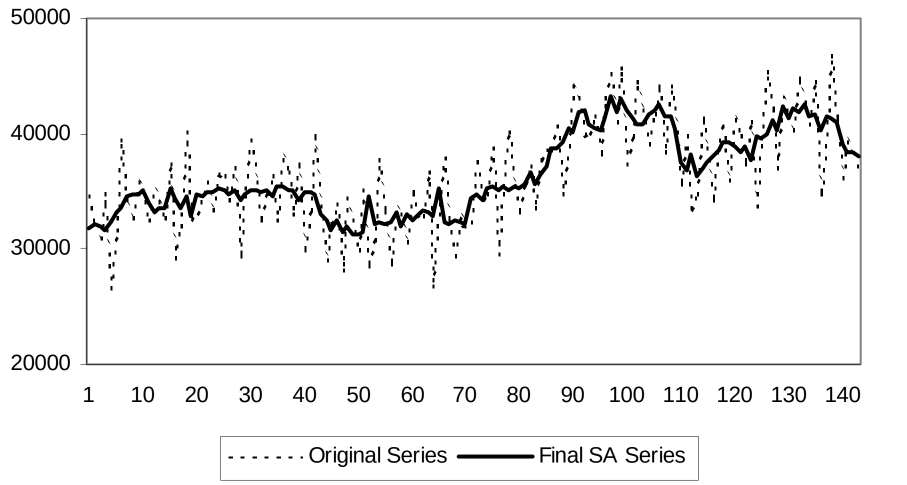
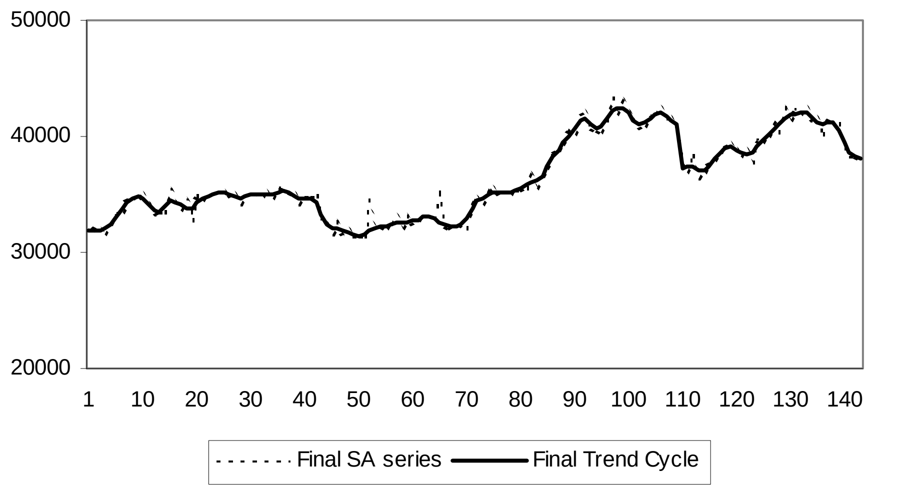


Fig. 7.2 Exports: Final Trend-Cycle



3.3 Import Series (I)

Starting with RSA = 8, the LY effect is clearly not significant. Moreover, the 6-variable TD specification is less significant than for the exports series, and the one-variable specification provides a better fit. The option RSA = 4, that yields the Airline model, seems rather satisfactory, except for a marginally significant EE, which is judged spurious. Further, an AO outlier is detected towards the end of the series, with a t-value equal to the threshold level set by default by the program (for 144 observations, equal to $t = 3.235$). This borderline significance of an outlier near the end of the series often causes model instability in AMI; in the present case, the instability concerns mostly the choice of a (1,0,1) or a (0,1,1) structure for the seasonal part in the multiplicative ARIMA model. It seem thus a good case for applying the value RSA = 1, adding the test for the one-variable TD specification (ITRAD = -1), and modelling the level. The model obtained can be expressed as

$$I(t) = OUT_i(t) + CAL_i(t) + x_i(t) \quad ,$$

where, using similar notation as in the exports series case,

$$OUT_i(t) = -4121\, d_1(t) - 4485\, \frac{1}{1-B}\, d_2(t) + 3067\, d_3(t)$$

(t-values): (-4.5) (-4.3) (3.2)

with $d_1(20) = 1$ (4/91), $d_2(102) = 1$ (2/98), $d_3(136) = 1$ (12/00), and $d_1(t) = d_2(t) = d_3(t) = 0$ otherwise. The first and third outliers are AO's; the second is a negative LS. The calendar effect is given by

$$CAL_i(t) = 218\, TD(t) \quad ,$$

(t-value): (9.4)

with TD denoting the one-variable specification, and the ARIMA model for the linearized series is equal to

$$\nabla \nabla_{12}\, x_i(t) = (1 - .330\, B)\,(1 - .790\, B^{12})\, a_i(t) \quad , \qquad (3.2)$$

(t-values): (-3.9) (-8.2)

with $\sigma_a^i = 1140$, which implies that the SE of the 1-period-ahead forecast is in the order of 3 - 4% of the level of the series. The summary diagnostics are contained in the second row of Table1; again, all tests are comfortably passed. Figures 8.1 to 8.4 display the residuals, residual ACF, the series forecast function and the linearized series and preadjustment component.

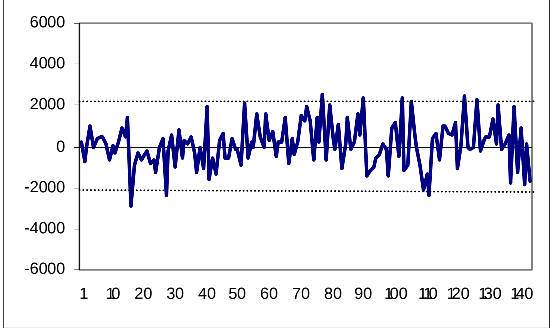


Fig.8.1 Residuals

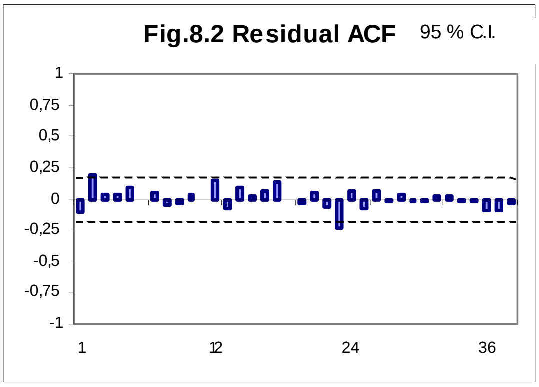


Fig.8.2 Residual ACF

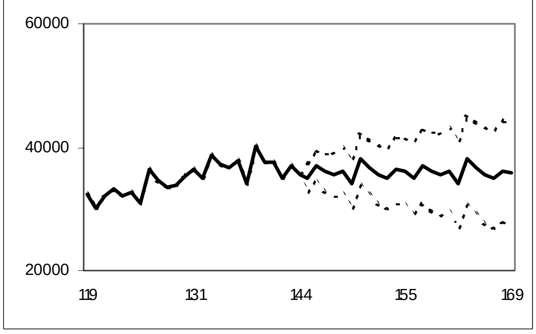


Fig. 8.3 Forecast Function

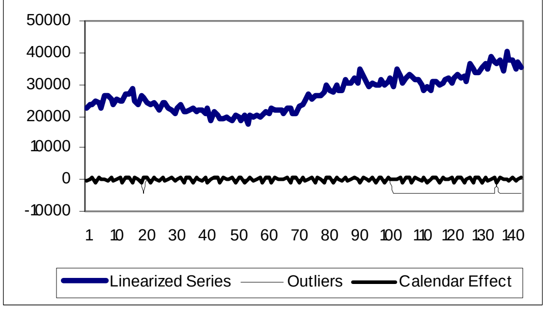


Fig.8.4 Preadjustment and Linear Series

SEATS decomposes model (3.2) into trend-cycle, seasonal and irregular components; no transitory component is now present. The models for the trend-cycle and seasonal components are

$$\nabla^2 p(t) = (1 + .020\ B - .980\ B^2)\ a_p(t) =$$
$$= (1 - .980\ B)(1 + B)\ a_p(t) \qquad ;$$

$$S\ s(t) = (1 + 1.5\ B + 1.63\ B^2 + 1.592\ B^3 + 1.409\ B^4 + 1.16\ B^5 +$$
$$+ .88\ B^6 + .594\ B^7 + .348\ B^8 + .113\ B^9 - .038\ B^{10} - .346\ B^{11})\ a_s(t) \quad .$$

The irregular component is white noise, and the model for the SA series is given by

$$\nabla^2 n(t) = (1 - 1.312\ B + .325\ B^2)\ a_n(t) =$$
$$= (1 - .332\ B)(1 - .980\ B)\ a_n(t) \qquad .$$

The trend-cycle and SA series follow IMA (2,2) models; the first one displays a spectral zero for the π-frequency, while the spectral zero for the seasonal component occurs for a frequency between the last two harmonics. The spectra of the components and the squared gains of the WK filters are also shown in the set of figures 3 and 4. Some characteristics of the decomposition (SD of the components innovation, SE of the estimation error, size and convergence of revisions in the concurrent estimator, and significance of seasonality) are given in the second row of Tables 2 to 5. Figures 9.1 to 9.5 present the stochastic decomposition of the linearized series; figures 10.1 to 10.3, the forecasts of the trend-cycle, seasonal and calendar components, and figures 11.1 and 11.2 the trend-cycle component and SA series in the final decomposition of the original series $y(t)$.

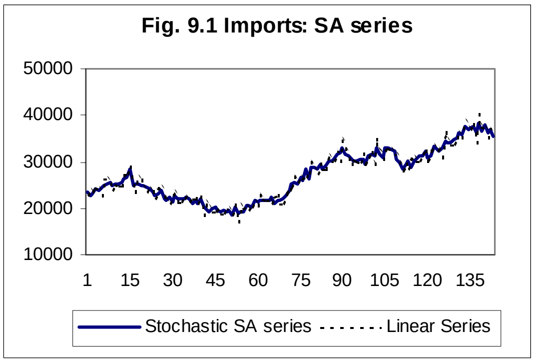


Fig. 9.1 Imports: SA series
50000
40000
30000
20000
10000
1 15 30 45 60 75 90 105 120 135
Stochastic SA series
Linear Series

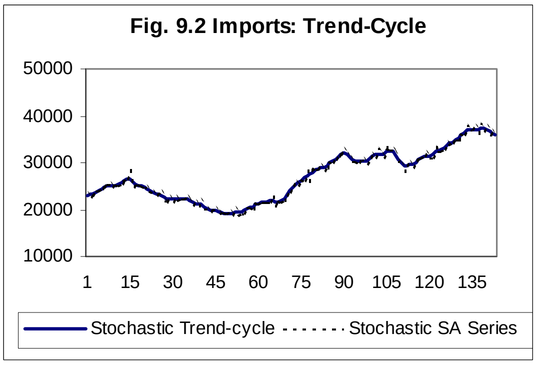


Fig. 9.2 Imports: Trend-Cycle
50000
40000
30000
20000
10000
1 15 30 45 60 75 90 105 120 135
Stochastic Trend-cycle
Stochastic SA Series

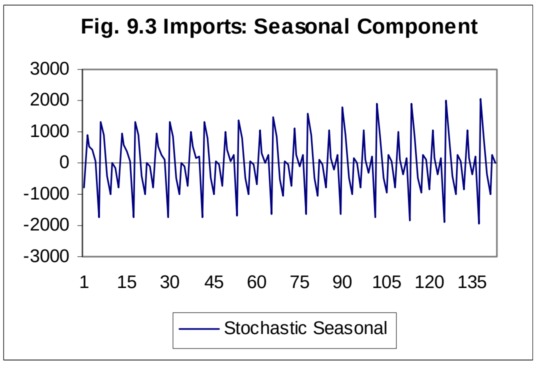


Fig. 9.3 Imports: Seasonal Component
3000
2000
1000
0
-1000
-2000
-3000
1 15 30 45 60 75 90 105 120 135
Stochastic Seasonal

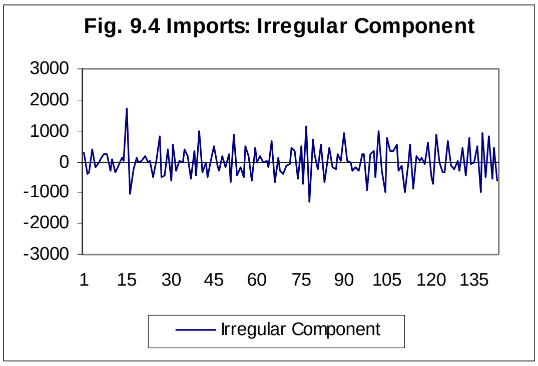


Fig. 9.4 Imports: Irregular Component
3000
2000
1000
0
-1000
-2000
-3000
1 15 30 45 60 75 90 105 120 135
Irregular Component

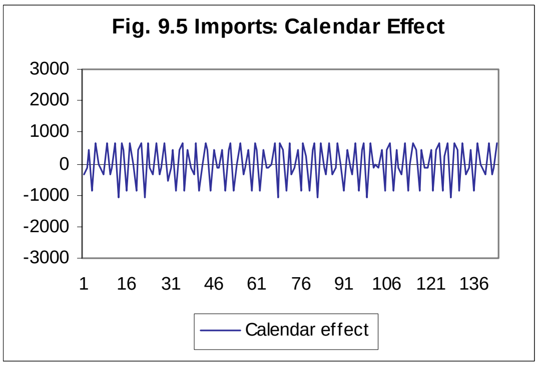


Fig. 9.5 Imports: Calendar Effect
3000
2000
1000
0
-1000
-2000
-3000
1 16 31 46 61 76 91 106 121 136
Calendar effect

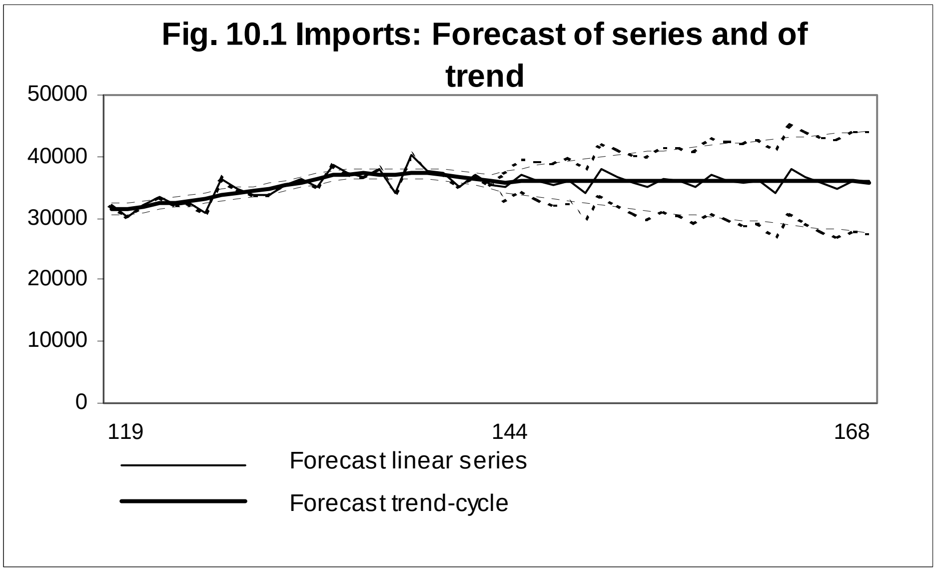


Fig. 10.1 Imports: Forecast of series and of trend
50000
40000
30000
20000
10000
0
119
144
168
Forecast linear series
Forecast trend-cycle

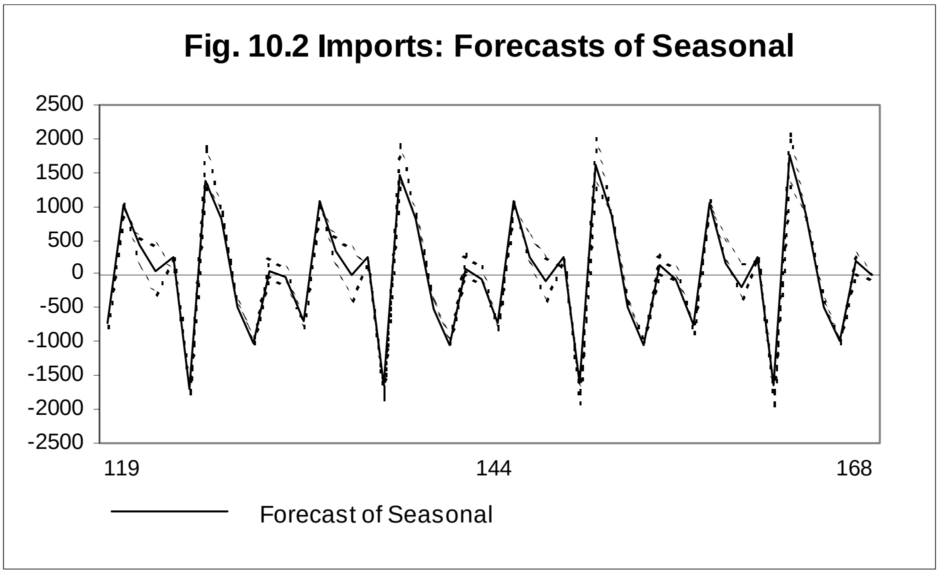


Fig. 10.2 Imports: Forecasts of Seasonal
2500
2000
1500
1000
500
0
-500
-1000
-1500
-2000
-2500
119
144
168
Forecast of Seasonal

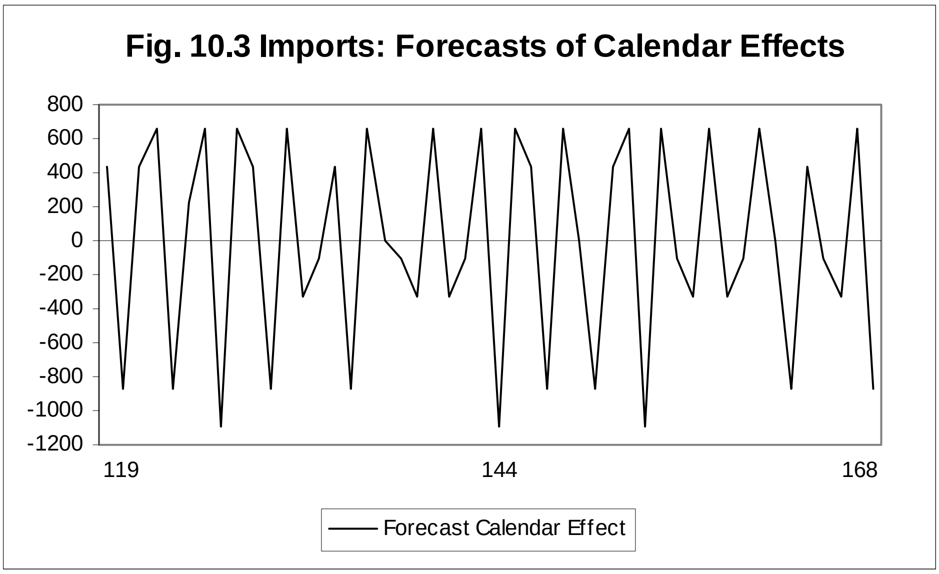


Fig. 10.3 Imports: Forecasts of Calendar Effects
800
600
400
200
0
-200
-400
-600
-800
-1000
-1200
119
144
168
Forecast Calendar Effect

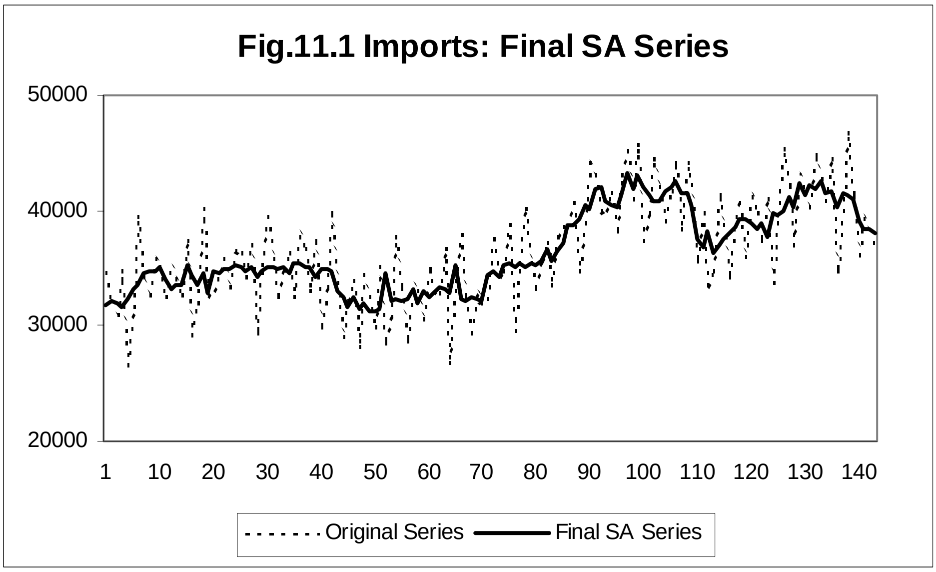


Fig.11.1 Imports: Final SA Series
50000
40000
30000
20000
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140
Original Series
Final SA Series

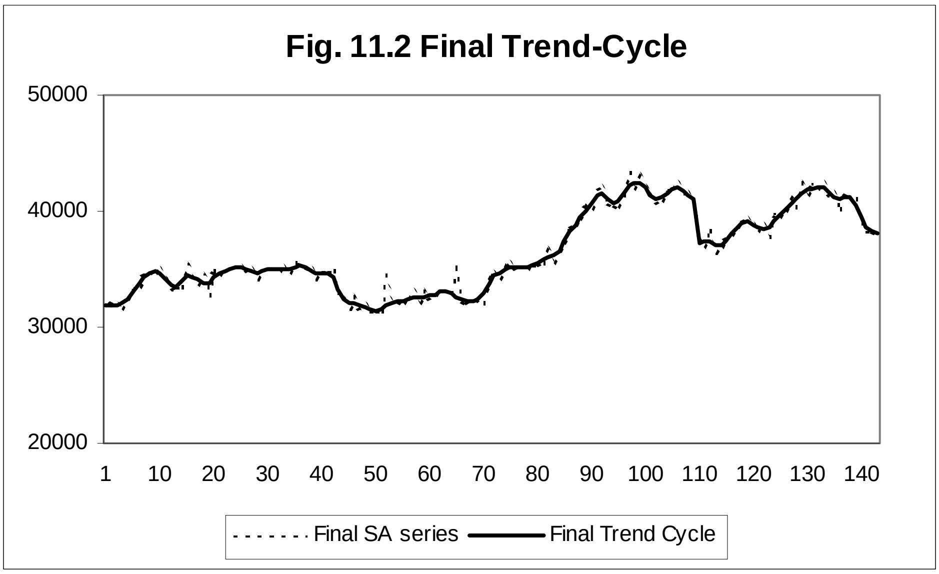


Fig. 11.2 Final Trend-Cycle
50000
40000
30000
20000
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140
Final SA series
Final Trend Cycle

3.4 Balance of Trade Series (BT)

Using RSA = 8, the LY effect is not significant; using next RSA = 6, the results are satisfactory. The model obtained is given by

$$BT(t) = OUT_b(t) + CAL_b(t) + x_b(t) \quad ,$$

where

$$OUT_b(t) = 3499 \frac{1}{1-B} d_1(t) - 4022\, d_2(t) \quad ,$$

(t-values): (3.5) (-4.3)

with $d_1(92) = 1$ (4/97), $d_2(114) = 1$ (2/99), and $d_1(t) = d_2(t) = 0$ otherwise,

$$CAL_b(t) = -777\, TD_1(t) - 302\, TD_2(t) + 197\, TD_3(t) - 242\, TD_4(t) +$$

(t-value): (-4.1) (-1.7) (1.1) (-1.3)

$$+126\, TD_5(t) + 874\, TD_6(t) \quad ,$$

(.7) (4.8)

$$\nabla\ \nabla_{12}\ x_b(t) = (1 - .452\, B)\ (1 - .533\, B^{12})\ a_b(t) \quad , \qquad (3.3)$$

(t-values): (-5.4) (-5.5)

with $\sigma_a^b = 1250$ (which represents, on average, a SE of the 1-period ahead forecast of about 12% of the level of the series). The third row of Table1 contains the summary diagnostics of the fitting, and Figures 12.1 to 12.4 exhibit the residuals, the residual ACF, the linearized series, the preadjustment component, and the series forecasts.

Model (3.3) decomposes into

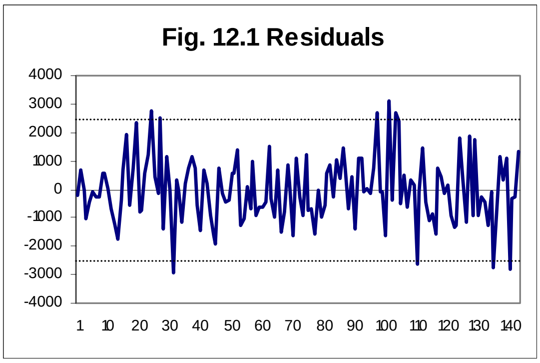
$$\nabla^2 p(t) = (1 + .051\, B - .949\, B^2)\ a_p(t) =$$
$$= (1 - .949\, B)(1 + B)\ a_p(t) \quad ;$$

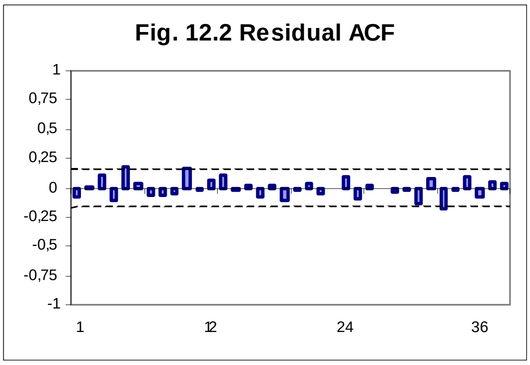
$$S\, s(t) = (1 + 1.346\, B + 1.373\, B^2 + 1.273\, B^3 + 1.065\, B^4 + .819\, B^5 +$$
$$+ .562\, B^6 + .315\, B^7 + .11\, B^8 - .079\, B^9 - .202\, B^{10} - .472\, B^{11})\ a_s(t) \quad ;$$

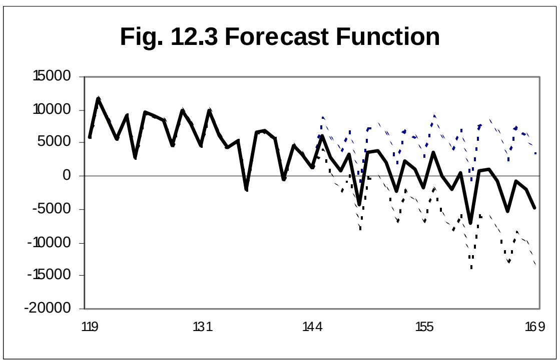
with u (t) a white noise variable, and the SA series follows the model

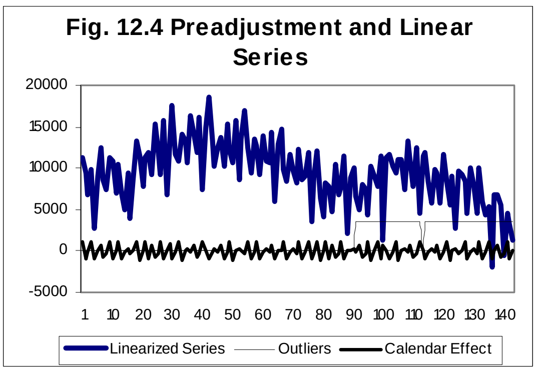
$$\nabla^2 n(t) = (1-1.413\, B + .441\, B^2)\ a_n(t) =$$
$$= (1-.464\, B)(1-.949\, B)\ a_n(t) \quad .$$

The spectral decomposition and the squared gains of the WK filters are given in the set of figures 3 and 4. As in the previous cases, some features of the decomposition are given in Tables 2 to 5 (third column). Figures 13.1 to 13.5 present the estimators of the stochastic components, figures 14.1 to 14.3, the forecasts of the components, and figures 15.1 and 15.2 the final adjustment, once the TRAMO and SEATS results are put together.









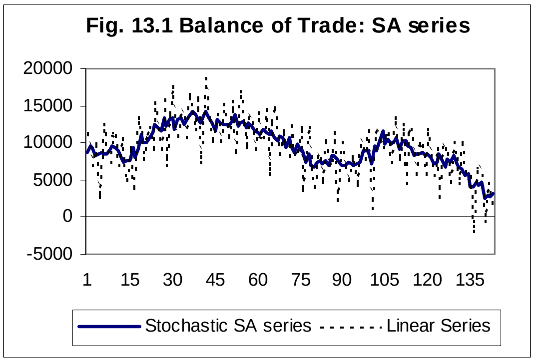


Fig. 13.1 Balance of Trade: SA series
20000
15000
10000
5000
0
-5000
1 15 30 45 60 75 90 105 120 135
Stochastic SA series
Linear Series

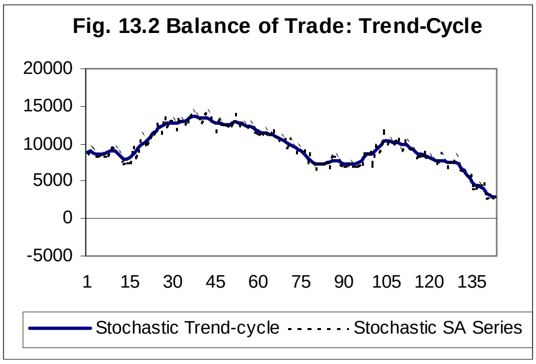


Fig. 13.2 Balance of Trade: Trend-Cycle
20000
15000
10000
5000
0
-5000
1 15 30 45 60 75 90 105 120 135
Stochastic Trend-cycle
Stochastic SA Series

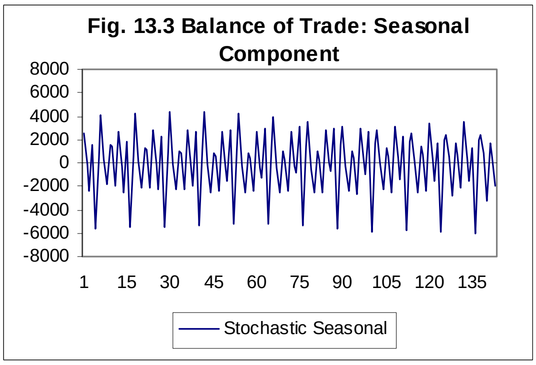


Fig. 13.3 Balance of Trade: Seasonal Component
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
1 15 30 45 60 75 90 105 120 135
Stochastic Seasonal

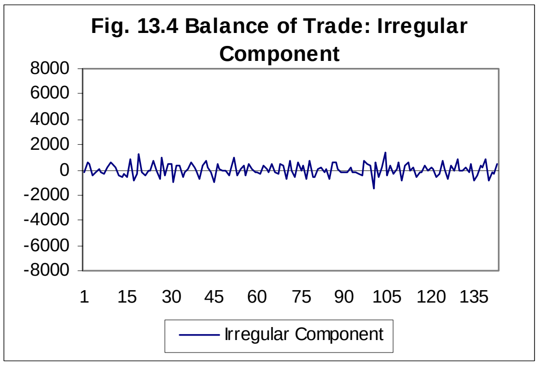


Fig. 13.4 Balance of Trade: Irregular Component
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
1 15 30 45 60 75 90 105 120 135
Irregular Component

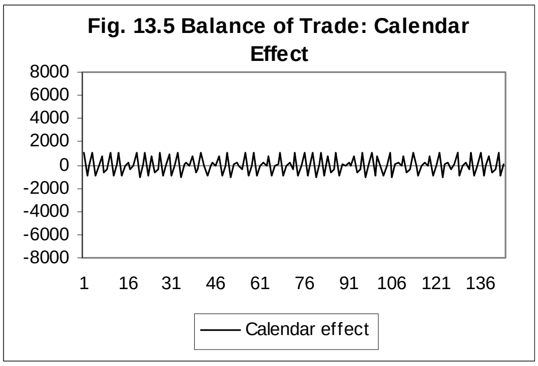


Fig. 13.5 Balance of Trade: Calendar Effect
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
1 16 31 46 61 76 91 106 121 136
Calendar effect

Fig. 14.1 Balance of Trade: Forecast of series and of trend

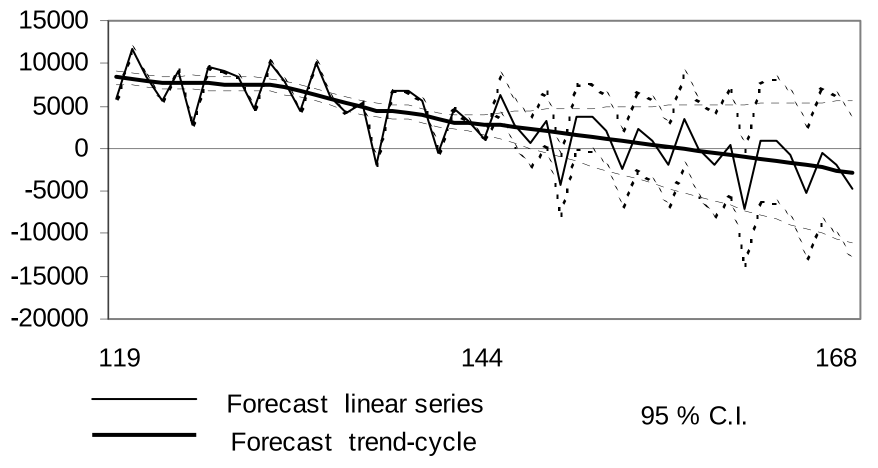


Fig. 14.2 Balance of Trade: Forecast of Seasonal

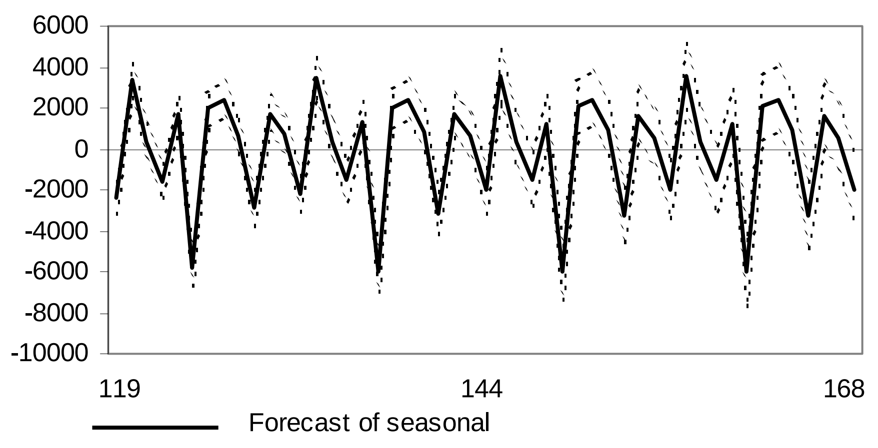
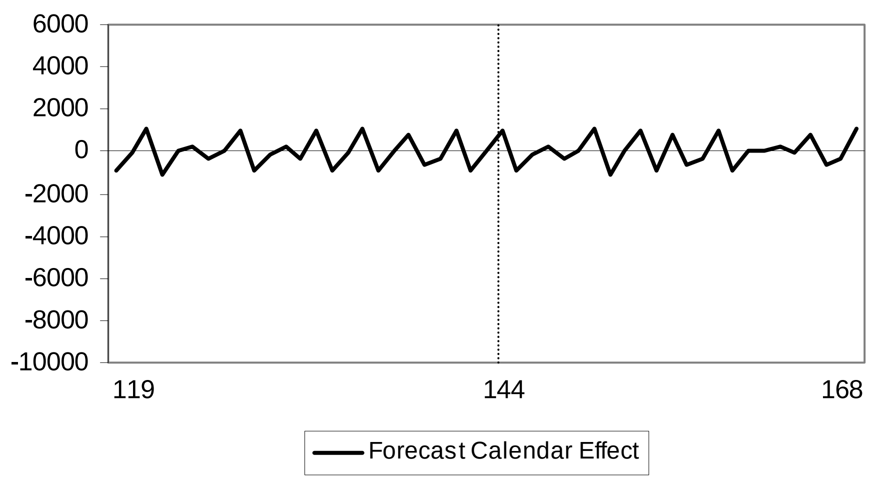


Fig. 14.3 Balance of Trade: Calendar Effect



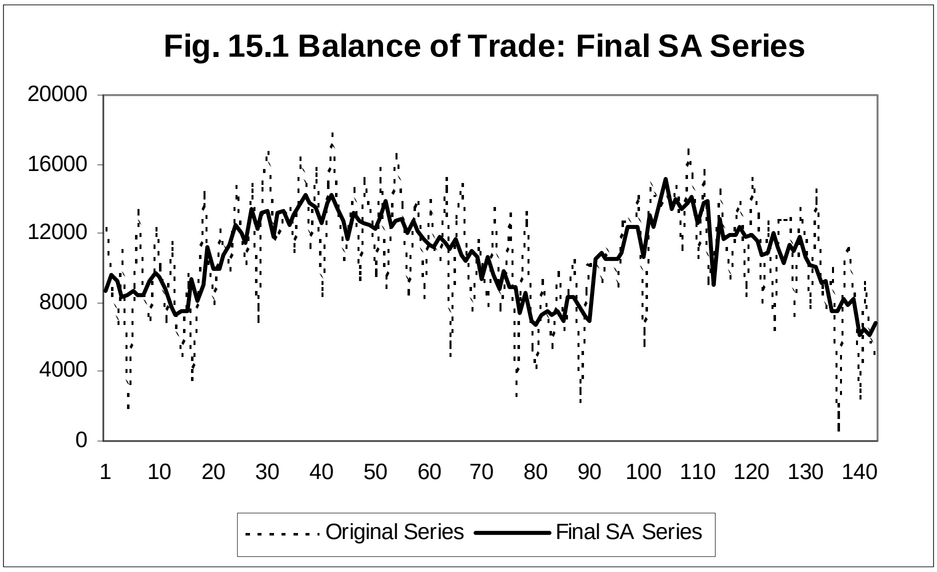


Fig. 15.1 Balance of Trade: Final SA Series
20000
16000
12000
8000
4000
0
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140
Original Series
Final SA Series

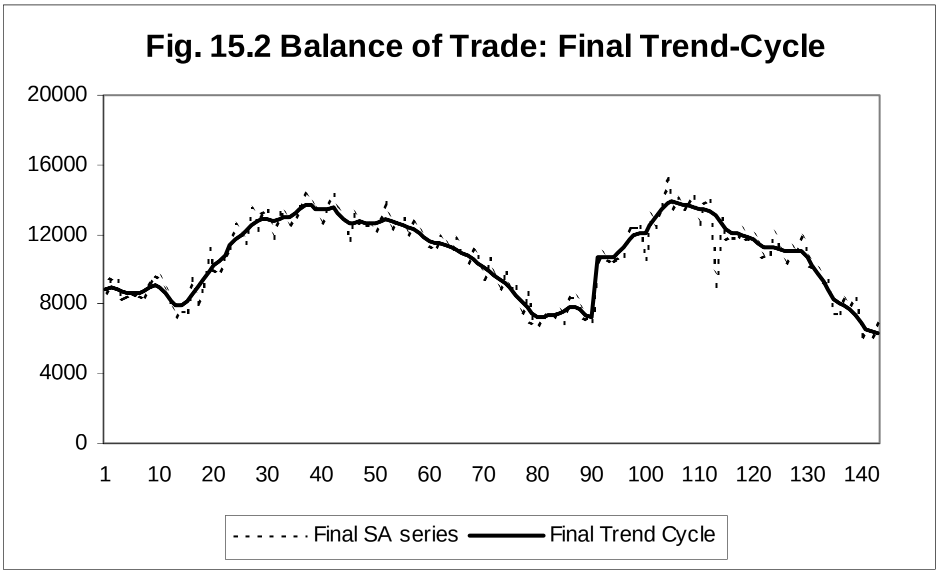


Fig. 15.2 Balance of Trade: Final Trend-Cycle
20000
16000
12000
8000
4000
0
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140
Final SA series
Final Trend Cycle

3.5 Some Remarks on the Models

Concerning preadjustment, the models for exports and imports contain 3 outliers (two AO and one LS outlier each) and the model for the balance of trade series contains only two (one AO, one LS outlier). The number of outlier is not excessive, and none of them is exceedingly large. The 8 outliers are displayed in Figure 16; it is noteworthy that none of the outlier dates is shared by two of the series. For the calendar effect, three different specifications are used. The effect is highly significant in all three cases.

As for the stochastic series, rewriting the ARIMA model for the export series as

$$\nabla \; \nabla_{12} \; x_e(t) = (1 + .323\ B)^{-1} \; (1 - .742\ B^{12}) \; a_e(t)$$
$$= (1 - .323\ B + .104\ B^2 - .037\ B^3 + ...) \; (1 - .742\ B^{12}) \; a_e(t) \qquad ,$$

it is seen to be relatively close to an Airline-type model. Thus the models for the three series are similar and the diagnostics, for the three cases, are excellent.

The model for the SA export series is an ARIMA (1,2,3) model that, after simple manipulation, is seem to be close to the IMA (2,2) model

$$\nabla^2 \; n(t) = (1 - .323\ B) \; (1 - .978\ B) \; a_n(t) \qquad .$$

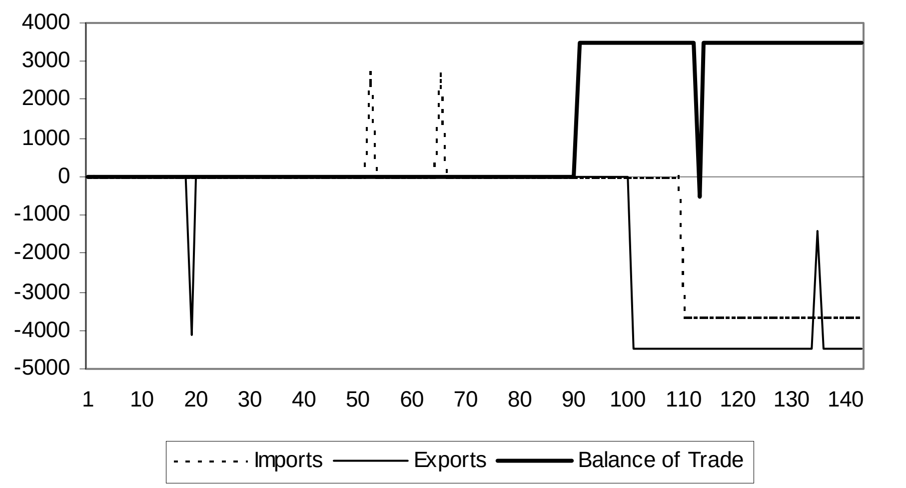
Therefore, the three models for the SA series contain an MA root very close to B = 1. Canceling this root with one of the differences, the three models become an “IMA (1,1) plus drift” model. Given that the remaining MA parameter is relatively small, the three SA series are not far from the popular “random-walk plus drift” model.

Comparing the standard deviation of the component innovations, Table 2 shows that the balance of trade series contains the most stable trend and the most unstable seasonal component, while the imports series presents the most stable seasonal component. As for the size of the estimation and revision errors for the SA series and trend-cycle, both errors are largest for the balance of trade series, and smallest for the exports series, although convergence of the preliminary estimator for the exports and imports series is relatively slow. Finally, seasonality is clearly significant for the three series, in particular for the exports series (followed by the imports one).

It is often the case that identification of the ARIMA model does not yield a clear-cut unique solution, and that more than one model may seem appropriate. When the decomposition of the series is a relevant concern, comparison of the SEATS results may help in the selection. As an example, it was already mentioned that the (1,1,0)

$(0,1,1)_{12}$ model identified for the exports series is close to a $(0,1,1)\ (0,1,1)_{12}$ alternative model. The results of the latter model (with the same outliers and calendar effect) are given in the fourth row of the tables. Table 2 shows that the trend-cycle is equally stable for the two models, and that the seasonal component is more stable for the case of the first model. Table 3 indicates that the alternative model implies larger estimation errors and larger revisions for, both, the SA series and the trend-cycle. Although the differences are not drastic, they all point to the same conclusion: the $(1,1,0)\ (0,1,1)_{12}$ model obtained with RSA = 8 outperforms the alternative model. (Adding the transitory component improves the performance of the trend-cycle and SA series).

Fig. 16 Total Outlier Effects



4. DIRECT VERSUS INDIRECT ADJUSTMENT.

Direct adjustment of the three series with TRAMO-SEATS run in a (quasi) automatic mode yields sensible decompositions in the three cases. Not having any a priori information on the series (i.e., knowing only the numerical values), one could feel comfortable accepting the results. But there is, of course, a very important relationship between the series: by definition,

$$BT(t) = E(t) - I(t) \quad .$$

Thus another obvious way to obtain the SA series for BT is an indirect adjustment, whereby the SA imports series are subtracted from the SA exports series. If the models for the series were “true”, Geweke (1978) could provide a rationale for indirect adjustment. Indirect adjustment has also the important virtue of preserving identities. On the other hand, it is a delicate question to decide at what measurement level disaggregation starts. Perhaps more relevantly, it is a well-known empirical fact that often aggregate series display a more regular behavior (ultimately, in accordance with the Central Limit Theorem). Further, ad-hoc enforcement of the constraints may affect revisions, to the point of inducing non-convergence.

The SA series and trend-cycle obtained with direct and indirect adjustment of the balance of trade series are displayed in Figures 17.1 and 17.2. For both components, the difference between direct and indirect adjustment is large. In both cases, the mean of the difference can be assumed zero, and the two standard deviations are very close, in the order of 4% of the average series level. The direct estimators, most notably for the trend-cycle case, are considerably smoother.

The discrepancy between direct and indirect estimator can be due to differences in the filters applied to the stochastic series, and to differences in the preadjustment components. Concerning the difference in filters, if the series are adjusted without preadjustment, using in the three cases the Airline model, estimating the θ_1 and θ_2 parameters, direct and indirect adjustment yield the results of Figures 18.1 and 18.2. The differences between direct and indirect adjustment are now seen to be much smaller. As before, for the SA series and trend-cycle, the differences can be assumed zero-mean and equal variance, with the standard error representing about 1% of the average level of the series. As could be expected, however, the diagnostics of the “pure” Airline-model fit are, for the three series, unacceptable.

Preadjustment has consisted, for the three series, of two types of corrections: one, for trading day effect; the other, for outliers. As seen in Figure 19, outlier correction is the source of the major discrepancies. Keeping in mind the relationship between the three variables, the model-based procedure could be modified in order to present better aggregation properties. For example, the 6-variable TD specification could be used for the three variables; the December 2000 outlier in the imports series, that causes model instability and is borderline significant, could be ignored; further, it is easily seen that, for the imports series, another borderline outlier is a LS for 10/98, close to the 11/98 LS in the import series, so that a ramp outlier for the two months in both series is highly significant and improves results. Be that as it may, it is nevertheless the case that, in general terms, the better we adjust a series within a univariate framework, the more likely it is that the aggregation properties of the decomposition deteriorate (i.e., that the difference between direct and indirect adjustment increases).

In particular, preadjustment is based on tests for the significance of several variables (for example, Easter, trading day, outliers, regression or intervention variables, …). In so far as these tests are necessareally 0-1 decisions (i.e., if some statistics is smaller than a critical value, the variable is droped, otherwise, included,) they introduce a nonlinear element in model building that may strongly affect aggregation.

Fig. 17.1 Direct and indirect adjustment: Reg ARIMA

Fig. 17.2. Direct and indirect trend-cycle: Reg ARIMA

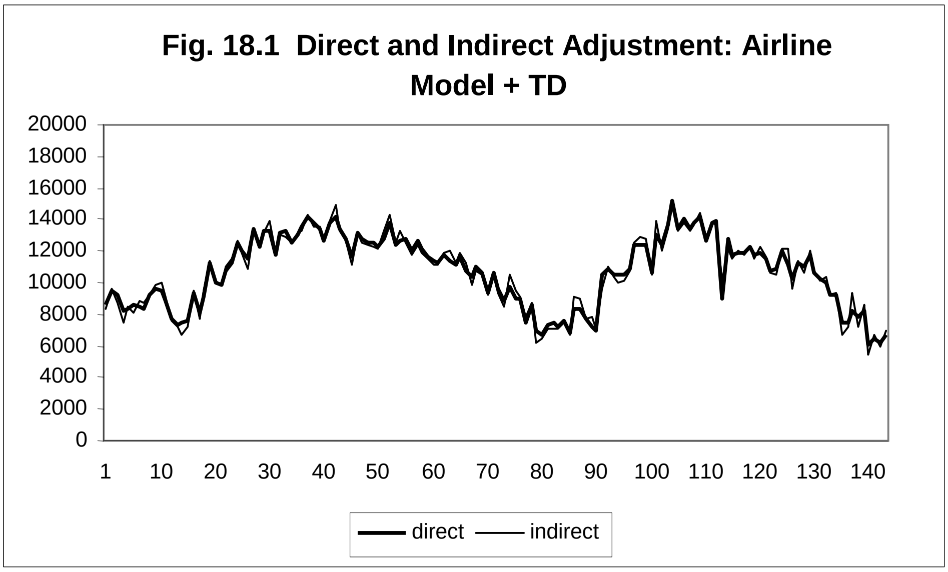


Fig. 18.1 Direct and Indirect Adjustment: Airline Model + TD
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140
direct
indirect

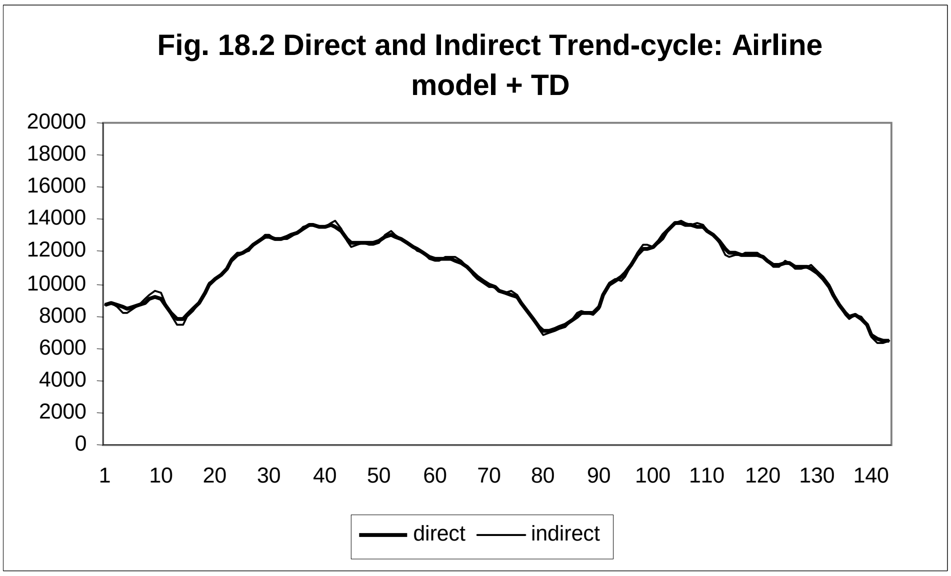


Fig. 18.2 Direct and Indirect Trend-cycle: Airline model + TD
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140
direct
indirect

The aggregation problem, however, is not simply the product of an imperfect methodology. It is also the result of a definitional ambiguity. If the series $X(t)$ is the aggregate of $x_1(t)$ and $x_2(t)$, as in $X(t) = x_1(t) + x_2(t)$, what is meant by "seasonally adjusted $X(t)$"? Is it the sum of the SA components? Is it the best direct adjustment of the aggregate series? We have seen how, under a "best-univariate-model" strategy, direct and indirect adjustment may produce relatively important differences. There are more fundamental reasons, even at the most basic conceptual level, that may imply different results. I proceed to illustrate the conceptual difficulty with some very simple examples.

Assume $x_1(t)$ and $x_2(t)$ are two series observed every semester, that follow the models

$$x_1(t) = a(t) + a(t-1) \quad ,$$
$$x_2(t) = b(t) - b(t-1) \quad ,$$

with $a(t)$ and $b(t)$ denoting two uncorrelated series of w.n. innovations, with variances $V_a = V_b = 1$. The spectra of the two series are equal to

$$g_1(\omega) = \frac{1}{\pi}(1 + \cos\omega) \quad ,$$
$$g_2(\omega) = \frac{1}{\pi}(1 - \cos\omega) \quad ,$$

for ω measured in radians and $-\pi \le \omega \le \pi$. The first spectrum presents a peak for $\omega = 0$, and decreases monotonically until it becomes zero for $\omega = \pi$. Thus x_1 can be seen as a pure trend. The second spectrum presents a peak for $\omega = \pi$ (the once-a-year seasonal frequency) and, moving to the left, decreases monotonically until it becomes zero for $\omega = 0$. Thus x_2 can be seen as a pure seasonal component. As a consequence, it is evident that the SA x_1 series is $x_1(t)$ itself (there is no seasonality to remove). On the other hand, the SA x_2 series is always zero, given that $x_2(t)$ is a pure seasonal component. Therefore, the sum of the two SA series is equal to $x_1(t)$.

Moving to the aggregate series $X(t)$, we just concluded that correct indirect adjustment would yield $x_1(t)$ as the SA series. Does this make sense? Given that $g_1(\omega) + g_2(\omega) = \text{constant}$, $X(t)$ is a white-noise series. As such, it contains no seasonality, and the SA $X(t)$ series should simply be the series itself ($= x_1(t) + x_2(t)$). Thus direct adjustment would seem to be the proper answer and, by construction, indirect adjustment would give a different one.

The second example is even simpler. Assume that a country, divided in 55 regions, hold an important fair every April, and that this fair has a significant impact in the economy of the country, mostly concentrated in the region where it is held. Further, assume the regions alternate hosting the fair (in an alphabetical, random, or whatever manner). The SA regional series would not remove the peaks due to the fair, because they are not seasonal. The national-level series would show a peak every April, that should be removed when seasonally adjusting. As before, direct adjustment would seem to provide the correctly adjusted aggregate, which would be different from the one obtained with an indirect procedure.

In summary, it is not clear to me that seasonal adjustment (or trend-cycle estimation) should preserve aggregation or balancing constraints among the original series: there are conceptual and methodological reasons that could justify departures. As a consequence, given the present state of the art, my preferred solution (possibly, Politically Uncorrect) would be the following:

1) Do as best as we can at each level of aggregation.

2) Insert a footnote in the table that says: "Because the component seasonalities may interact, and because seasonal adjustment is a non-linear transformation of the original series, aggregation constraints may not be preserved."

Clearly, interaction between series should be better handled in a multivariate framework. But reliable and efficient multivariate models, that capture series interactions properly, including seasonal ones, are not in the horizon of real-world applications. Besides, because full disaggregation is, in general, impossible, the aggregation problem will always be present, at one or another level.

Table 1. ARIMA fit: Summary diagnostics

	$t(\mu_a)$	$Q_a(24)$	N_a	t_a (skew)	t_a (kur)	$Q_{as}(2)$	$Q_{a2}(24)$	t_a (runs)
Exports	-.13	11.93	.90	-.26	-.92	1.92	20.18	.91
Imports	.45	27.76	.22	.35	-.31	3.89	22.26	-.53
Balance	-.83	21.86	.07	.02	.26	2.04	20.00	.36
Exports (alternative)	-.19	14.86	1.23	-.09	-1.12	2.13	21.65	1.64
CV (95%)	$\lvert t \rvert < 2$	< 34	< 6	$\lvert t \rvert < 2$	$\lvert t \rvert < 2$	< 6	< 34	$\lvert t \rvert < 2$

1) $t(\mu_a)$ is the t-value associated with H_0 : the mean of the residuals is zero.
2) $Q_a(24)$ is the "portmanteau" Ljung-Box test for residual autocorrelation, computed with 24 autocorrelations (in all cases, asymptotically distributed (a.d.) as $\chi^2(22 \text{ d.f.})$).
3) N_a is the Bowman-Shenton test for Normality of the residuals (a.d. as $\chi^2(2 \text{ d.f.})$).
4) t_a (skew) is the t-value associated with H_0 : skewness (residuals) = 0.
5) t_a (kur) is the t-value associated with H_0 : kurtosis (residuals) = 3.
6) Q_{as} (2) is the Pierce test for the presence of seasonality in the residual autocorrelation, (a.d. as $\chi^2(2 \text{ d.f.})$).
7) Q_{a2} (24) is the McLeod and Li, (1983) test on linearity of the process versus bilinear or ARCH-type structures (a.d. as $\chi^2(22 \text{ d.f.})$).
8) t_a (runs) is the t-value associated to H_0 : signs of the residuals are random.

The 95% critical value for each test is given in the last row.

Table 2. Standard Deviation of Component Innovation

	Trend-cycle	Seasonal Component	Irregular Component	SA series
Exports	336	146	623 (*)	893
Imports	343	129	678	1028
Balance	254	291	674	945
Exports (alternative)	336	152	554	897

(*) Includes the variance of the transitory component

Table 3. Estimation Standard Errors: Concurrent Estimator

	Total Estimation Error		Revision	
	Trend-cycle	SA series	Trend-cycle	SA series
Exports	516	410	374	292
Imports	597	416	418	295
Balance	609	561	464	399
Exports (alternative)	557	428	395	305

Table 4. Convergence of Estimators: Percentage Reduction in Revision Error Variance

	After 1 year of additional data		After 5 more years of additional data	
	Trend-cycle	SA series	Trend-cycle	SA series
Exports	60.3	25.3	88.0	77.3
Imports	68.4	20.7	87.7	69.2
Balance	79.3	44.8	98.3	95.5
Exports (alternative)	63.7	26.3	89.6	78.9

Table 5. Significance of Seasonality: Number of Months per year with significant seasonality (95% level)

	Historical Estimator	Preliminary Estimator (last year)	Forecasts (next year)
Exports	11	11	11
Imports	6	6	6
Balance	9	9	9
Exports (alternative)	11	11	11

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