COMBINING FILTER DESIGN WITH MODEL-BASED FILTERING (WITH AN APPLICATION TO BUSINESS-CYCLE ESTIMATION)

2004

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Documentos de Trabajo
N.º 0417

BANCO DE ESPAÑA

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(*) Thanks are due to David Findley and William Bell for their helpful comments, and to Nieves Morales and Jorge Carrillo for their secretarial and research assistance.

Documentos de Trabajo. N.º 0417
2004

ISSN: 0213-2710 (print)
ISSN: 1579-8666 (on line)
Depósito legal:
Imprenta del Banco de España

Abstract

Filters used to estimate unobserved components in time series are often designed on a priori grounds, so as to capture the frequencies associated with the component. A limitation of these filters is that they may yield spurious results. The danger can be avoided if the so-called ARIMA-model-based (AMB) procedure is used to derive the filter. However, parsimony of ARIMA models typically implies little resolution in terms of the detection of hidden components. It would be desirable to combine a higher resolution with consistency with the structure of the observed series.

We show first that for a large class of a priori designed filters, an AMB interpretation is always possible. Using this result, proper convolution of AMB filters can produce richer decompositions of the series that incorporate a priori desired features for the components, and fully respect the ARIMA model for the observed series. (Hence no additional parameter needs to be estimated.)

The procedure is discussed in detail in the context of business-cycle estimation by means of the Hodrick-Prescott filter applied to a seasonally adjusted series or a trend-cycle component.

Keywords: Time Series; Filtering and Smoothing; ARIMA models; Trend and Cycle Estimation; Hodrick-Prescott Filter.

JEL Classification: C22, C80, E32, E37.

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1 Introduction and Summary

Filters used to estimate unobserved components (**UC**) –also called "signals"– in economic time series are often designed on a priori grounds, so as to capture the frequencies that should be associated with the signal of interest. We shall refer to them as a-priory designed (**APD**) filters, and their design is independent of the particular series at hand. It is well known that a limitation of APD filters is that they may produce spurious results (a trend, for example, could be extracted from white noise).

The spuriousness problem can, in principle, be avoided if the filter is derived following a model-based approach. The series features are captured through an ARIMA model, models for the components are derived, and the Wiener-Kolmogorov filter is used to obtain the Minimum Mean Squared Error (**MMSE**) estimator of the components. We shall refer to this approach as ARIMA-model-based (**AMB**) filtering. AMB filtering also presents some drawbacks. First, it may provide components that display poor band-pass features. Second, parsimony of the ARIMA models typically identified for economic series implies little resolution in terms of UC detection, so that the AMB decomposition cannot go much beyond the standard "trend-cycle + seasonal + irregular" decomposition. Thus, it would be nice to combine a higher resolution with lack of spuriousness and consistency with the structure of the overall observed series.

It is first seen that, for a fairly wide class of APD filter that are symmetric and linear, an AMB interpretation is always possible, whereby the signal obtained is the MMSE estimator of white noise in the decomposition of an ARIMA model (straightforward to obtain from the APD filter). Given that the signal of interest will not be, in general, white noise, the previous interpretation does not provide a sensible model, but allows for a Wiener-Kolmogorov representation of the APD filter. This representation permits us to integrate the APD filter within the AMB approach. An important case is the following.

To avoid contamination with undesired frequencies, estimation of a signal often implies two steps: the APD filter is applied to series that have already been filtered. (For example, the business cycle can be estimated on the seasonally adjusted series or on the trend-cycle component; sampling error may be estimated on the SA series or on the irregular component; calendar effects can be estimated with filters applied to the detrended series.) Thus, in the first step, a basic component is estimated and, in the second step, the APD is applied to this estimator.

If the first step is performed using an AMB approach, it is seen that the two-step estimator of the signal is also the MMSE estimator of a component in a full UC model, where the models for the components are sensible and incorporate elements reflecting the desirable features of the components, as well as elements that guarantee consistency with the observed series model. The two-step procedure accepts thus a full model specification and the components can be estimated in a single step. In this way, it becomes possible to increase the resolution of AMB filters, while preserving the parsimony of the overall model (crucial for forecasting).

The result is discussed in detail in the context of business-cycle estimation with the Hodrick-Prescott (**HP**) filter applied to the trend-cycle or Seasonally Adjusted (**SA**) series. It is seen that there is an infinite number of admissible decompositions of the trend-cycle into a long-term trend and a (business-) cycle component, where the former captures the frequencies in a narrow band around zero, and the cycle is a standard ARMA (2,2) linear stationary stochastic cycle, with the AR roots associated with a cyclical frequency. Reparametrizing the HP filter in terms of the period (τ_0) for which the gain of the filter is .5 (i.e., the cutting point between periods mostly associated with the trend and those mostly associated with the cycle), it is seen that the choice of a particular τ_0 identifies a unique decomposition. The models corresponding to this decomposition are derived and discussed.

2 Filter Design and Arima-Model-Based Filtering

2.1 Unobserved Components and Linear Filters

Consider the problem of estimating an unobserved component hidden in an observed time series (i.e., the problem of "signal extraction"). Obvious examples are Seasonal Adjustment, and Trend or Cycle estimation. The series variation that should be excluded from the signal of interest will be denoted "noise" (for example, the SA series could be the signal and the seasonal component the noise). Thus we consider the "signal plus noise" decomposition $x_t = m_t + c_t$, where x_t is the observed series, c_t the "signal", m_t the non-signal (or "noise"), which in general will not be white, and the two UC are orthogonal. In order to avoid phase effects that would distort historical dating of turning points, we shall obtain the historical (or final) estimator of the signal with a two-sided symmetric linear filter, as in

$$\hat{c}_t = \nu_k \; x_{t-k} + ... + \nu_0 \; x_t + ... + \nu_k \; x_{t+k} \tag{2.1}$$

Let B and F denote the Backward and Forward operators, such that $B z_t = z_{t-1}$, and $F z_t = z_{t+1}$, respectively. We can write (2.1) as

$$\hat{c}_t = \nu(B, F) \, x_t \tag{2.2}$$

where

$$\nu(B, F) = \nu_0 + \sum_{j=1}^{k} \nu_j \; (B^j + F^j) \tag{2.3}$$

The weights in $\nu(B, F)$ are supposed to capture the "desired" features of the signal. Given that the features of a trend, a seasonal, or a cyclical component are often better described in the frequency domain, we obtain the Fourier Transform (**FT**) of the filter (2.3), which implies replacing $(B^j + F^j)$ in (2.3) by $(2 \cos(j\,\omega))$, where ω denotes the frequency in radians ($0 \le \omega \le \pi$). This transformation yields the gain function of the filter

$$G(\omega) = \nu_0 + 2 \sum_{j=1}^{k} \nu_j \; \cos(j\,\omega) \tag{2.4}$$

The gain will determine how much the different frequencies will contribute to the signal. If $G(\omega_0) = 0$, the frequency ω_0 will be fully ignored; when $G(\omega_0) = 1$, the frequency ω_0 will be fully transmitted.

A cyclical frequency, ω, is easily translated into the period τ of the associated cycle through

$$\tau = 2\pi / \omega \tag{2.5}$$

The period τ denotes the number of units of time needed for the completion of a full cycle. Hence, for example, for the two extreme values of the frequency:

- $\omega = 0 \;\Rightarrow\; \tau \to \infty \;\Rightarrow$ Trend frequency
- $\omega = \pi \;\Rightarrow\; \tau = 2 \;\Rightarrow$ 2-period cycle.

Figure 1 plots the gain of a filter aimed at capturing a trend, while Figure 2 that of a filter aimed at removing the previous trend.

Figure 1: Filter for estimating a trend

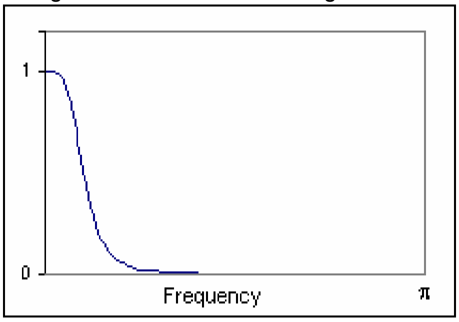
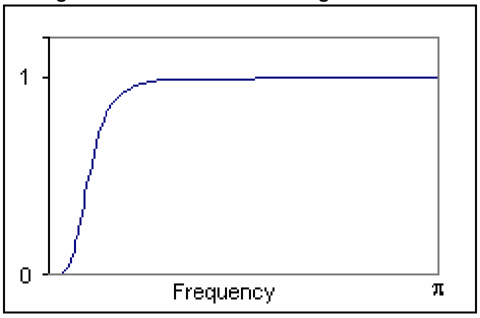
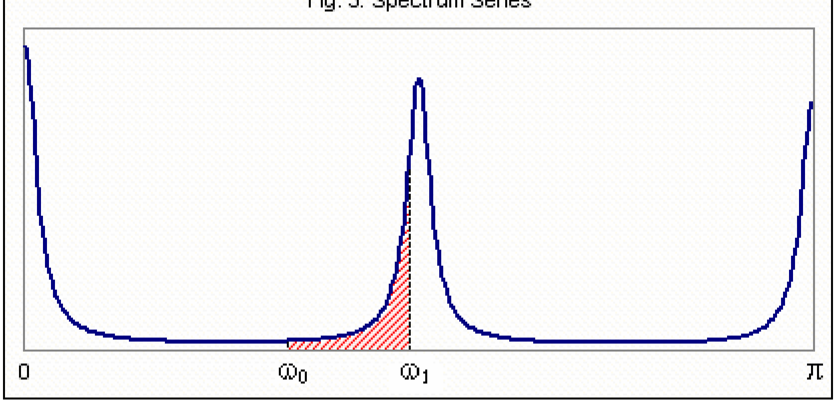


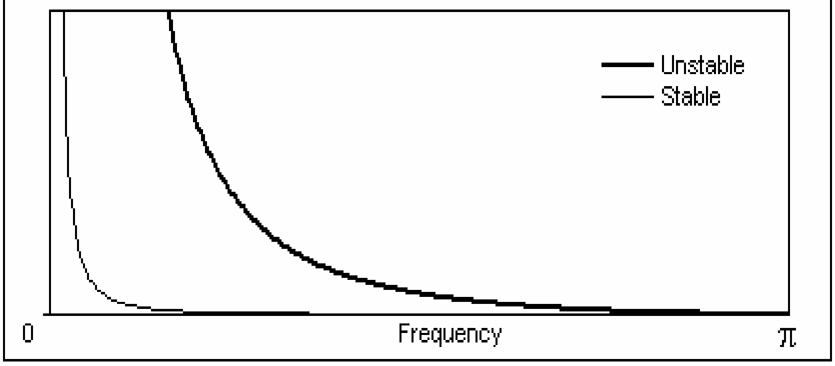
Figure 2: Filter for removing a trend



APD filters are often designed, on *a-priori* grounds, so as to capture (as best as possible) the series variation associated with certain frequencies, namely, those that characterize the signal of interest. The filter is applied to an observed series x_t that is assumed the output of an ARIMA model. Let $g_x(\omega)$ denote the spectrum (or pseudo-spectrum when there are unit AR roots) of x_t. In the stationary case, $g_x(\omega)$ decomposes the variance of x_t according to frequency. For example, in Figure 3 the shaded area represents the variance associated with the frequency interval (ω_0, ω_1). The peaks of $g_x(\omega)$ are associated with trend and seasonal AR roots.



We shall also use the "term "spectrum" to refer to the pseudo-spectrum because both will be used in a similar way in the following sense. If, for example, the peak for $\omega = 0$ is very wide, there will be a lot of stochastic variability in the trend. The trend will thus be highly stochastic (or "moving"). On the contrary, if the peak is narrow, the trend will have little stochastic variability and be stable. Two (extreme) examples are illustrated in figures 4-6.



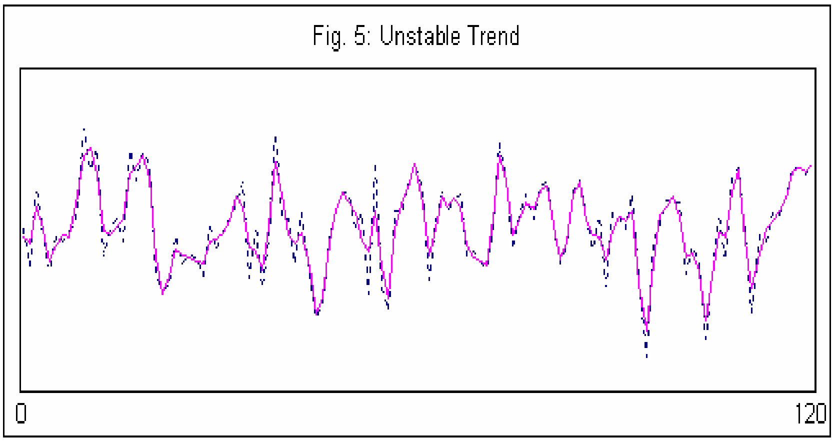


Fig. 5: Unstable Trend

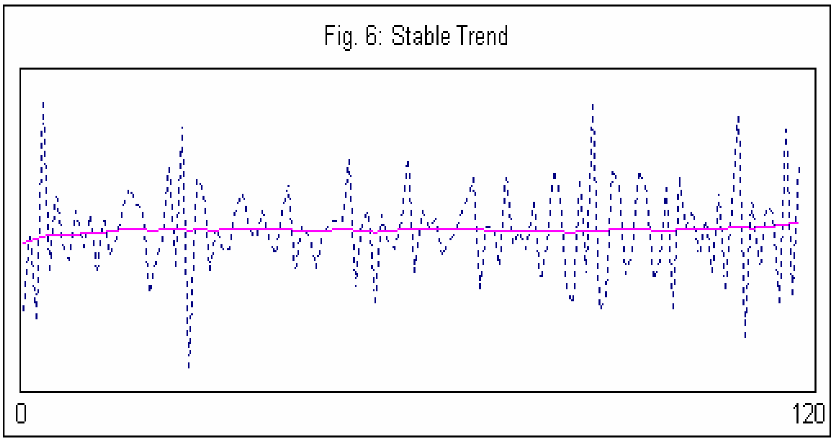


Fig. 6: Stable Trend

Allowing for unit AR roots, the FT of the (pseudo-) Autocovariance Generating Function (ACGF) –see Hatanaka and Suzuki (1967)– of the two sides of (2.2) yields:

$$g_{\hat{c}}(\omega) = [G(\omega)]^2 \, g_x(\omega) \tag{2.6}$$

where $g_x(\omega)$ is the spectrum of x_t, $[G(\omega)]^2$ is the Squared Gain (SG), which determines which parts of $g_x(\omega)$ are passed on to the spectrum of the signal, and $g_{\hat{c}}(\omega)$ is the spectrum of the estimated signal $\hat{c}_t$. The SG provides information concerning the filter; information concerning the signal obtained is contained in its spectrum $g_{\hat{c}}(\omega)$. "A priori" design may produce a filter with an appealing SG. But it can be wrongly applied to a series. As a simple example, a trend filter (Figure 7a) applied to a white-noise series (Figure 7b) will produce a trend component (Figure 7c), and hence a spurious result.

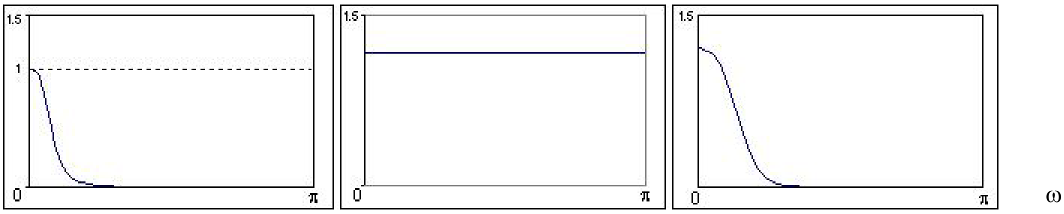


Fig. 7a SG of Trend Filter | Fig. 7b Spectrum of white-noise series → | Fig. 7c Spectrum of trend in white-noise.

Therefore, the filter should depend on the particular series being analyzed. This consideration, added to the spuriousness danger, fostered an alternative approach to filtering: the ARIMA - Model - Based (AMB) approach. Spuriousness is avoided by decomposing the series in such a way that its specific features are respected. These features are summarized in the ARIMA model identified for the series. From this ARIMA model, the UC models are derived in such a way that they aggregate into the model for the observed series. The signal is estimated with the Wiener-Kolmogorov (WK) filter, which provides the MMSE estimator and, under our normality assumptions, the conditional expectation of the signal given the data [see Hillmer and Tiao (1982), Burman (1980), and Gómez and Maravall (2001)]. We shall follow the AMB approach, as enforced in programs SEATS [Gómez and Maravall (1996)] and TSW [Caporello and Maravall (2004)]. The programs can be freely downloaded from the Bank of Spain web site *www.bde.es*.

Other efficient approaches to the estimation of signals in UC models are available [examples are Harvey (1989), García-Ferrer and del Hoyo (1992), Gersh and Kitagawa (1983), and Engle (1978)]. These approaches differ from the AMB one in several respects. In particular, no identification of an ARIMA model for the observed series is made and the models for the components are specified “a priori”.

2.2 Wiener-Kolmogorov Filter

Consider the decomposition of x_t into two uncorrelated components, as in

$$x_t = m_t + c_t \tag{2.7}$$

where the signal c_t follows the model

$$\phi_c(B)\, c_t = \theta_c(B)\, a_{ct} \quad , \qquad a_{ct} \sim wn(0, V_c) \tag{2.8}$$

and the model for the observed series is given by

$$\phi(B)\, x_t = \theta(B)\, a_t \quad , \qquad a_t \sim wn(0, V_a) \tag{2.9}$$

where “wn” denotes a white-noise (i.e., normally identically independently distributed) variable, V_c and V_a are the variances of a_{ct} and a_t, and $\theta(B)$ is an invertible polynomial. We assume that $\phi(B)$ can be factorized as

$$\phi(B) = \phi_c(B)\, \phi_m(B) \tag{2.10}$$

with $\phi_c(B)$ and $\phi_m(B)$ containing the AR roots that will be assigned to the signal and non-signal respectively. Suppose, first, that model (2.9) is stationary and define the MA expressions

$$\psi_c(B) = \theta_c(B) / \phi_c(B) \quad ; \qquad \psi(B) = \theta(B) / \phi(B)$$

The WK estimator of the signal for a full realization $(x_{-\infty}, \dots, x_{\infty})$ is given by

$$\begin{aligned} \hat{c}_t &= \left[\frac{ACGF(c_t)}{ACGF(x_t)} \right] x_t = \left[k_c \frac{\psi_c(B)\, \psi_c(F)}{\psi(B)\, \psi(F)} \right] x_t \\ &= \nu(B,F)\, x_t \quad ; \qquad k_c = V_c / V_a \end{aligned} \tag{2.11}$$

Considering (2.10), it is obtained that the WK filter is equal to

$$\nu(B,F) = k_c \frac{\theta_c(B)\,\phi_m(B)}{\theta(B)} \frac{\theta_c(F)\,\phi_m(F)}{\theta(F)} \tag{2.12}$$

a centered, symmetric, and convergent filter. The convergence does not depend on the roots of the AR polynomials, and in fact expression (2.12) can be extended to the nonstationary case (Bell, 1984). Notice that, writing the model for m_t (the non-signal) as

$$\phi_m(B)\, m_t = \theta_m(B)\, a_{mt} \quad ; \qquad a_{mt} \sim w.n.(0, V_m) \tag{2.13}$$

then, letting $k_m = V_m / V_a$, (2.7) implies the following identity

$$\begin{aligned} &\theta_m(B)\,\theta_m(F)\,\phi_c(B)\,\phi_c(F)\,k_m + \\ &+\theta_c(B)\,\theta_c(F)\,\phi_m(B)\,\phi_m(F)\,k_c = \theta(B)\,\theta(F) \end{aligned} \tag{2.14}$$

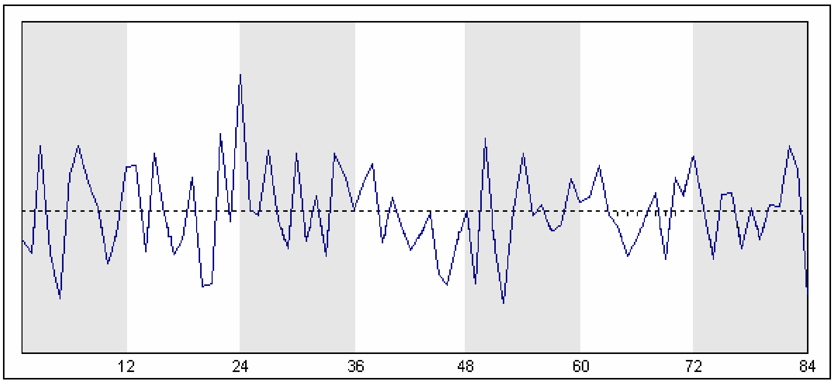
2.3 Some Limitations of Arima-Model-Based Filtering

We mentioned some problems with APD filtering. There are also limitations and ambiguities in the AMB approach even in the infinite realization (in practice, historical estimation) case. Ambiguity is due to the fact that we do not have clear, universally accepted, definitions of the components. As a simple example, consider the seasonal AR (1) model:

$$(1+\phi_{12} B^{12})\, x_t = a_t \tag{2.15}$$

with $\phi_{12} = -.3$. The model displays correlation for seasonal lags, but this correlation is very small. Figure 8 shows an example of a realization of (2.13): the seasonal effect is certainly not discernible.

Figure 8. Stationary Seasonal AR(1)



Does a series following model (2.15) contain a seasonal component? Possibly one should say "not really". Seasonality is meant to capture something that reappears periodically in a more systematic manner. But then, what about the case $\phi_{12} = -.8$? Seasonality now is longer lasting, but still considerably erratic. Where should the cutting point be? In Hillmer, Bell, and Tiao (1983) and Maravall (1983) it is argued that only seasonal components with seasonal AR unit roots should be considered. Still the seasonal root $(1-.9B^{12})$ has a seasonal effect that persists for many years.

2.4 A Basic Underidentification Problem

Unobserved component models require, in general, identification restrictions [see, for example, Maravall (1985)]. The AMB approach proceeds as follows: Consider the model for x_t given by (2.9), and its decomposition into model (2.8) for c_t plus model (2.13) for m_t. Given that x_t is observed, model (2.9) can be identified in the usual way. The problem is then to derive models (2.8) and (2.13). The AR factorization (2.10) identifies the polynomials $\phi_c(B)$ and $\phi_m(B)$. What remains is identification of the MA polynomials $\theta_c(B)$, $\theta_m(B)$, and of the variances V_c and V_m. These parameters should be determined from the identity (2.14), and it is straightforward to see that there will be an infinite number of solutions. In order to reach identification, in the two component case, the AMB approach assumes first that the model for the signal is "balanced", that is, the order of its AR polynomial (including unit roots) is equal to the order of its MA polynomial. Second, within the infinite decompositions that satisfy (2.14) and have a balanced signal, the one with the smoothest signal is selected. This is done through the "canonical" assumption, which requires the signal to be free of white-noise [see Box, Hillmer, and Tiao (1978), and Pierce (1978)]. It can be seen that a canonical signal will display a spectral zero, or, equivalently, a unit MA root. Putting together (2.10), (2.14), and the previous two assumptions (balanced and canonical signal), a single decomposition of model (2.9) into models of the type (2.8) and (2.13) is obtained.

It is a fact, however, that standard ARIMA modelling favors parsimonious models, as simple as possible. Yet the simple model may hide a more complex structure. A very simple example that illustrates the point is the following. Consider a biannual series x_t that is the sum of two components with models $p_t = b_t + b_{t-1}$, and $s_t = c_t - c_{t-1}$, where b_t and c_t are uncorrelated white-noise variables with variances $V_b = V_c = 1$. The associated spectra are

$$g_p(\omega) = \frac{1}{\pi}(1+\cos\omega) \quad ; \qquad g_s(\omega) = \frac{1}{\pi}(1-\cos\omega) \tag{2.16}$$

The spectrum of p_t presents a peak for $\omega = 0$ and decreases monotonically until it becomes zero for $\omega = \pi$. Thus p_t can be seen as a trend-cycle component. As for s_t, the spectrum displays a zero for $\omega = 0$ and increases monotonically reaching a peak for $\omega = \pi$ (the once-a-year seasonal frequency for biannual data). Thus s_t can be seen as a seasonal component. The MMSE estimators obtained with the WK filter are

$$\hat{p}_t = \frac{1}{2}x_t + \frac{1}{4}(x_{t-1} + x_{t+1})$$

$$\hat{s}_t = \frac{1}{2}x_t - \frac{1}{4}(x_{t-1} + x_{t+1})$$

Given that p_t and s_t are non-invertible, the estimator of the irregular component is $\hat{u}_t = 0$. The AMB approach applied to x_t would not provide this result. From (2.16), it is seen that $g_x(\omega) = g_p(\omega) + g_s(\omega) = 2/\pi$, and hence x_t turns out to be simply white noise. Therefore, the AMB decomposition should yield $\hat{p}_t = \hat{s}_t = 0$, $\hat{u}_t = x_t$.

The difficulty in detecting hidden components is particularly noticeable in the range of cyclical frequencies. ARIMA identification relies heavily in the use of differences as a way of reaching stationarity, and it is well-known that differencing often affects (sometimes very strongly) the cyclical frequencies. As a consequence, the AMB method will only be able to extract an aggregate trend-cycle component, and separate identification of the trend and cycle will require additional assumptions. As an example, suppose that a series x_t is known by analysts to be cyclical, but that standard ARIMA identification yields the IMA (2,2) structure

$$\nabla^2 x_t = (1 + \theta_1 B + \theta_2 B^2)a_t \tag{2.17}$$

The AMB decomposition of x_t would yield a (canonical) IMA (2,2) trend-cycle and a white-noise irregular component. As shall be seen in Section 4.7, this trend-cycle model can be split into the sum of uncorrelated longer-term trend (an ARIMA (2,2,2) model) plus an ARMA (2,2) cyclical component, both with sensible spectral shapes. Model (2.17) and the UC ("trend + cycle") model are observationally equivalent, but in the absence of a priori information, ARIMA identification will always choose the parsimonious model (2.17), which shows no evidence of cyclical behavior.

3 Relationship between APD and AMB Filters

From the previous discussion we conclude that it would be desirable to mix the virtues of the AMB and the APD approaches in such a way that: a) there would be consistency with the observed series (no spurious results); b) filters and components would have desirable properties; c) the model-based structure could be preserved.

It is well-known that some important APD filters have been given a model-based interpretation (at least, as an approximation). In this interpretation, the filter can be seen as the one that provides the MMSE estimator of a component in a particular UC model. This interpretation may provide insights into the type of series for which the filter might be more appropriate [examples are the X11 interpretations of Cleveland-Tiao (1976) and Burridge-Wallis (1984)]. It might simply offer an alternative algorithm to compute the signal with the Kalman or WK filters and can be of help in improving the filter design [Pollock, (2003)]. We see next that, under fairly general conditions, the mapping "symmetric linear filter $\rightarrow$ AMB filter" is, feasible. This will allow us to incorporate the desired ad-hoc/model-based mixture.

3.1 "Naïve" Model-Based Interpretation

Assume the APD filter (2.2) is symmetric. Thus, if *B* is a root, B^{-1} is also a root, and $\nu(B,F)$ can be factorized as

$$\nu(B,F) = A(B)\ A(F)\ k_c \tag{3.1}$$

with $A(B) = 1 + \sum_{j=1}^{k} a_j B^j$. We shall further assume that the coefficients of A(B) are real numbers, so that (3.1) can be interpreted as an ACGF (and the filter gain satisfies $0 \le G(\omega) \le 1$). Symmetric linear filters that satisfy this "admissibility" condition will be denoted SAL filters. We shall center attention to APD filters of the SAL class. As shown by (2.12), AMB filters will always belong to this class. From (2.2) and (3.1), the estimator of the signal can be expressed as

$$\hat{c}_t = \left[k_c\ \ A(B)\ A(F)\ \right] x_t \tag{3.2}$$

which always accepts the following AMB interpretation.

<u>**Result 1**</u>

The estimator (3.2) can be seen as the MMSE estimator of white noise in the decomposition of x_t into orthogonal signal (m_t) + noise (c_t), as in (2.7), when x_t follows the model

$$\mathbf{A(B)\,x_t = a_t} \tag{3.3}$$

with a_t and c_t white noises such that $V_c / V_a = k_c$.

More generally, if $A(B) = A_N(B) / A_D(B)$, then $\hat{c}_t$ is the MMSE estimator of the noise in a series that follows the ARIMA model

$$A_D\ (B)\ x_t = A_N\ (B)\ a_t \tag{3.4}$$

(The result follows from straightforward application of the WK filter to a white noise signal when the model for the series is (3.3) or (3.4).)

This result gives a very simple way to find a AMB-type algorithm for SAL filters. The algorithm is based on the (artificial) assumption that c_t is white noise, which implies that the (artificial) model for the "non-signal" m_t would be of the type

$$A(B)\ m_t = \theta_m(B)\ a_{mt} \quad ; \quad a_{mt} \sim w.n.(0, V_m) \tag{3.5}$$

where $\theta_m(B)$, $k_m\ (= V_m / V_a)$, and k_c are determined from the identity

$$\theta_m(B)\ \theta_m(F)\ k_m + A(B)\ A(F)\ k_c = 1 \tag{3.6}$$

The algorithm is efficient, and (3.6) guarantees consistency with the overall series. But the models behind the algorithm do not provide a realistic interpretation, because the observed series will not follow in general model (3.3), nor would we expect the cycle to be white noise. This "signal + noise"- decomposition interpretation of a symmetric filter will be called the "naïve" model-based interpretation.

3.2 Mixed Estimation

To simplify expressions, we introduce the following notation: for a finite-order polynomial in B with real coefficients, say P(B), $\|P(B)\|^2 = P(B)\ P(F)$. Suppose we wish to apply a symmetric APD filter, say (3.1), to estimate some signal (c_t) in x_t, but that the filter should be applied to the series clean of seasonality (perhaps also of noise). Consider the decomposition

$$x_t = \hat{n}_t + \hat{s}_t \tag{3.7}$$

where $\hat{n}_t$ and $\hat{s}_t$ are the seasonally adjusted (SA) series and seasonal component estimators respectively. We can follow a two-step procedure: First, AMB filtering to estimate the SA series. Second, APD filtering of the SA series to estimate the signal.

In the first step, we start with an ARIMA model identified for x_t, say (2.9). From this, we derive the models for the SA series (n_t) and seasonal component (s_t), say

$$\phi_n(B)\ n_t = \theta_n(B)\ a_{nt} \quad , \quad a_{nt} \sim w.n.(0, V_n) \tag{3.8}$$

$$\phi_s(B)\ s_t = \theta_s(B)\ a_{st} \quad , \quad a_{st} \sim w.n.(0, V_s) \tag{3.9}$$

with a_{st} uncorrelated with $a_{nt'}$, $\phi(B) = \phi_n(B)\ \phi_s(B)$, and $x_t = n_t + s_t$. Finally, the WK estimator $\hat{n}_t$ is obtained:

$$\hat{n}_t = \frac{V_n}{V_a} \left\| \frac{\theta_n(B)\ \phi_s(B)}{\theta(B)} \right\|^2 x_t$$

In the second step, we apply the APD filter to $\hat{n}_t$,

$$\begin{aligned} \hat{c}_t &= k_c\ A(B)\ A(F)\ \hat{n}_t = \\ &= k_n\ k_c \left\| \frac{A(B)\ \theta_n(B)\ \phi_s(B)}{\theta(B)} \right\|^2 x_t \\ &= \nu_c(B,F)\ x_t \end{aligned} \tag{3.10}$$

The estimator of m_t is $\hat{m}_t = \hat{n}_t - \hat{c}_t = \left[1 - k_c \; A(B) \, A(F) \right] \hat{n}_t$, or, using (3.6),

$$\hat{m}_t = \left[k_m \;\; \theta_m(B) \; \theta_m(F) \right] \; \hat{n}_t \tag{3.11}$$

It is easily verified that the sum of the 3 WK estimators $\hat{m}_t + \hat{c}_t + \hat{s}_t$ yields the ARIMA model for x_t.

3.3 Direct Estimation

<u>Result 2</u>

The 2-step estimators $\hat{m}_t$, $\hat{c}_t$, and $\hat{s}_t$ accept a non-naïve AMB interpretation, in the sense that they can be seen as the direct MMSE estimators of m_t, c_t, and s_t which follow sensible models and aggregate into the ARIMA model for x_t.

Specifically, consider the UC model given by

$$x_t = m_t + c_t + s_t \tag{3.12}$$

where x_t and s_t follow models (2.9) and (3.9), respectively, and the models for the cycle c_t and trend m_t are

$$c_t = A(B) \;\; \psi_n(B) \;\; a_{ct} \tag{3.13}$$

$$m_t = \theta_m(B) \;\; \psi_n(B) \;\; a_{mt} \tag{3.14}$$

where $\psi_n(B) = \theta_n(B) / \phi_n(B)$, $a_{ct} \sim wn(0, k_c V_n)$, $a_{mt} \sim wn(0, k_m V_n)$, and a_{ct} is uncorrelated with $a_{mt'}$. Direct application of the WK filter to the full UC model yields the 2-step estimators of the mixed approach.

Let $n_t = m_t + c_t$, and denote by $\delta_n(B)$, $\delta_m(B)$, and $\delta_c(B)$ the polynomials with the AR unit roots of the models for n_t, m_t, and c_t. Assuming that $\delta_m(B)$ and $\delta_c(B)$ do not share a root in common, then $\delta_n(B) = \delta_m(B) \;\; \delta_c(B)$. Thus

$$\psi_n(B) = \frac{\theta_n(B)}{\varphi_n(B) \;\; \delta_m(B) \;\; \delta_c(B)}$$

where $\varphi_n(B)$ is the stationary AR polynomial in the model for n_t. Given that the filter (3.1) is aimed at removing m_t, A(B) will have zeros for the frequencies associated with the unit roots of $\phi_m(B)$, so that we can factorize A(B) as $A(B) = a(B) \;\; \delta_m(B)$. In expression (3.13) there will be cancellation of unit roots, and the model for c_t can be rewritten as

$$\varphi_n(B) \;\; \delta_c(B) \;\; c_t = a(B) \;\; \theta_n(B) \;\; a_{ct}$$

The model for the cycle component contains APD filter elements [A(B), and k_c in (3.13)] that will capture desirable features of the filter, as well as series-dependent elements [$\psi_n(B)$ and V_n in (3.13)] that will impose consistency with the observed series model. (If the SA series n_t is replaced by the trend-cycle p_t, the discussion extends trivially.)

Remark: The approach to Result 2 is closely related to the derivation of the "Consistency with the Data" check of Bell and Hillmer (1984), developed in the context of AMB seasonal adjustment.

4 An application to trend and Cycle estimation with the Hodrick-Prescott filter

ADP filters have often been used in the context of trend extraction for business-cycle analysis [Hodrick and Prescott (1980), Baxter and King (1999), Pollock (2000), Canova (1998)]. We focus on the Hodrick and Prescott (HP) filter, which has been the center of considerable attention [Kydland and Prescott (1990), Cogley and Nason (1995), Gómez (2001), Harvey and Trimbur (2003), and Kaiser and Maravall (2001)].

4.1 Model-Based Implementation of the Hodrick-Prescott Filter

The so-called Hodrick-Prescott (HP) filter is an APD filter that decomposes the series, as in (2.7), into a relatively long-term trend (m_t) plus a cycle (c_t), often called "business cycle". The filter is a particular case of the Butterworth family of filters [see Gómez (2001)], and can be derived as the solution of the minimization of the Loss Function

$$LF = \left[\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=3}^{T} (\nabla^2 m_t)^2 \right]$$

where the first term penalizes poor fit and the second term penalizes lack of smoothness. The parameter λ balances the relative importance of the two and determines thus the relative smoothness of m_t (larger values of λ will imply smoother trend series).

The HP filter can also be derived from a "model based"- type algorithm [King-Rebelo (1993)] whereby the cycle is obtained as the estimator of the noise in an UC model $x_t = m_t + c_t$, with

$$\begin{aligned} &\nabla^2 m_t = a_{mt} , \qquad a_{mt} \sim w.n.(0, V_m) \\ &c_t \sim w.n.(0, V_c) , \quad V_c / V_m = \lambda \end{aligned} \tag{4.1}$$

This UC model implies that

$$\nabla^2 x_t = a_{mt} + \nabla^2 c_t \tag{4.2}$$

which can be expressed as an IMA (2,2) model, say

$$\nabla^2 x_t = (1 + \theta_1^{HP} B + \theta_2^{HP} B^2)\, b_t = \theta_{HP}(B)\, b_t \tag{4.3}$$

where θ_1^{HP}, θ_2^{HP}, and V_b are easily obtained from λ (see the Appendix). Accordingly, the HP filter can also be obtained as the WK filter that provides the estimator of c_t (assumed w.n.), when the series follows model (4.3). (This is simply a particular case of Result 1.) The WK filters to obtain $\hat{m}_t$ and $\hat{c}_t$ are:

$$\hat{m}_t = k_m \frac{1}{\theta_{HP}(B)\, \theta_{HP}(F)} x_t = \nu_{HP}^m (B,F)\, x_t \tag{4.4a}$$

$$\hat{c}_t = k_c \frac{\nabla^2 \overline{\nabla}^2}{\theta_{HP}(B)\, \theta_{HP}(F)} x_t = \nu_{HP}^c (B,F)\, x_t \tag{4.4b}$$

where $\overline{\nabla} = 1 - F$ and $k_m = V_m / V_b$, $k_c = V_c / V_b$. From (4.2) and (4.3), the following identity between ACGF has to hold

$$\theta_{HP}(B)\, \theta_{HP}(F) = k_m + \nabla^2 \overline{\nabla}^2 k_c \tag{4.5}$$

Therefore, expression (4.4a) can be rewritten as

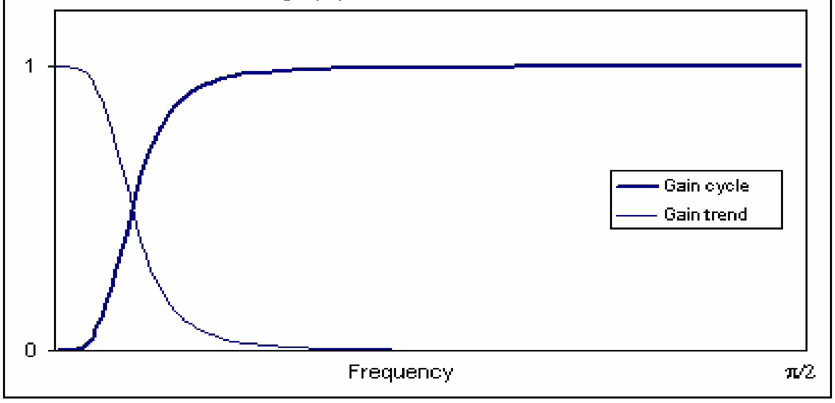
$$\hat{m}_t = \left[\frac{1}{1+\lambda \nabla^2 \bar{\nabla}^2} \right] x_t$$

where we have used, from (4.1), $k_c / k_m = V_c / V_m = \lambda$. The gain of the trend filter is the FT of the term in brackets, which yields

$$G_m(\omega) = \frac{1}{1+4\lambda(1-\cos\omega)^2} \tag{4.6}$$

and $G_c(\omega) = 1 - G_m(\omega)$. Both gains are represented (for $\lambda = 1600$) in Figure 9, for the range $0 \le \omega \le \pi/2$.

Fig. (9): Gains of HP Filter



Despite this "model-based" representation, the filter is an APD filter and the danger of spuriousness becomes an issue [as shown in Maravall (1995)].

Notwithstanding academic criticism [see, for example, Harrey and Jaeger (1993)], the HP filter has become the most widely used procedure to estimate business cycles in applied work [see, for example, International Monetary Fund (1993), Giorno et al. (1995), European Commission (1995), and European Central Bank (2000)]. Can this be rationalized within an AMB perspective? To answer the question, we start by reviewing some very basic concepts having to do with the cycle.

4.2 Basic Model for a Cycle

a. Simplest case: Deterministic Model

A standard expression for a deterministic cycle is $c_t = A^t \cos(\omega t + B)$, where A is the Amplitude, B is the Phase and ω is the Frequency (number of cycles per unit of time) measured in radians. An equivalent representation to the previous cosine function is given by the second order difference equation:

$$c_t + \phi_1 c_{t-1} + \phi_2 c_{t-2} = 0 \tag{4.7}$$

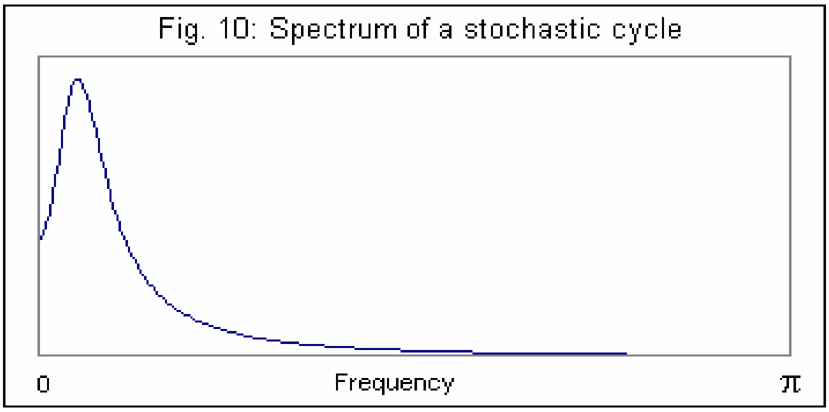
or $(1+\phi_1 B + \phi_2 B^2)\, c_t = \phi_c(B) c_t = 0$, when the roots of $\phi_c(B) = 0$ are complex and associated with the frequency ω. In this deterministic case, the spectrum of c_t degenerates into a single spike for ω.

b. Linear Stochastic Cycles

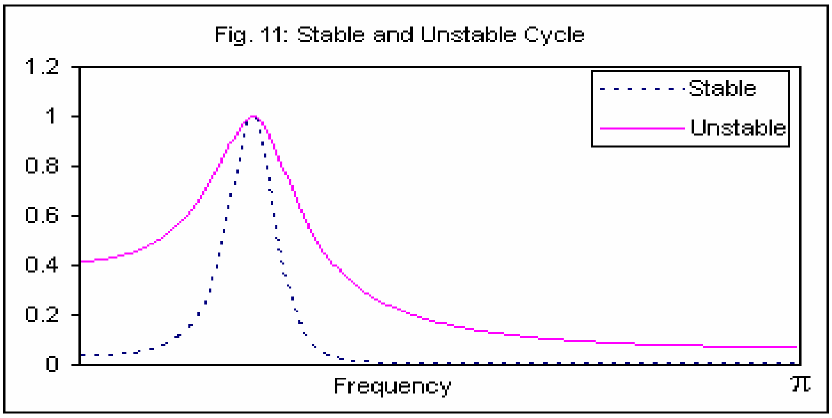
Economic cycles typically do not behave in a deterministic way. Period and amplitude, are not constant, and evolve with some randomness. One way to incorporate this randomness is by introducing every period a stochastic shock, as in, for example,

$$c_t + \phi_1 \ c_{t-1} + \phi_2 \ c_{t-2} = a_{ct} \quad ; \qquad a_{ct} \sim \text{w.n.} (0, \ V_c) \qquad (4.8)$$

Thus, a) every period the "deterministic equilibrium" (4.7) is perturbed by a stochastic shock (with zero mean and moderate variance). b) The shocks will affect the cycle characteristics (for example, a sequence of positive shocks may increase the duration of an expansion). What is obtained now is a distribution of frequencies (or periods) around the value ω_0 (or τ_0) of the deterministic equation. This distribution of frequencies is precisely the spectrum of the AR(2) model (4.8), a typical spectrum of a stochastic cycle (Figure 10).



The spectrum of the cycle provides centrality measures (mode, mean, median), confidence intervals around theses measures, and an idea of how stable or moving the cycle is. In Figure 11, the cycle with the narrower peak will produce cyclical oscillations with periods closer on average to the modal value.

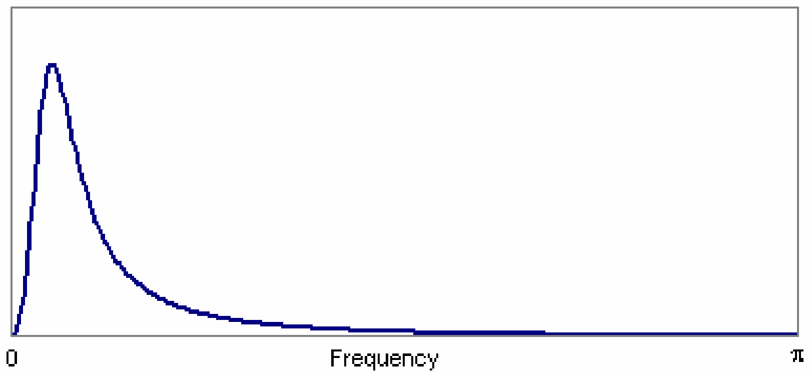


The stochastic shock perturbing equation (4.7) can be different from white noise and allow for some autocorrelation. A more general stochastic cycle can be represented by the ARMA (2, Q) model

$$\phi_c \ (B) \, c_t = \theta_c \ (B) \, a_{ct} \qquad (4.9)$$

with $\phi_c(B)$ containing complex roots associated with a cyclical frequency, and often Q = 2. The MA part may affect the width or the minima of the cycle spectrum. For example, if $\theta_c(B) = 1 - B^2 = (1-B)(1+B)$, the cycle spectrum will display zeros for $\omega = 0$ and $\omega = \pi$, as in Figure 12.

Fig. 12: Spectrum of Doubly Canonical Cyclical Component



The presence of spectral zeros will make the cycle component "canonical", so that no additive white noise can be extracted from it.

c. An Apparent Paradox

Economists have known for a long time that many economic series are cyclical. Yet despite some exceptions, estimation of ARIMA models for macroeconomic series seldom evidences cyclical effects (complex AR roots for cyclical frequencies). Should we reject in these cases the presence of cycles? Or, given that differencing may strongly affect cyclical frequencies, does this means that when we difference we cannot identify cycles? To this issue we turn next.

4.3 A Modified Hodrick-Prescott Filter

We introduce a change in the parametrization. The filter, as presented in Section 4.1, depends on a parameter λ that does not have an easy interpretation. Knowing λ, the gain $G(\omega)$ of the filter is given by (4.6), and using (2.5), it can alternatively be expressed as a function of the period (Figures 13a and b).

Fig. 13a: Gain as function of frequency

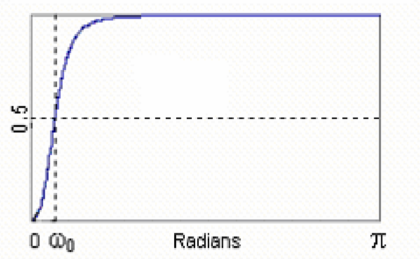
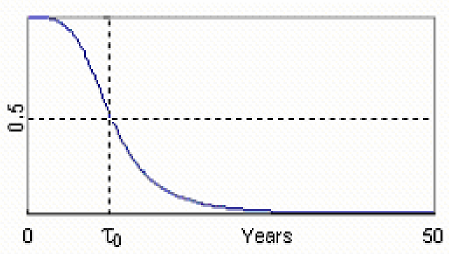


Fig. 13b: Gain as function of period



Consider ω_0 and τ_0, the values for which the gain equals ½, i.e.,

$$G(\omega_0 = 2\pi/\tau_0) = .5 \quad (4.10)$$

Heuristically, for $\tau > \tau_0$, $(\omega < \omega_0)$, most of the series will go to the trend, and for $\tau < \tau_0$, $(\omega > \omega_0)$, most of the series will go to the cycle. Thus τ_0 (or ω_0) represent the "cutting point" for the trend-cycle partition. In fact, ω_0 is the parameter used by engineers to characterize the filter in its Butterworth expression. The relationship between the two parameters, λ_0 and τ_0, is obtained from solving (4.10), which yields, considering (4.6),

$$\tau_0 = 2\pi / a\cos\left(1 - \frac{1}{2\sqrt{\lambda_0}}\right) \tag{4.11}$$

The parameter τ has a more direct interpretation than λ. For example, "cycles with periods beyond 10 years should be mostly assigned to the trend" is a more easy-to-understand assumption than "$\lambda = 1600$". Therefore, for business-cycle analysis a sensible strategy to apply the HP filter could be:

i. A priori choice of the cutting point τ_0
ii. Obtain $\lambda_0 = f(\tau_0)$ though (4.11)
iii. Obtain $\theta_{HP}(B)$ and V_b as described in the Appendix
iv. Apply the WK filter to obtain $\hat{m}_t$ and $\hat{c}_t$

Following Kaiser and Maravall (2001), two modifications will be made to the standard application of the HP filter.

– It has often been pointed out that the behavior of the estimated cycle for the end periods is highly unstable. This instability is partly due to the fact that the HP is a two-sided filter, and hence is subject to revisions as more data become available. Preliminary estimators can be obtained with the WK filter applied to the available series extended with forecasts and backcasts. Standard application of the HP filter can be seen to be the same as the WK implementation, with the series extended with forecasts and backcasts generated by the (fixed) model (4.3), which will in general be poor. When the series is extended with an appropriate ARIMA model, end-point stability is significantly increased. In what follows we assume that the filter is always applied to appropriately extended series.

– As with "seasonal noise", there does not seem much point in leaving highly transitory noise in the series that is input to the HP filter. Thus we shall apply the filter to the trend-cycle component (or noise-free SA series), which shall be denoted p_t.

When these modifications are incorporated, we shall refer to the resulting filter as the "Modified Hodrick-Prescott" (MHP) filter.

4.4 Two-Step Estimation of the Cycle

Assume that the series follows the general ARIMA model

$$\phi(B)\,\nabla^d\,\nabla_r^{d_r}\,x_t = \theta(B)\,a_t \quad ; \qquad a_t \sim w.n.(0, V_a)\,, \tag{4.12}$$

where r denotes the number of observations per year, ∇ and ∇_r denote the regular and seasonal differencing, d and d_r are nonnegative integers (in practice, d = 0, 1, 2, d_r = 0, 1,) $\phi(B)$ is a stationary autoregressive polynomial in B, and $\theta(B)$ is an invertible moving average polynomial in B.

a. First Step

If u_t denotes the noise contained in the series, and s_t its seasonal component, we consider the decomposition of x_t into orthogonal components, as in

$$x_t = p_t + s_t + u_t \tag{4.13}$$

where the first component p_t is the signal of interest for the posterior extraction of the cycle, namely the trend-cycle component. To estimate p_t we follow the AMB procedure. The AR polynomials of the component models are determined from the factorization of the AR polynomial of the ARIMA model for x_t according to the following rule. Let ω denote the frequency of a root expressed in radians. If $\omega \in [0, 2\pi/r)$, the root is allocated to the trend-cycle; if ω is a seasonal frequency (for example $\omega = 2\pi j/r$, $j = 1, ..., 6$, for monthly series,) the root is allocated to the seasonal component; finally, when $\omega \in (2\pi/r, \pi)$ and is not a seasonal frequency, the root is allocated to the irregular component. Thus cycles with period longer than a year will be part of the trend-cycle component, while cycles with periods shorter than a year will go to the irregular one. Following this rule, the polynomial $\phi(B)$ can be factorized as $\phi(B) = \phi_p(B)\,\phi_s(B)\,\phi_u(B)$, and model (4.12) can be rewritten as $[(\phi_p(B)\,\nabla^D)(\phi_s(B)\,S^{d_r})\,(\phi_u(B))]\;x_t = \theta(B)\,a_t$, where $D = d + d_r$, S is the annual aggregation operator $S = 1 + B + \ldots + B^{r-1}$, and use has been made of the identity $\nabla_r = \nabla S$. The first parenthesis groups the trend-cycle AR roots, and the second and third parenthesis group the seasonal and the irregular AR roots, respectively. The components will have models of the type

$$\phi_p(B)\,\nabla^D\,p_t = \theta_p(B)\,a_{pt} \quad , \qquad a_{pt} \sim w.n.(0, V_p) \tag{4.14a}$$

$$\phi_s(B)\,S^{d_r}\,s_t = \theta_s(B)\,a_{st} \quad , \qquad a_{st} \sim w.n.(0, V_s) \tag{4.14b}$$

$$\phi_u(B)\,u_t = \theta_u(B)\,a_{ut} \quad , \qquad a_{ut} \sim w.n.(0, V_u) \tag{4.14c}$$

with the variables a_{pt} , a_{st} , a_{ut} mutually uncorrelated. Consistency between the "reduced form" model (4.12) and the "structural model" (4.14a, b, c) requires that the MA polynomials $\theta_p(B)$, $\theta_s(B)$, $\theta_u(B)$, and the variances V_p, V_s, V_u, satisfy the identity

$$\begin{aligned} \theta(B)\,a_t = {} & \phi_s(B)\,S^{d_r}\,\phi_u(B)\,\theta_p(B)\,a_{pt} + \\ & + \phi_p(B)\,\nabla^D\,\phi_u(B)\,\theta_s(B)\,a_{st} + \\ & + \phi_p(B)\,\phi_s(B)\,\nabla^D\,S^{d_r}\,\theta_u(B)\,a_{ut}\,. \end{aligned} \tag{4.15}$$

Applying (2.12), the WK estimators of the trend-cycle, seasonal and irregular components are given by

$$\hat{p}_t = \frac{V_p}{V_a} \left\| \frac{\theta_p(B)\,\phi_s(B)\,\phi_u(B)\,S^{d_r}}{\theta(B)} \right\|^2 x_t \tag{4.16a}$$

$$\hat{s}_t = \frac{V_s}{V_a} \left\| \frac{\theta_s(B)\,\phi_p(B)\,\phi_u(B)\,\nabla^D}{\theta(B)} \right\|^2 x_t \tag{4.16b}$$

$$\hat{u}_t = \frac{V_u}{V_a} \left\| \frac{\theta_u(B)\phi_p(B)\phi_s(B)\nabla^D S^{d_r}}{\theta(B)} \right\|^2 x_t \qquad (4.16c)$$

and it is straightforward to verify that $x_t = \hat{p}_t + \hat{s}_t + \hat{u}_t$.

b. Second Step

In the MHP procedure, the trend-cycle estimator $\hat{p}_t$ is used as input to the HP filter. From (4.4b), (4.5) and (4.16a),

$$\hat{c}_t = k_c \frac{(1-B)^2 (1-F)^2}{\theta_{HP}(B)\theta_{HP}(F)} \hat{p}_t = \qquad (4.16d)$$

$$= k_c \frac{V_p}{V_a} \left\| \frac{\theta_p(B)\phi_s(B)\phi_u(B)\nabla^2 S^{d_r}}{\theta_{HP}(B)\ \theta(B)} \right\|^2 x_t \quad ,$$

$$\hat{m}_t = k_m \frac{V_p}{V_a} \left\| \frac{\theta_p(B)\phi_s(B)\phi_u(B) S^{d_r}}{\theta_{HP}(B)\ \theta(B)} \right\|^2 x_t \qquad (4.16e)$$

For a finite sample, extending the series x_t with backcasts and forecasts computed with the correct model (4.12), the above expressions provide the MHP two-step estimators of the cycle (c_t) and trend (m_t), respectively.

4.5 A Complete Unobserved Component Model

In the MHP two-step procedure, a full decomposition of the series is finally obtained, namely

$$x_t = \hat{m}_t + \hat{c}_t + \hat{s}_t + \hat{u}_t \qquad (4.17)$$

where the estimators in the r.h.s. of the equation are given by the expressions (4.16b-e). The question is: can these estimators be the direct MMSE estimators of the UCs in a full decomposition of the series of the type

$$x_t = m_t + c_t + s_t + u_t \qquad (4.18)$$

where m_t, c_t, s_t, and u_t are the (orthogonal) trend, cycle, seasonal, and irregular components, all with sensible models that aggregate into the ARIMA model (4.12) identified for the series x_t? The answer is in the affirmative, and follows from Result 2.

Result 3

Let x_t be an observed series that follows the general ARIMA model (4.12). Consider the UC model consisting of the aggregate equation (4.18), the models for the seasonal and irregular components (4.14b, c) (obtained from the standard AMB decomposition of x_t, as in the first of the two-step procedure,) plus the following models for the trend and cycle components:

$$\theta_{HP}(B)\nabla^{D} m_t = \psi_p(B) a_{mt} \quad , \quad a_{mt} \sim w.n.(0, k_m V_p / V_a); \qquad (4.19)$$

$$\theta_{HP}(B) c_t = \psi_p(B) \nabla^{2-D} a_{ct} \quad , \quad a_{ct} \sim w.n.(0, k_c V_p / V_a); \qquad (4.20)$$

where $\psi_p(B) = \theta_p(B) / \phi_p(B)$, and a_{st}, a_{ut}, a_{mt}, and a_{ct} are mutually uncorrelated. Then, the MMSE estimators of m_t, c_t, s_t, and u_t in the full model are the MHP two-step estimators (4.16 b-e).

(The result follows from direct application of the WK filter to the complete UC model.) Further, $\hat{m}_t + \hat{c}_t = H(B,F)\left[k_m + \nabla^2 (1-F)^2 k_c\right] x_t$, where

$$H(B,F) = \frac{V_p}{V_a} \left\| \frac{\theta_p(B)\phi_s(B)\phi_u(B) S^{d_s}}{\theta_{HP}(B)\,\theta(B)} \right\|^2$$

and, considering (4.5), it is obtained that

$$\hat{m}_t + \hat{c}_t = \frac{V_p}{V_a} \left\| \frac{\theta_p(B)\phi_s(B)\phi_u(B) S^{d_s}}{\theta(B)} \right\|^2 x_t$$

or, according to (4.16a), $\hat{m}_t + \hat{c}_t = \hat{p}_t$. Similarly, from (4.2) and (4.3), it is straightforward to show that the components also satisfy $m_t + c_t = p_t$, with p_t given by (4.14a). Thus aggregation of the four components or of the four estimators yields the ARIMA model for the observed series.

Some features of the complete UC model are worth mentioning.

i. The argument has been made for the historical estimators, which can be assumed for the central years of a long-enough series. Estimation of the signal at the end points of the series is equal to the application of the full filter to the series extended with forecasts and backcasts. End-point estimation of the trend and cycle in the 2-step procedure requires forecasts and backcasts of the trend-cycle component, while the full UC model requires forecasts and backcasts of the observed series x_t. The two extension procedures however are identical because the forecasts of p_t are obtained by extending further the series x_t with more forecasts and backcasts. In both procedures, the forecasts of x_t are computed with the identified model. Having the same filter and the same extended series, the preliminary trend and cycle estimators obtained with the 2-step method will be identical to the direct estimators in the full UC model. (Notice that MMSE forecasts of the cycle can be obtained in the same way as end-point estimators. Thus the forecasts will also be identical.)

ii. A similar result can be derived when the estimator of the SA series $\hat{n}_t$ is used as input of the HP filter. However, part of the irregular (or transitory noise) component will be absorbed by m_t and (mostly) c_t, and the cyclical signal will be contaminated by noise.

iii. The model is obtained from the AMB decomposition by simply splitting the trend-cycle component into separate (long-term) trend and cycle components, with the split determined by the choice of the "cutting point" τ_0 (or λ_0) for the HP filter.

iv. The seasonal and irregular components are those of the standard AMB decomposition. What are new are the trend and cycle models. These models share the polynomials $\theta_{HP}(B)$ and $\psi_p(B)$, but given that the shared AR roots are stationary, the estimators MSE will be bounded and converge to a finite value (Pierce, 1979).

v. The models for the trend and cycle components incorporate "a priori" and series-dependent features. The first ones ($\theta_{HP}(B)$, k_m, and k_c) are determined by the parameter τ_0 (or λ_0,) and reflect desirable features of the filter (broadly, how to split the frequencies between trend and cycle). The polynomial $\psi_p(B)$ and the variance V_p are series dependent, and guarantee consistency with the model identified for the series.

vi. Given that p_t is obtained from the AMB decomposition of x_t, both components, trend and cycle, have to be canonical and will display a spectral zero for $\omega = \pi$.

vii. The order of integration at the zero frequency of the trend will be equal to that of the observed series.

viii. The cycle will be stationary as long as d < 3. The spectrum of the cycle will have the shape of a distribution skewed to the right (for quarterly or monthly series), and with a well-defined mode. Besides the spectral zero for $\omega = \pi$, when d < 2 the spectrum will contain an additional zero for $\omega = 0$ (and hence, will be doubly canonical).

ix. We have concluded that the MHP 2-step procedure is the same as MMSE estimation of the components in a full UC model, and that the reduced form of this model is the ARIMA model identified for the observed series. The two models are observationally equivalent; they will fit equally well the data, and have the same likelihood and forecast functions. One may disagree with the specification of the components, but the results cannot be properly called spurious.

4.6 First Example: The Cycle in the Airline Model

We consider the so-called "Airline model", popularized by Box and Jenkins (1970), which has been found appropriate for many economic series. For quarterly series the model is given by

$$\nabla \nabla_4 x_t = (1+\theta_1 B)(1+\theta_4 B^4) a_t \tag{4.21}$$

with $|\theta_1| < 1$ and $-1 < \theta_4 < 0$. Setting $V_a = 1$, and $\theta_1 = -.4$, $\theta_4 = -.6$ (the values used by Box and Jenkins), the AMB decomposition of x_t with program SEATS into (4.13) yields the following models for the components.

$$\nabla^2 p_t = (1 + .119\,B - .881B^2)\,a_{pt} \quad , \qquad V_p = .064 \tag{4.22a}$$

$$S\,s_t = (1 - .046\,B - .496\,B^2 - .458\,B^3)\,a_{st} \quad , \qquad V_s = .019 \tag{4.22b}$$

$$u_t = w.n.(0, V_u = .305) \quad . \tag{4.22c}$$

For the second step, in order to split the trend-cycle (p_t) into trend (m_t) plus cycle (c_t), the polynomial $\theta_{HP}(B)$, as well as k_c and k_m are needed. Setting $\lambda = 1600$, it is obtained that (see the Appendix)

$$\theta_{HP}(B) = 1 - 1.7771B + .7994\,B^2 \quad , \qquad V_b = 2001.4 \tag{4.23}$$

so that $k_m = 1/2001.4$ and $k_c = 1600/2001.4$. The models for the trend and cycle can now be specified as

$$(1 - 1.777\,B + .799\,B^2)\,\nabla^2 m_t = (1 + .119\,B - .881 B^2)\,a_{mt} \tag{4.24}$$

$$(1 - 1.777\,B + .799\,B^2)\,c_t = (1 + .119\,B - .881 B^2)\,a_{ct} \tag{4.25}$$

with $V_m = .32\ (10\ -4)$ and $V_c = .0512$. The model for m_t is I(2) while the model for c_t is stationary; both are noninvertible due to a spectral zero at $\omega = \pi$. The AMB spectral decomposition of x_t into p_t and s_t is presented in Figure 14 (the spectrum of u_t is a constant,) and the spectral decomposition of p_t into m_t and c_t is displayed in Figure 15. Although the spectrum of p_t does not exhibit any peak for a cyclical frequency, it can be split into a smooth nonstationary peak around the zero frequency (m_t), and a stationary spectrum with a well-defined peak for a cyclical frequency (c_t). The period associated with this peak is approximately 13 years. Figure 16 exhibits the squared gains of the filters to estimate the trend-cycle, trend, and cycle, all of which display sensible shapes.

Fig. 14: Spectrum Series, trend-cycle and seasonal

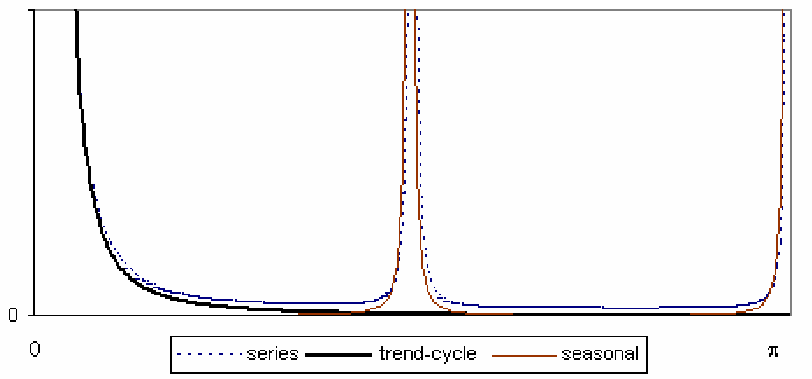
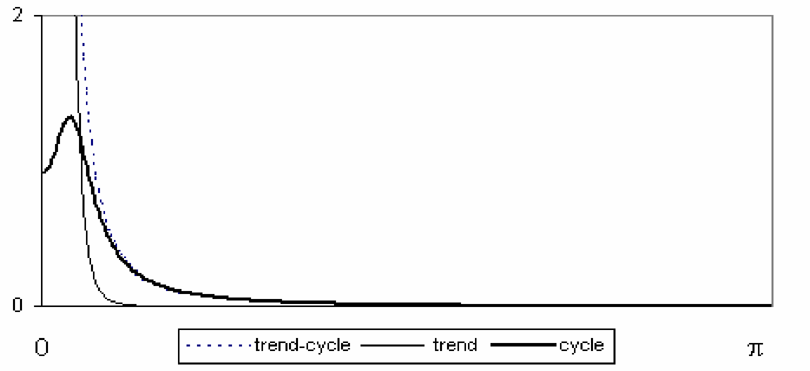
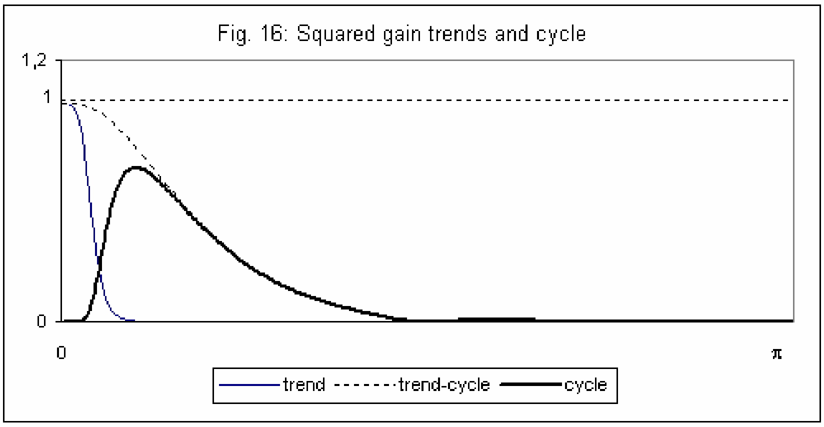


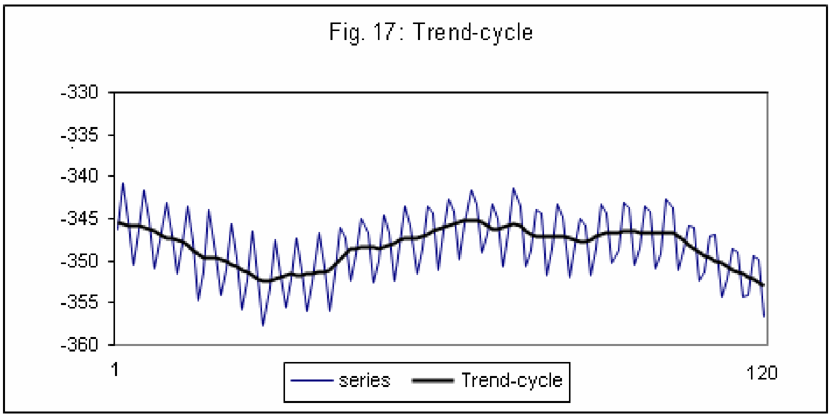
Fig. 15: Spectrum trend and cycle

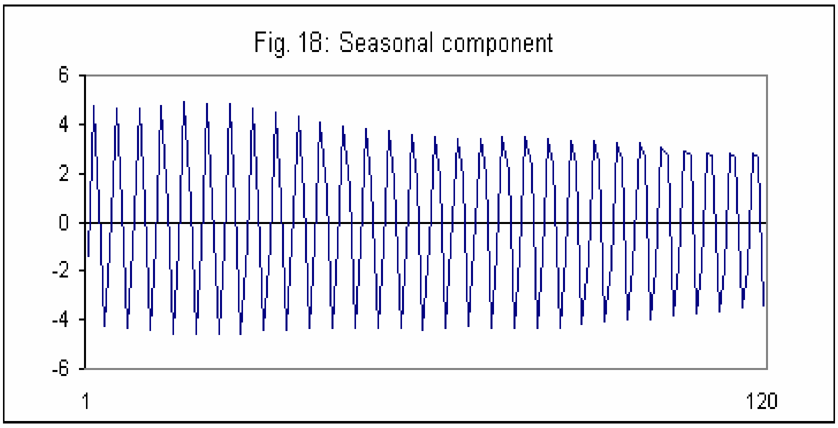




Figures 17 to 21 provide an example: the decomposition of a Spanish quarterly economic indicator over a 30 year period. For the cycle, the 95% confidence interval implied by the revision error has also been included. The standard error (SE) of the revision for the concurrent estimator of the cycle is about 1/3 of the SE of its one-period-ahead forecast error, and it takes about 3 years for the revision to become negligible.

Forecasts of the cycle (and associated SE) can be obtained in the same way as end-point (preliminary) estimators. However, due to the size of the SE, and to the fact that the stationary model for the cycle implies a forecast function that converges to zero, these forecasts are of limited interest in practice.





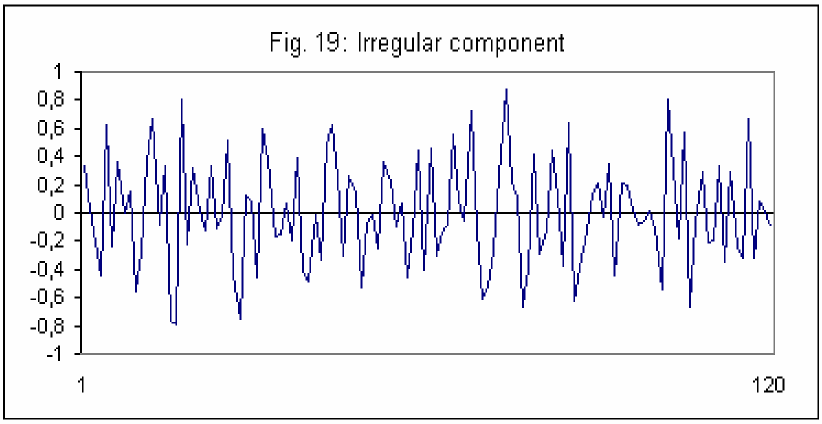


Fig. 19: Irregular component
1
0,8
0,6
0,4
0,2
0
-0,2
-0,4
-0,6
-0,8
-1
1
120

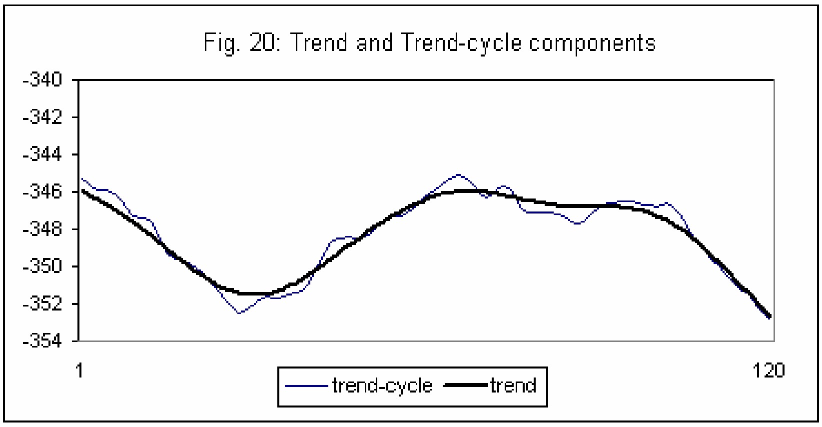


Fig. 20: Trend and Trend-cycle components
-340
-342
-344
-346
-348
-350
-352
-354
1
120
trend-cycle
trend

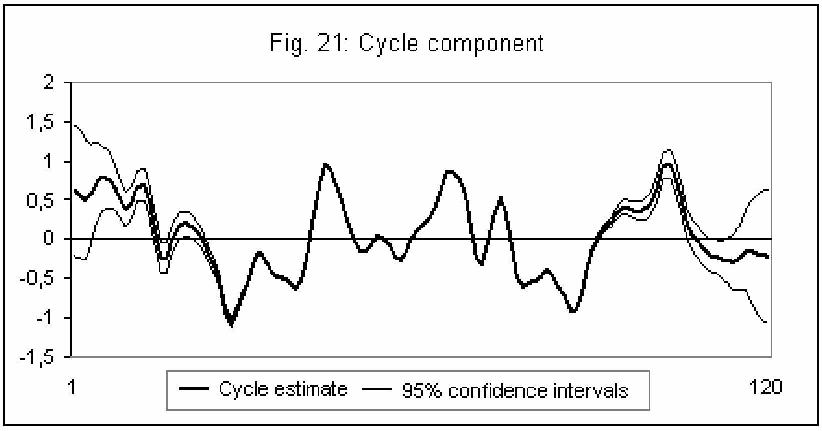


Fig. 21: Cycle component
2
1,5
1
0,5
0
-0,5
-1
-1,5
1
120
Cycle estimate
95% confidence intervals

It is of interest that the cycle obtained in the 2-step procedure, that mixes data-consistency with ad-hoc desirable features, turns out to be an ARMA (2,2) model with the AR roots associated with a cyclical frequency: that is, a linear stochastic process of the type discussed in section 4.2b. This model, given by (4.25), has $\theta_{HP}(B)$ as the AR polynomial, which is determined **a priori** from τ_0 (or λ_0). This a priori choice will shape the eventual ACF of c_t, its eventual forecast function, and strongly influence its spectrum.

On the other hand, the MA polynomial $\theta_p(B)$ and V_p in model (4.25) are determined from the model for p_t, obtained in the AMB decomposition of the model for the observed series. Factorization of $\theta_p(B)$ yields $\theta_p(B) = (1+B)(1-.958\,B)$. The first root implies a spectral zero for $\omega = \pi$, and the second root implies a spectral (local) minimum close to zero for $\omega = 0$.

4.7 A Remark on Identification

Incorporation of the HP filter to the AMB procedure implies decomposing the trend-cycle component (p_t) into orthogonal trend (m_t) and a stationary cycle (c_t). Considering (4.22a), (4.24), and (4.25), this decomposition is of the type

IMA (2,2)	=	ARIMA (2,2,2)	+	ARMA (2,2)
trend-cycle		long-term trend		cycle

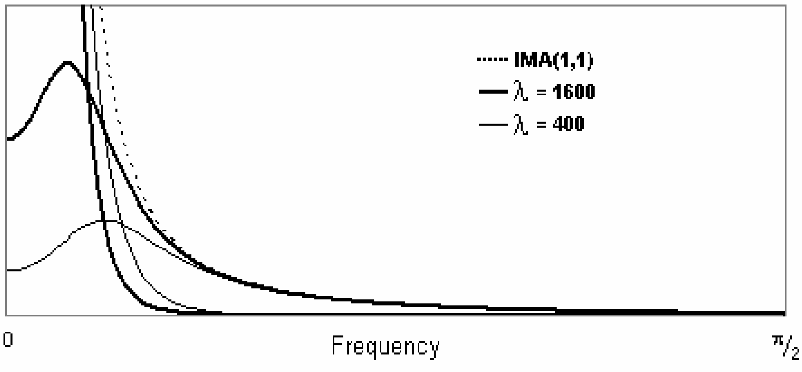
Given the l.h.s. of this identity (i.e., he IMA (2,2) model for p_t), the r.h.s. decomposition depends on $\theta_{HP}(B)$, k_m, and k_c, all determined from the HP-filter parameter λ. Therefore, for each value λ in R^+, a different "trend + cycle" decomposition of the same trend-cycle component will be obtained. Specifying a particular value of λ, a particular decomposition is obtained. For example, setting $\lambda = 400$ and applying the algorithm in the Appendix, yields a model similar to the one in the previous example (for which $\lambda = 1600$), but with the new set of parameters

$$\theta_{HP}(B) = 1 - 1.6857\,B + .7284\,B^2 ; \tag{4.26}$$

$$k_c = .7284 \;;\quad k_m = .00182$$

Thus (4.22b and c) remain unchanged in the new UC model, but the AR polynomial in (4.24) and (4.25) will now be (4.26), and $V_c = k_c V_p / V_a = .0012$, $V_m = k_m V_p / V_a = .0466$. Figure 22 compares the spectra of the two decompositions of p_t obtained for the two values of λ. In both cases, the sum of the trend and the cycle spectra yields the same aggregate spectrum: that of the IMA (2,2) model for p_t given by (4.22a), the dotted line in the figure.

Fig. 22: Trend/Cycle descomposition of IMA (1,1)



The basic identification problem in terms of the cycle and trend components can be seen as the choice of an appropriate value for λ or τ. At this stage, desirable features can be introduced: for example, a priori choice of the cycle period τ_0 that is the cutting point between periods that belong mostly to the cycle or to the trend. Setting $\tau = \tau_0$ identifies a particular decomposition.

4.8 Second Example: Stationary Series

Although the naïve model-based derivation of the HP filter, given by (4.1), implied an I(2) trend, Result 3 holds for any order of integration. Consider, for example, the stationary AR(1) model $(1-.8\,B)\,x_t = a_t$, $V_a = 1$. The AMB decomposition yields $x_t = p_t + u_t$, with the following models for the components $(1-.8\,B)\,p_t = (1+B)\,a_{pt}$, $V_p = .247$; $u_t = w.n.$, $V_u = .309$.

Therefore, the complete unobserved component model is given by $x_t = m_t + c_t + u_t$, where the components follow the models

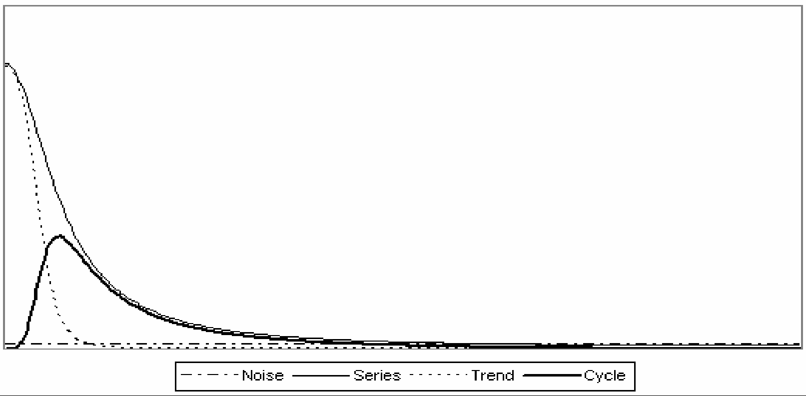
$$(1-.8B)\,\theta_{HP}(B)\,m_t = (1+B)\,a_{mt} \qquad a_{mt} \sim w.n.(0, V_m)$$

$$\theta_{HP}(B)\,c_t = (1+B)\,(1-B)^2\,a_{ct} \qquad a_{ct} \sim w.n.(0, V_c)$$

$$u_t = w.n.(0, V_u)\;;$$

with $V_m = k_m\,V_p / V_a$ and $V_c = k_c\,V_p / V_a$. Assuming annual data and $\lambda = 7$ (the value approximately equivalent to the quarterly value of 1600), the associated parameters in $\theta_{HP}(B)$, plus k_m and k_c are given in Table A of the Appendix. The spectral decomposition of the AR(1) is shown in Figure 23 and represents a sensible decomposition of a trend-cycle into separate trend and cycle.

Fig. 23: MHP "Trend plus cycle": Spectral Descomposition of an AR(I)



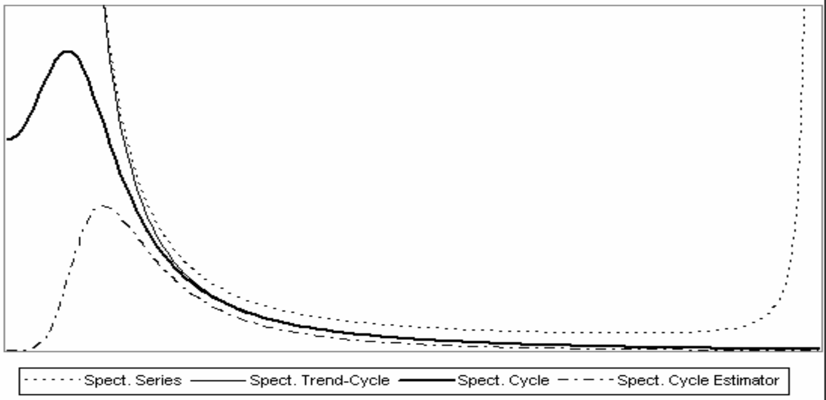
4.9 Distortion in MMSE Estimation of the Cycle Component

MMSE (historical) estimation of c_t in the full UC model provides an expression of the type $\hat{c}_t = \eta(B,F)\,x_t$, and, from the ARIMA model for x_t, one can obtain $\hat{c}_t$ as a filter applied to a_t, say $\hat{c}_t = \xi(B,F)\,a_t$. Back to the Airline model example of Section 4.6, after simplification, it is obtained that

$$\hat{c}_t = \left[\left(k_c\,V_p\right) \frac{\theta_p(B)}{\theta_{HP}(B)} \; \frac{\theta_p(F)\,\overline{\nabla}\,\overline{\nabla}_4}{\theta_{HP}(F)\,\theta(F)} \right] a_t \qquad (4.27)$$

where $\overline{\nabla} = 1-F$, $\overline{\nabla}_4 = 1-F^4$ and $\theta(F) = (1-.4\,F)\,(1-.6\,F^4)$. The spectrum of $\hat{c}_t$ is shown in Figure 24. Comparison of the spectrum of the component (4.25) with that of its estimator (4.27) illustrates a well-known feature of MMSE estimation (see, Nerlove, Grether and Carvalho, 1979): the estimator underestimates the variance of the component. For the case of the cyclical component, this loss of variance affects mostly the lower frequencies. As a result, the estimator inflates the relative importance of the higher frequencies and the spectral peak is pushed to the right, implying a shorter period. Therefore, when interpreting an estimated cycle in the model-based framework, one should be aware that MMSE estimation will bias downwards the modal period implied by the theoretical model for the cycle.

Fig. 24 Spectra of Cyclical Component and its MMSE Estimator



APPENDIX: COMPUTATION OF θ_1^{HP}, θ_2^{HP}, AND V_b FOR HP FILTER GIVEN λ

WK implementation of the HP filter requires the IMA(2,2) specification (4.3) for x_t, namely, the parameters θ_1^{HP}, θ_2^{HP}, and V_b. Given λ, they can be obtained as follows. (All square roots are taken with their positive sign.) Compute sequentially:

$$(1) \quad a = 2, \quad b = \frac{1}{\sqrt{\lambda}}, \quad k = -b^2, \quad s = ab$$

$$(2) \quad z = \sqrt{\frac{1}{2\lambda}\left(1+\sqrt{1+16\lambda}\right)}; \quad r = \frac{s}{z}$$

$$(3) \quad m_1 = \frac{-a+r}{2} \quad , \quad n_1 = \frac{z-b}{2}$$

$$(4) \quad m_2 = \frac{-a-r}{2} \quad , \quad n_2 = \frac{-z-b}{2}$$

(5) Of the two complex numbers $(m_1 + in_1)$ and $(m_2 + in_2)$ pick up the one with smallest modulus. Let this number be $R = M + iN$; then,

$$(6) \quad \theta_1^{HP} = 2M, \; \theta_2^{HP} = M^2 + N^2, \; V_b = \frac{1+6\lambda}{1+\left(\theta_1^{HP}\right)^2 + \left(\theta_2^{HP}\right)^2}$$

Notice that implementation of the WK filter requires k_c and k_m, computed as $k_c = \lambda / V_b$ and $k_m = 1/V_b$.

The following table presents the values of $\theta_{HP}(B)$, V_b, and the period τ associated with a filter gain equal to .5, for several values of λ. The first three values comprise the standard quarterly value $\lambda = 1600$, and the monthly and annual values implied by temporal aggregation following the criterion of Maravall and del Río (2001), which preserves the period of the cycle associated with a gain of .5. [This criterion yields values that are close to those proposed by Ravn and Uhlig (2002).]

TABLE A: WK FILTER PARAMETERS FOR DIFFERENT VALUES OF λ

	FREQUENCY OF OBSERV.	λ	θ_1	θ_2	V_b	τ (approx.)
Approximately equivalent values under aggregation	Monthly	130 000	-1.9255	.9282	140 050	120
	Quarterly	1 600	-1.7771	.7994	2 001.4	40
	Annual	7	-1.1706	.4137	16.92	10
E-Views	Annual	100	-1.5583	.6382	156.68	20
OECD …	Monthly	14 400	-1.8710	.8788	16 385	69

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