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Abstract

A common finding in empirical studies using micro data on consumer and producer prices is that hazard functions for price changes are decreasing. This means that a firm will have a lower probability of changing its price the longer it has kept it unchanged. This result is at odds with standard models of price setting. In this note a simple explanation is proposed: decreasing hazards may result from aggregating heterogeneous price setters. We show analytically the form of this heterogeneity effect for the most commonly used pricing rules and find that the aggregate hazard is (nearly always) decreasing. Results are illustrated using Spanish producer and consumer price data. We find that a very accurate representation of individual data is obtained by considering just 4 groups of agents: one group of flexible Calvo agents, one group of intermediate Calvo agents and one group of sticky Calvo agents plus an annual Calvo process.

Key words: hazard function, price setting models, heterogeneous agents, mixture models.
***JEL* Codes:** C40, D40, E30.

1 Introduction

A common finding in empirical studies using micro data on consumer and producer prices is that hazard functions for price changes are decreasing[1] (see figure 1). This means that a firm will have a lower probability of changing its price the longer it has kept it unchanged. This result is at odds with standard theoretical models of price setting.
The explanation to this puzzle proposed in this paper is that unconditional decreasing hazards are due to the aggregation of heterogeneous price setters, and thus decreasing hazards are not necessarily evidence against standard models (e.g. Taylor, Calvo or truncated Calvo). The intuition is as follows. By definition, the probability of observing price changes is lower for firms with sticky price schemes than for firms following flexible pricing rules, while the aggregate hazard considers price changes for all firms. Therefore, when the aggregate hazard function is obtained, the share of price changes corresponding to firms with more flexible pricing rules decreases as the horizon increases and, consequently, the hazard rate also decreases[2].

In this paper we formalise this idea by analysing the consequences for the aggregate hazard rate of the coexistence of firms with different pricing rules. In particular, we show that if micro data are generated by heterogeneous firms then the aggregate hazard is (nearly always) decreasing. We provide analytical expressions for these heterogeneity effects in the most widely used pricing models.

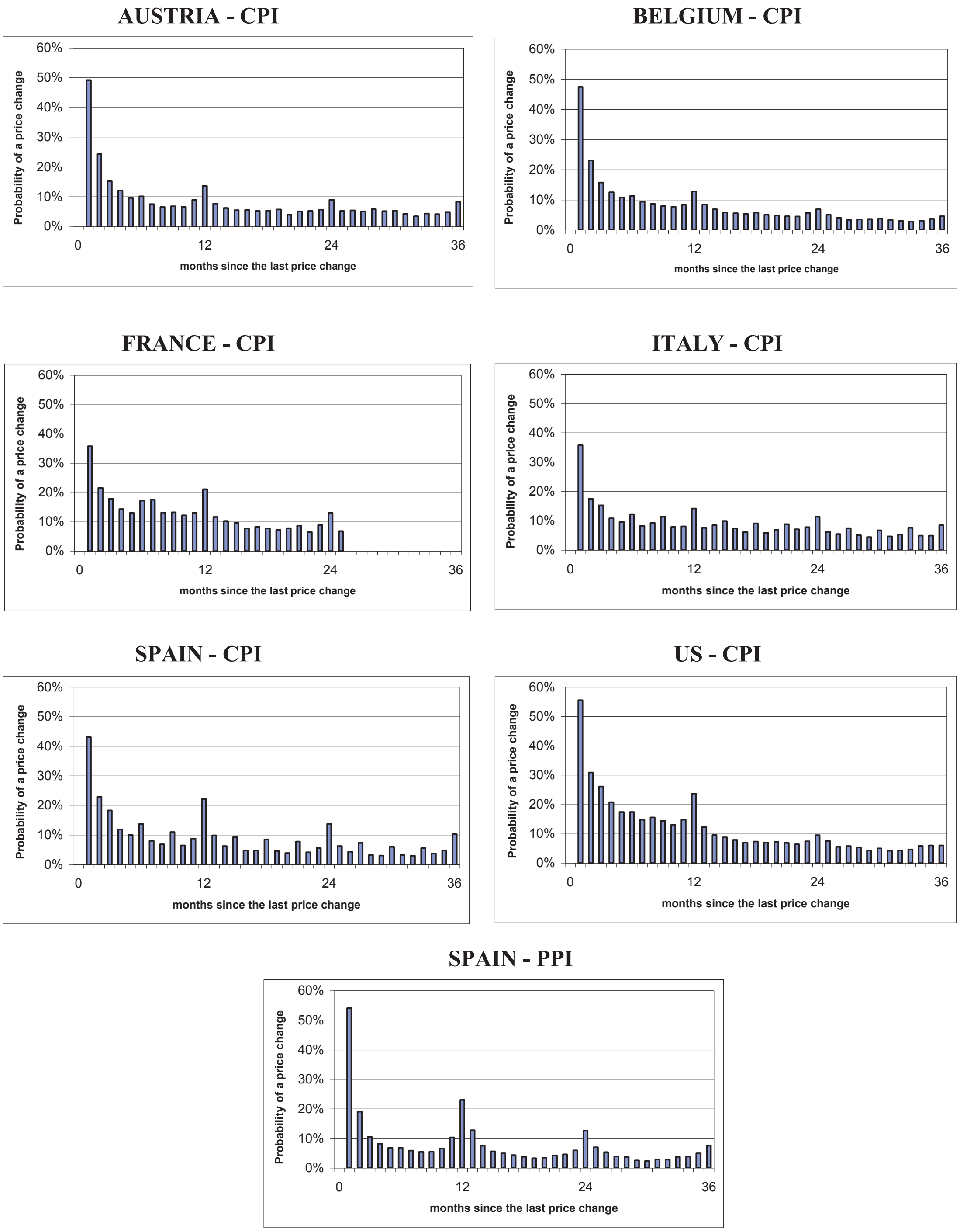
Moreover, in the empirical section, we test some of these theoretical results using Spanish consumer (CPI) and producer price (PPI) micro data. We take a parsimonious approach, assuming that the aggregate economy is composed of Calvo agents with different average price duration[3], and let the data determine the optimal number of groups. In particular, we estimate a finite mixture of Calvo models considering 1, 2, 3, 4 and 5 groups and then choose the optimal model according to several model selection criteria.

[1] A more detailed description of the empirical evidence on consumer prices (CPI) can be found in Baumgartner et al (2004) for Austria, Aucremanne and Dhyne (2004) for Belgium, Fougère et al (2004) for France, Veronese et al (2004) for Italy, Alvarez and Hernando (2004) for Spain, Campbell and Eden (2004) and Klenow and Krystov (2004) for the United States. In addition, Dhyne et al (2004) review the empirical evidence for CPI across euro area countries. Finally, empirical evidence on producer prices is found in Alvarez et al (2004) for Spain and in Stahl (2004) for Germany.

[2] Conditional estimations of the hazard function of price spells by Aucremanne and Dhyne (2004), Dias, Robalo Marques and Santos Silva (2004) and Fougère, Le Bihan and Sevestre (2004) indicate that accounting for product heterogeneity of products indeed reduces the negative slope of the hazard function. These results are consistent with the hypothesis that the declining overall hazard is mainly a result of aggregation.

[3] The Calvo (1983) model of time dependent price setting involves a simple analytical expression for the hazard function (as well as the density and survival), which requires estimating only one parameter per group. An alternative would be Taylor's (1980) model. However, in this context, this model is less parsimonious since it requires having as many groups as exit times or actual spell durations observed in the data.

Figure 1: International evidence on decreasing hazard functions for price changes



We find the most adequate description of both the CPI and PPI data is a model with 4 Calvo groups of agents: one group with a very flexible pricing rule that results in an average duration slightly over 1 month; another group with intermediate flexibility (average price-duration is around 10 months); and, a group with very sticky prices, which are kept constant on average for more than 3 years; plus a group of firms with an annual Calvo pricing rule, with an average price duration of around a year and a half. In terms of the relative size of the groups, the largest is the intermediate group, accounting for around 50% of the production value in the case of the PPI and 57% of consumer's expenditure in the case of the CPI. The flexible and sticky Calvo groups are roughly similar in size in terms of the share of PPI (slightly above 20%). In the case of CPI, the share of flexible Calvo agents (13%) is lower than the share of the sticky Calvo group (18%). Finally, the annual Calvo group is the smallest one, accounting for 7% of PPI and 12% of CPI. An analysis of the composition of the groups in terms of the different types of goods and services provides interesting results. Specifically, we observe that the flexible pricing rule is used mostly by producers of energy and intermediate goods and by retailers of food products; the intermediate rule is used by all producers and retailers, except energy producers and retailers of unprocessed food; and the sticky and annual Calvo pricing rules are mainly used by producers of capital and consumer durable goods and by retailers of non-energy goods and services.

The structure of this paper is as follows. Section 2 presents the analytical expression of the hazard for the aggregate economy. Section 3 shows these results for the Calvo, Taylor and Dotsey, King and Wolman's price-setting mechanisms. Section 4 presents the results of an empirical application for Spanish producer and consumer price data as well as the econometric methodology used. Finally, section 5 concludes.

2 General case

The aim of this section is to present the relationship between the (change in the) hazard rate of an aggregate economy and the (change in the) hazard rate of the groups of agents composing it. We use throughout a discrete time approach since this is the one most frequently used for price setting models.

First of all, it is assumed that the aggregate economy is composed of two groups of agents with different hazard functions, with sizes s_1 and s_2, respectively.

The hazard rate is the probability that a price will change in period k, provided that it has remained constant during the previous $k-1$ periods[4]. More formally, the hazard rate for group i

[4] k is the elapsed time since start of the price spell.

is given by

$$h^i(k) = \frac{f^i(k)}{S^i(k)}$$

where $f^i(k)$ is the density function, which measures the frequency of firms adjusting prices in period k and $S^i(k)$ is the survival function, which measures the frequency of firms which have kept their prices constant during the previous $k-1$ periods.

For the aggregate economy, the aggregate frequency of firms changing prices in period k and the aggregate frequency of firms not having adjusted prices in the previous $k-1$ periods are given by

$$f(k) = \lambda f^1(k) + (1-\lambda) f^2(k)$$

$$S(k) = \lambda S^1(k) + (1-\lambda) S^2(k)$$

where $\lambda = \frac{s_1}{s_1+s_2}$ is the share of firms of group 1 in the economy as a whole. That is, the density function and the survival function of the aggregate economy are convex linear combination of the respective functions for each of the groups of firms, with fixed weights equal to the relative size of each group.

In turn, the hazard rate of the aggregate economy in period k can be expressed as

$$h(k) = \beta(k) h^1(k) + [1-\beta(k)]\, h^2(k)$$

where the weight $\beta(k) = \left(\frac{S^1(k)}{S(k)}\right)$ is a function of k and thus not constant along the hazard. Therefore, the aggregate hazard is a convex linear combination of individual hazards, although the weights vary with k.

It is straightforward to show[5] that the change in this aggregate hazard, for a given change in k, is equal to

$$\frac{\Delta h(k)}{\Delta k} = \frac{\Delta h^1(k)}{\Delta k}\beta(k) + \frac{\Delta h^2(k)}{\Delta k}\left[1-\beta(k)\right] + H(k) \tag{1}$$

[5] Note that

$$\frac{\Delta h(k)}{\Delta k} = \frac{\Delta h^1(k)}{\Delta k}\beta(k) + \frac{\Delta h^2(k)}{\Delta k}\left[1-\beta(k)\right] + \frac{\Delta \beta(k)}{\Delta k}\left[h^1(k) - h^2(k)\right] + \frac{\Delta \beta(k)}{\Delta k}\left[\frac{\Delta h^1(k)}{\Delta k} - \frac{\Delta h^2(k)}{\Delta k}\right]\Delta k$$

$$\frac{\Delta \beta(k)}{\Delta k} = \frac{\beta(k)\left[1-\beta(k)\right]}{1-h(k)\Delta k}\left[h^1(k) - h^2(k)\right]$$

where

$$
\begin{aligned}
H(k) &= -\beta(k)\,[1-\beta(k)]\left[h^1(k)-h^2(k)\right]^2 \varepsilon(k) \\
& \text{and} \\
\varepsilon(k) &= \left\{ \frac{1+[h^1(k)-h^2(k)]^{-1}\left[\frac{\Delta h^1(k)}{\Delta k}-\frac{\Delta h^2(k)}{\Delta k}\right]\Delta k}{1-h(k)\Delta k} \right\}
\end{aligned}
$$

This expression shows that the change in the hazard rate of an aggregate is a convex linear combination of the change in the hazard rates of its components plus a heterogeneity effect[6]. This heterogeneity effect is the discrete time version of the well known result in the duration analysis literature that not controlling for unobserved heterogeneity biases estimated hazard functions towards negative duration dependence (see Lancaster and Nickell(1980) or Heckman and Singer(1986)). In fact, $\varepsilon(k)$ converges to 1 as Δk tends to zero and the expression of $H(k)$ converges to the continuous time one (see Appendix A). Notice, however, that in the discrete time case the heterogeneity effect will be positive if $\varepsilon(k) < 0$. This contrasts with the continuous time result, where the heterogeneity effect cannot be positive.

Note that, for the three most widely used time dependent pricing rules (Calvo, truncated Calvo[7] and Taylor) the change in individual hazards is zero for all k, so that the slope is completely determined by the heterogeneity effect. For these models, this effect is never positive and so is the slope of the hazard[8].

A necessary and sufficient condition to have a downward sloping hazard is that the third term in equation (1) be larger than the sum of the first two terms

$$
\frac{\Delta h(k)}{\Delta k} \leq 0 \qquad iff \qquad \frac{\Delta h^1(k)}{\Delta k}\beta(k) + \frac{\Delta h^2(k)}{\Delta k}\,[1-\beta(k)] \leq -H(k) \tag{2}
$$

These results can be easily generalised for the case of N groups of firms. In fact, it can be shown that

$$
\begin{aligned}
\frac{\Delta h(k)}{\Delta k} &= \sum_{j=1}^{N} \beta^j(k)\frac{\Delta h^j(k)}{\Delta k} + H(k) \qquad (3) \\
\text{where } H(k) &= -\sum_{j=1}^{N-1}\sum_{l=j+1}^{N} \beta^j(k)\beta^l(k)\left[h^j(k)-h^l(k)\right]^2 \varepsilon_{jl}(k)
\end{aligned}
$$

[6] Note that the heterogeneity effect disappears if the hazards of the two groups are equal ($h^1(k) = h^2(k)$) or if, for a given k, there are no more firms belonging to one group ($\beta(k) = 0$ or $\beta(k) = 1$).

[7] See Wolman (1999) and Dotsey (2002).

[8] Except for the period k when truncation occurs for the truncated Calvo and Taylor cases, where the hazard increases.

Moreover, a similar necessary and sufficient condition to have a downward sloping hazard in this case is

$$\frac{\Delta h(k)}{\Delta k} \leq 0 \qquad if \qquad \sum_{j=1}^{N} \beta^j(k) \frac{\Delta h^j(k)}{\Delta k} \leq -H(k)$$

3 Particular cases

In this section we provide the expressions corresponding to the aggregation of different types of agents, each setting prices according to some of the most widely used models in the literature. First of all, we present results for the two most widely used time-dependent models in the literature, those of Calvo (1983) and Taylor (1980). Then, we propose a new time-dependent model to deal with the existence of firms with annual pricing rules. Finally, we present results based on the state dependent model of Dotsey, King and Wolman (1999)[9].

3.1 Calvo agents

The model of price setting introduced by the seminal work of Calvo (1983) has become one of the most widely used in the current macro literature on sticky prices, mainly due to its theoretical tractability and that is easy to test empirically[10]. This model of price setting assumes that there is a constant probability that a given price setter will change its price at any instant. This, together with the assumption that there are a large number of price setters who act independently, implies that there is a constant proportion of prices being changed at any instant.

The density, survival and hazard functions for this type of agents take the following functional forms

$$f^i(k)s_i = (1-\theta_i)\theta_i^{k-1}s_i$$

$$S^i(k)s_i = \theta_i^{k-1}s_i$$

$$h^i(k) = (1-\theta_i)$$

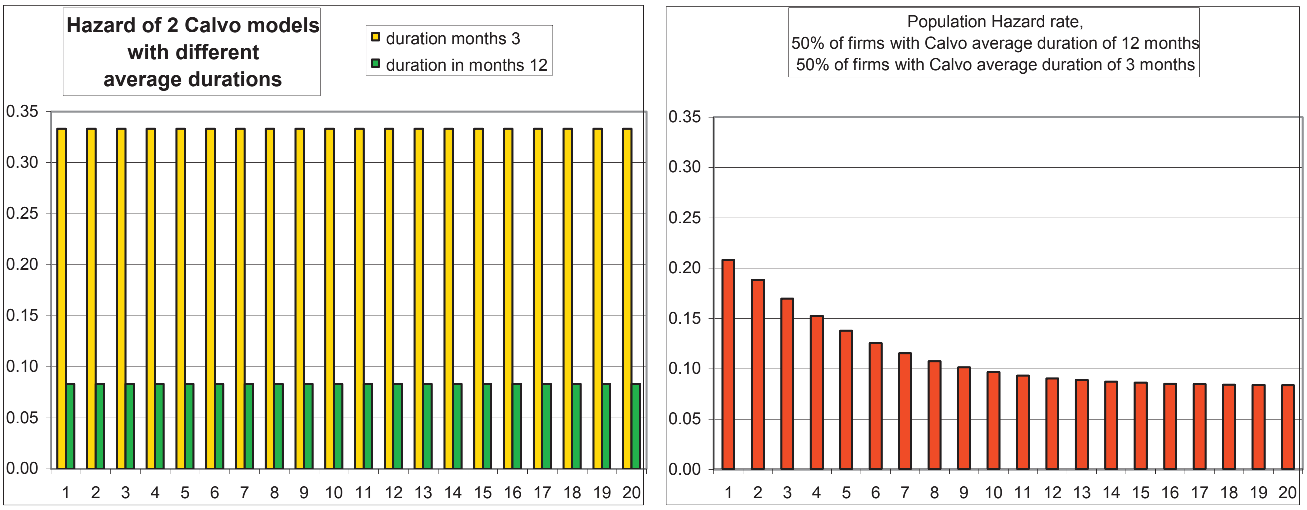
When we aggregate two groups of agents with Calvo price setting rules with different average price durations, the aggregate economy will have the following density, survival and hazard functions, respectively

$$f(k) = (1-\theta_1)\theta_1^{k-1}\lambda + (1-\theta_2)\theta_2^{k-1}(1-\lambda)$$

[9] In addition, in Appendix B we show results for the so-called Truncated Calvo model.

[10] Important contributions to this literature include Roberts (1995), Fuhrer and Moore (1995), King and Wolman (1996), Rotemberg and Woodford (1997), Galí and Gertler (1999), Galí, Gertler and López-Salido (2001) and Sbordone (2002).

Figure 2. Hazard of 2 groups with Calvo price setting and average price duration of 3 and 12 months, respectively



$$S(k) = \theta_1^{k-1}\lambda + \theta_2^{k-1}(1-\lambda)$$

$$\begin{aligned} h(k) &= \beta(k)(1-\theta_1) + [1-\beta(k)]\,(1-\theta_2) \\ \text{where } \beta(k) &= \left[1 + \left(\frac{\theta_2}{\theta_1}\right)^{k-1}\left(\frac{1-\lambda}{\lambda}\right)\right]^{-1} \end{aligned}$$

An interesting property of this model is that the aggregate hazard converges asymptotically to the hazard of the group with the longest average price duration[11], as can be seen in the right hand side of figure 2.

$$\lim_{k\to\infty} h(k) = \left(1-\overline{\theta}\right) \qquad where \quad \overline{\theta} = \max\{\theta_i\} \tag{4}$$

In this case, the change in the aggregate hazard as k changes is equal to

$$\frac{\Delta h(k)}{\Delta k} = H(k) = -\frac{\beta(k)\,[1-\beta(k)]}{1-h(k)\Delta k}\,(\theta_1-\theta_2)^2 \leq 0$$

That is, when the aggregate economy is composed of groups of Calvo agents, there is only a heterogeneity effect because the hazard is constant for all k and $0 \leq \beta(k) \leq 1$. Therefore, the change in the aggregate hazard will never be positive. Moreover, the change in the aggregate

[11] Note that:

$$\lim_{k\to\infty} \beta^i(k) = 1 \quad \& \quad \lim_{k\to\infty} \beta^j(k) = 0 \quad if \quad \theta_i > \theta_j$$

hazard as k changes converges asymptotically to zero, since the aggregate hazard converges to the hazard of the group with the longest average duration (see equation (4)). As an illustration, figure 2 presents in the left hand side the hazard functions of two groups of Calvo agents with durations of 3 and 12 months and in the right hand side the downward sloping hazard of the aggregate.

Results are easily generalized for the case of N groups of firms following different Calvo price setting rules. The change in the aggregate hazard as k changes is equal to

$$\frac{\Delta h(k)}{\Delta k} = H(k) = -\sum_{j=1}^{N-1} \sum_{l=j+1}^{N} \beta^j(k)\beta^l(k) \left[\frac{(\theta_j - \theta_l)^2}{1 - h(k)\Delta k}\right] \leq 0$$

and it is clear that the change in the aggregate hazard is only due to the heterogeneity effect and that the aggregate hazard will never be positive. Again, the aggregate hazard will converge asymptotically to the one of the group with longest average price duration.

3.2 Taylor agents

The model of price setting first introduced by the seminal work of Taylor (1980) is another model widely used in the current macro literature on sticky prices[12]. This model of price setting assumes that prices are set by multiperiod contracts, thus remaining constant for the duration of the contract.

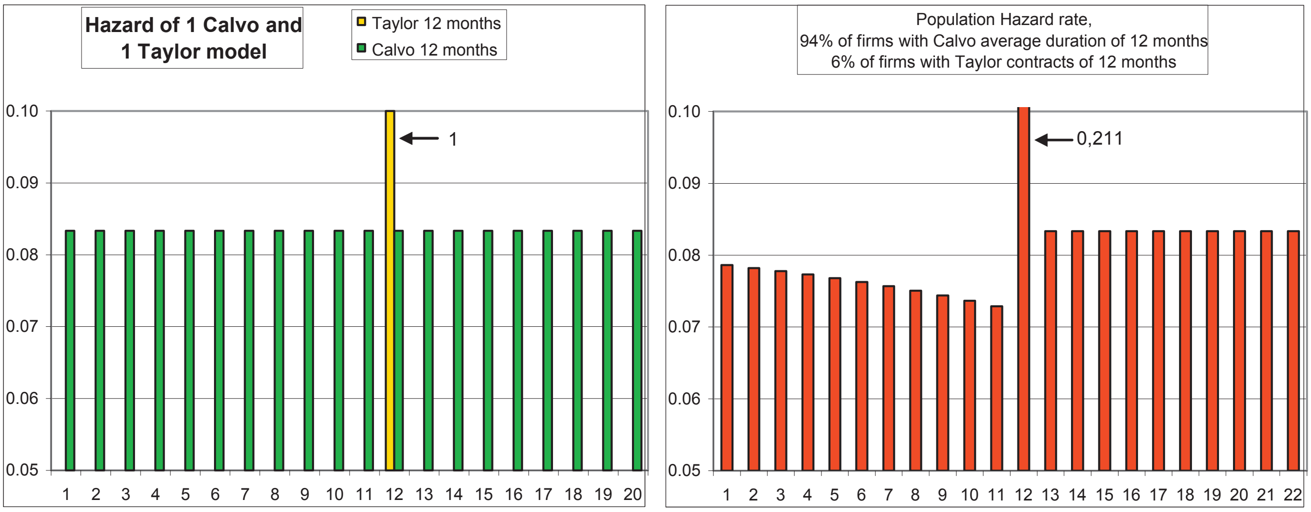
When one aggregates two groups of agents with Taylor contracts of different duration, $J_1 > J_2$, and sizes s_1 and s_2, respectively, the aggregate economy will have the following hazard function

$$h(k) = \begin{cases} 1-\lambda & for & k = J_2 \\ 1 & for & k = J_1 \\ 0 & for & \text{other } k \end{cases}$$

In this case the hazard rate is zero except in those periods in which the end of the Taylor contract occurs for one of the groups, that is, it is never decreasing. The same is true when the economy is composed of several groups of firms with different Taylor contracts.
Alternatively, when the aggregate economy is composed of two groups of firms, one setting prices according to a Calvo model and another setting prices according to a Taylor contract of length J,

[12]Important contributions to this literature include Erceg et al (2000), Chari, Kehoe and McGrattan (2000) and Coenen and Levin (2004).

Figure 3. Hazard of 1 Calvo and 1 Taylor



with sizes s_1 and s_2, respectively, the aggregate hazard takes the following form

$$h(k) = \begin{cases} \beta(k)(1-\theta) & for \quad k = 0, 1, ..., J-1 \\ \beta(k)(1-\theta) + [1-\beta(k)] & for \quad k = J \\ (1-\theta) & for \quad k > J \end{cases}$$

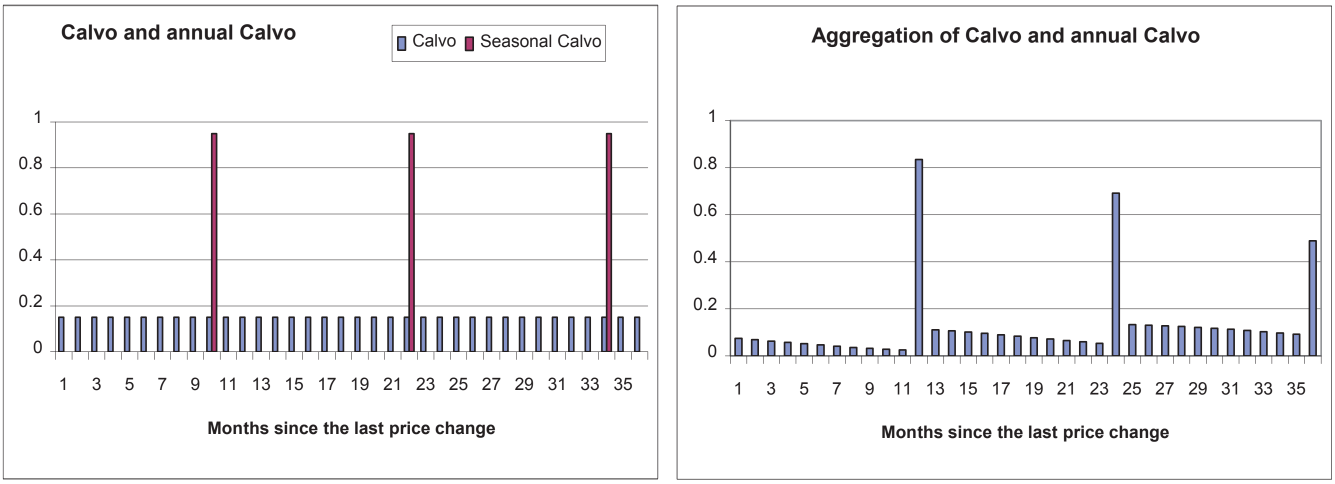
where $\beta(k) = \frac{s_1 \theta^{k-1}}{s_1 \theta^{k-1} + s_2}$. As shown in figure 3, this aggregate hazard will be decreasing for all k until the period in which Taylor contracts end. Note that hazard rates for horizons shorter than the length of the Taylor contract are lower than those for longer horizons.

3.3 Annual pricing agents

International evidence shows that aggregate hazard functions of price spells are characterised by local modes at durations of 12, 24, 36,... months (see figure 1), indicating that a fraction of firms apply annual pricing rules. This is in line with results of Álvarez and Hernando (2005) for Spain and Fabiani et al (2004) using the surveys on pricing behaviour that have been recently carried out for most euro area countries. A significant fraction of firms review their prices on a yearly basis and decide to change them on the basis of cost and demand developments. Specifically, modal and median number of price changes per year is one in eight out of the nine countries considered.

This stylized fact is easily accommodated theoretically by defining a group of agents with an annual Calvo rule, according to which these firms reset their prices every 12 months, but keep them constant in between. We propose a novel pricing rule to try to capture this behaviour.

Figure 4. Hazard of 1 Calvo and 1 Annual Calvo



Specifically, the frequency and survival functions for agents using this pricing rule are as follows

$$f(k) = (1-\theta)\,\theta^{int\left(\frac{k-1}{12}\right)} I_{12}, \qquad \text{where } I_{12} = \begin{cases} 1 & if \quad \frac{k}{12} = int\left(\frac{k}{12}\right) \\ 0 & \quad otherwise \end{cases}$$

$$S(k) = \theta^{int\left(\frac{k-1}{12}\right)}$$

which generate the following hazard function

$$h(k) = (1-\theta)\, I_{12}$$

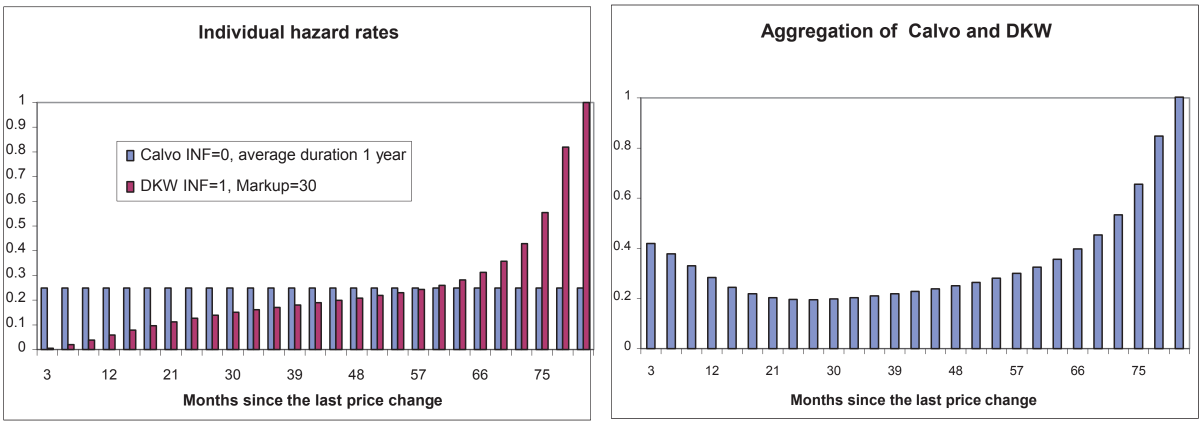
Note that one year Taylor contracts are a special case of this pricing scheme with probability of price change equal to one.

When the aggregate economy is composed of two groups of agents, one setting prices according to a standard Calvo mechanism with parameter θ and another setting prices according to an annual Calvo with parameter θ_s, with sizes s_1 and s_2, respectively, the aggregate hazard function takes the following form

$$h(k) = \ \beta(k)(1-\theta) + [1-\beta(k)]\,(1-\theta_s)\, I_{12} \qquad for \qquad k = J$$

where $\beta(k) = \frac{s_1\theta^{k-1}}{s_1\theta^{k-1}+s_2\theta_s^{int\left(\frac{k-1}{12}\right)}}$. As shown in figure 4, the slope of this aggregate hazard is decreasing for all months except for the multiples of 12, when the agents with annual Calvo rules change their prices. Comparing values of the hazard function for periods multiples of 12 also shows a

Figure 5. Hazard of 1 Calvo and 1 DKW model



decreasing pattern.

3.4 Dotsey, King and Wolman agents

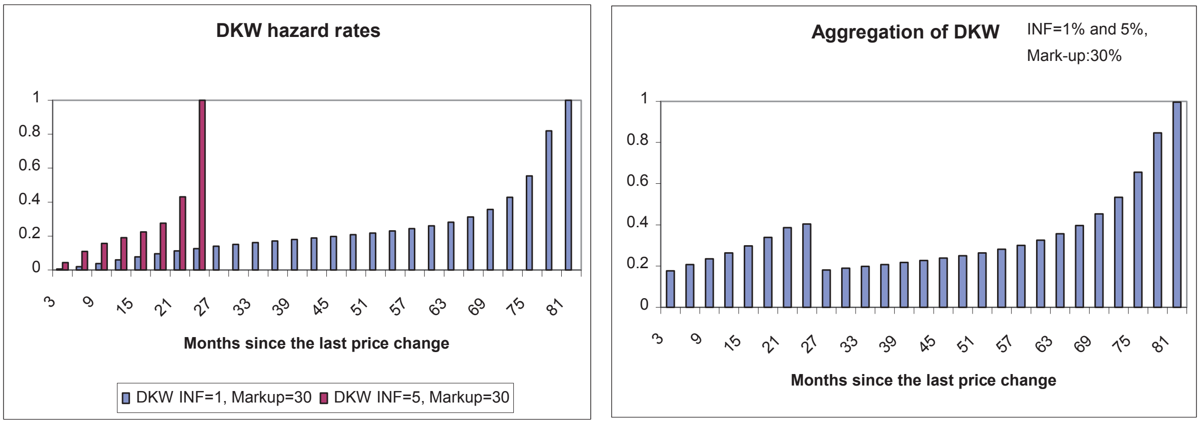
In the models of price setting analyzed so far, firm's pricing decisions are time-dependent, that is, they do not depend on any of the state variables determining the situation of the firm. In contrast to these models, Dotsey, King and Wolman (1999) (DKW henceforth) present a theoretical state-dependent pricing framework[13], in which every firm faces each period a different fixed cost of adjusting its nominal price, which is drawn independently over time. At the start of each period there is a discrete distribution of firms which last adjusted its price k periods ago. The number of firm types is determined endogenously and will vary with factors such as the average inflation rate or the elasticity of product demand. When inflation is high, firms choose to maintain a given price for fewer periods, because inflation erodes its relative price. Positive inflation means that the benefits of adjusting prices are higher for firms whose prices were set further in the past (which then suffer higher accumulated inflation), and this translates into higher adjustment probabilities for such firms. As a consequence, the hazard rate is increasing.

This model is very interesting and intuitive but also analytically complex and difficult to test empirically. In fact, it is not possible to derive closed form expressions for the hazard function (or the density and survival functions). Nevertheless, numerical expressions of the hazard rate can be obtained through simulations for a given underlying distribution of menu costs of adjusting prices and for given steady state values of the other variables of the model.

In order to show the implications of having some agents in an economy behaving in a DKW

[13] Another example of state-dependent pricing rules is Golosov and Lucas (2003).

Figure 6. Hazard of two different DKW models



state dependent manner, we present two types of simulations. The first one corresponds to the aggregation of one group of Calvo agents -with constant hazard rate- and another one of DKW agents -with increasing hazard rate- (see left hand side of figure 5). As was shown in equation (1), the slope of the aggregate hazard of this economy will have two components: the weighted average of the hazard rates of each group plus a heterogeneity effect. As can be seen in the right hand side of figure 5, the aggregate hazard declines initially since the (negative) heterogeneity effect dominates the upward sloping hazard of DKW agents. However, for longer horizons the upward sloping hazard effect dominates.

The second simulation aggregates two groups of DKW agents with different steady state inflation rates. As can be seen in the left hand side of figure 6, the higher the inflation rate the shorter the length of time prices remains unchanged. In this case, the heterogeneity effect is very moderate and the aggregate hazard is again (nearly always) increasing.

These two examples illustrate the fact that heterogeneity does not necessarily lead to decreasing hazards. In addition, they show that it is difficult to obtain an aggregate hazard decreasing for all horizons when some of the agents in the economy face an increasing hazard, since the weighted average of the individual slopes eventually dominates the (negative) heterogeneity effect.

4 Empirical results

In this section, we review the international evidence on unconditional hazard functions for price changes and test empirically the theoretical results derived in the previous sections.

Available international evidence on unconditional hazard functions (see figure 1 and references in the introduction) employing consumer price and producer price micro data suggests the following three stylised facts:

F1: Hazard functions are downward sloping.

F2: A large fraction of firms change their prices monthly or even more frequently.

F3: An important number of firms review their prices once a year and change them every 12, 24, 36 ... months.

As explained in the theoretical sections above, it is possible to build price setting models that allow for these stylized facts. For this purpose, we take the most parsimonious approach possible and consider only time dependent representations of Calvo price setting processes. The main reason for using the Calvo model is that it is analytically simple, easier to estimate[14], and, at the same time, easily reconciled with the stylized facts (see section 3.1). Alternatively, the Taylor model of price setting could be used. However, this model is less parsimonious in this context, since it requires having as many groups of agents as exit times or actual spell durations observed in the data.

A model based on Calvo price setting consistent with the stylized facts would be as follows:
- F1 can be explained as the result of the aggregation of several heterogeneous agents. In fact, a simple way to incorporate this stylised fact into the analysis is to specify two (or more) agents with different Calvo price setting rules (see figure 2 for an example).
- F2 can be easily accommodated assuming that there is a fraction of firms with highly flexible Calvo pricing rules or, alternatively, one-month Taylor contracts.
- Finally, F3 suggests using an annual Calvo pricing rule like the one defined in section 3.3.

The hazard function that considers F1-F3 would be as follows

$$h(k;\theta,\pi) = \frac{\sum_{j=1}^{g-1} \pi_j (1-\theta_j)\theta_j^{k-1} + \left(1 - \sum_{j=1}^{g-1} \pi_j\right)(1-\theta_g)\theta_g^{\left(int\left(\frac{k-1}{12}\right)\right)} I_{12}}{\sum_{j=1}^{g-1} \pi_j \theta_j^{k-1} + \left(1 - \sum_{j=1}^{g-1} \pi_j\right)\theta_g^{\left(int\left(\frac{k-1}{12}\right)\right)}} \tag{5}$$

[14] One should always keep in mind that the hazard function is highly non-linear.

where the $j = 1, \ldots, g-1$, groups represent the different standard Calvo agents, the gth group represents the annual Calvo agents, θ_j, represent the Calvo parameters and π_j the weights of the different groups of agents.

In this section we have considered only time dependent representations of price setting processes. Although this framework may provide a reasonable description of the data at an aggregate level, particularly in a stable economic environment, evidence presented in Álvarez and Hernando (2004) and Álvarez, Burriel and Hernando (2004) points to the importance of state dependent elements such as inflation and fiscal developments when analysing pricing behaviour of individual firms in Spain[15]. However, the estimated impacts of these state dependent variables are moderate. Similarly, Klenow and Krystov (2004) show that a calibration of the DKW model for the U.S. provides impulse responses that are quite close to those of a simple time dependent model. It has to be stressed that closed form expressions for the hazard function of the DKW model do not exist, which renders its empirical implementation difficult[16]. Moreover, empirical hazard rates do not show an upward slope, as suggested by DKW model[17].

The most adequate econometric methodology to estimate a model like the one described by equation (5) is a finite mixture model. This will be described in the next section.

4.1 Econometric specification: Finite mixture models

This section briefly reviews the finite mixture models that are employed in the section below. Finite mixture models have been applied to a wide variety of data in the physical, social and medical sciences[18] since the seminal contribution of Pearson (1894). A finite mixture model represents a heterogeneous population consisting of g groups of sizes proportional to π_j $(j = 1, \ldots, g)$ and where the group from which each observation is drawn is unknown. The probability density function of the observed random variable y has the form

$$f(y; \pi, \theta) = \pi_1 f_1(y, \theta_1) + \pi_2 f_2(y, \theta_2) + ... + \pi_g f_g(y, \theta_g)$$

This is a weighted average of densities $f_1, \ldots, f_g$ with mixing weights $\pi_1, \ldots, \pi_g$ where $\pi_1 + \pi_2 + \ldots + \pi_g = 1$ and θ_j is a vector of the unknown parameters in f_j, which need not belong to the same parametric family.

[15] In future work we intend to estimate state dependent models that also use the economic theory based unobserved heterogeneity that is described in this section.

[16] In future work we plan to consider state dependent pricing models by estimating mixture modes with covariates.

[17] To try to capture an upward sloping component we have considered the discrete Weibull distribution as proposed by Nakagawa and Osaki (1975). Available estimates do not lead to a group of agents with an upward sloping hazard.

[18] See Titterington et al (1992) and Mclahlan and Peel (2000) for a review.

This framework fully uses the individual information available and is easily modified to take into account that duration data are typically censored[19]. Specifically, allowing for censoring, the log likelihood function for the finite mixture model above is given by:

$$l(y;\pi,\theta)=\sum_{NC}\log\left(\sum_{j=1}^{g}\pi_j f_j(y,\theta_j)\right)+\sum_{C}\log\left(\sum_{j=1}^{g}\pi_j S_j(y,\theta_j)\right)$$

where NC and C refer to non censored and censored price spells and f_j and S_j represent the density and survival functions, respectively. Since this log-likelihood function involves the log of a sum of terms that are highly non-linear functions of parameters and data, its maximization using standard optimization routines is not in general feasible. Therefore, we resort to the EM algorithm, as it is usual in the literature(see Dempster, Laird and Rubin, 1977)[20]. Specifically, we consider the data augmented with unobservable dummy variables that identify each group π_{ij} $= (\pi_{i1}, \ . \ . \ . \ , \pi_{ig})$, such that, for each i, $\pi_{ij} = 1$, for one j and $\pi_{ij} = 0$ for the rest). The log likelihood can then be written as

$$l(y;\pi,\theta)=\sum_{j=1}^{g}\sum_{i=1}^{n}\pi_{ij}\log\pi_j+\sum_{j=1}^{g}\sum_{i=1}^{n}\pi_{ij}\log f_j(y,\theta_j)+\sum_{j=1}^{g}\sum_{i=1}^{n}\pi_{ij}\log S_j(y,\theta_j)$$

and the EM approach computes ML estimates using the following algorithm.

1. Expectation (E) step. For given θ, compute π_{ij} (the estimated conditional probability of individual i belonging to group j) and π_j (marginal probabilities) using the formulae

$$\begin{aligned}\hat{\pi}_{ij} &= \begin{cases}\frac{\pi_j f_j(y_i;\theta_j)}{\Sigma_{j=1}^{g}\pi_j f_j(y_i;\theta_j)} & \text{if } y_i \text{ is uncensored}\\ \\ \frac{\pi_j S_j(y_i;\theta_j)}{\Sigma_{j=1}^{g}\pi_j S_j(y_i;\theta_j)} & \text{if } y_i \text{ is censored}\end{cases}\\ \text{and }\ \hat{\pi}_j &= \frac{1}{n}\Sigma_{i=1}^{n}\hat{\pi}_{ij}\end{aligned}$$

2. Maximization (M) step. For given values of π_{ij} and π_j, maximize the log likelihood function with respect to θ

Starting from initial estimates, the EM algorithm consists in iterating 1) and 2) until convergence. It can be shown that each iteration of the algorithm increases the likelihood and that it finally

[19] In what follows we do not take into account left censored observations. For ease of exposition we will refer to right censored observations simply as censored observations.

[20] Alternatively, Diebolt and Robert (1994) and Richardson and Green (1997) use Bayesian approaches to estimate finite mixtures employing Markov chain Monte Carlo (MCMC) methods.

maximizes it. In our applications we use as starting values minimum distance estimates of this model employing grouped data[21].

Our empirical strategy is agnostic with respect to the number of groups characterizing the data, that is, about the number of $j's$ in equation (5). The optimal number of different groups of Calvo agents is obtained by estimating the mixture model, separately for models including one to five groups of different Calvo agents (values of $j = 1 - 5$) and also augmented models incorporating a group of annual Calvo agents. Then, for each of these estimations we compute two model selection criteria: the Akaike Information criterion (AIC) and the Bayesian Information Criterion (BIC). These criteria are calculated as follows:

$$\begin{aligned} AIC &= -2\log\left[L(\widehat{\Psi})\right] + 2d \\ BIC &= -2\log\left[L(\widehat{\Psi})\right] + d\log(n) \end{aligned}$$

where d is the number of unknown coefficients estimated, n is the number of observations and $L(\widehat{\Psi})$ is the maximum likelihood for the set of unknown parameters estimated.

So far we have implicitly assumed that the researcher is interested in obtaining the mixture distribution of price spells. Nonetheless, other interests are likely to arise. First, one can also be interested in determining the number of firms[22] belonging to each particular group. In this case, the distribution of price spells cannot be directly used since, by definition, firms whose prices remain unchanged for long time periods contribute less price spells and are, therefore, underrepresented in terms of price spells. In the empirical section below, we estimate proportions of firms by randomly selecting one price spell for each price product trajectory and then applying the EM algorithm. Second, the use of the number of firms may also be considered misleading, since the value of production greatly varies across branches of activity. Therefore, when using the number of firms one is overrepresenting firms with low production value. In applied work, production (or expenditure) weighted shares are likely to be the main object of interest.

As stated above, the presented framework considers that each price spell is the result of one simple

[21] The minimum distance estimates can be obtained in the following manner. If $\overline{h} = [h(1), ..., h(k)]$ denotes the empirical hazard and $h(\pi, \theta) = [h(1, \pi, \theta), ..., h(k, \pi, \theta))$ the hazard corresponding to a given theoretical model, then the minimum distance estimator is the result of the following optimization problem

$$\pi^{MD}, \theta^{MD} = \underset{\{\pi,\theta\}}{\arg\min} \left[\overline{h} - h(\pi,\theta)\right]' \Omega^{-1} \left[\overline{h} - h(\pi,\theta)\right]$$

where Ω^{-1} is a weighting matrix.

[22] Firms and retailers generally manufacture (sell) different products. However, for ease of exposition, we refer to firms (retailers) instead of manufacturing (selling) units of a specific good or service.

Table 1. Producer Prices: selection of the number of different Calvo agents

Producer prices								
	standard + 1 annual Calvo				standard Calvo			
number of models	firms		spells		firms		spells	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
1	144469	144476	1203134	1203144	130709	130711	1153457	1153460
2	133104	133116	914830	914847	123544	123552	906843	906853
3	121257	121274	897292	897316	122468	122480	902244	902261
4	125501	125523	902373	902404	124889	124906	920091	920115
5	122716	122743	901480	901518	123852	123874	907579	907609

price setting rule. We do not, however, directly observe to which group the observation precisely belongs, although a model-based clustering procedure may be designed to classify the different price spells into groups with different price setting behaviour. Specifically, for each individual, we compute the conditional probability of belonging to a given price setting group. This probability, along with a classification rule[23], allows us to assign firms or price spells to different pricing rules. The analysis of these clusters can be very informative. For example, we can determine the relationship of these price setting groups with some other variables such as the type of good, which allows us to compute the (value-added or expenditure) weighted shares of the different types of agents. Moreover, as will be exploited in future work, duration models for the different economic theory based clusters may be built.

4.2 Results for producer price data

In this section we try to account for the three abovementioned stylised facts in explaining the empirical hazard for Spanish producer price data. The dataset on which we compute the hazard function contains over 1.6 million price records for a 7 year period (1991:11-1999:2) and covers over 99% of the production value of the PPI. This dataset is also employed in Álvarez, Burriel and Hernando (2004), where a detailed explanation can be found.

[23]One possibility is to assign each observation to the group for which the maximum conditional probability is obtained. This is known in the literature as maximum a posteriori (MAP) rule and is the most widely used procedure in the non-Bayesian literature.

Table 2. Producer Prices: estimation of price setting models

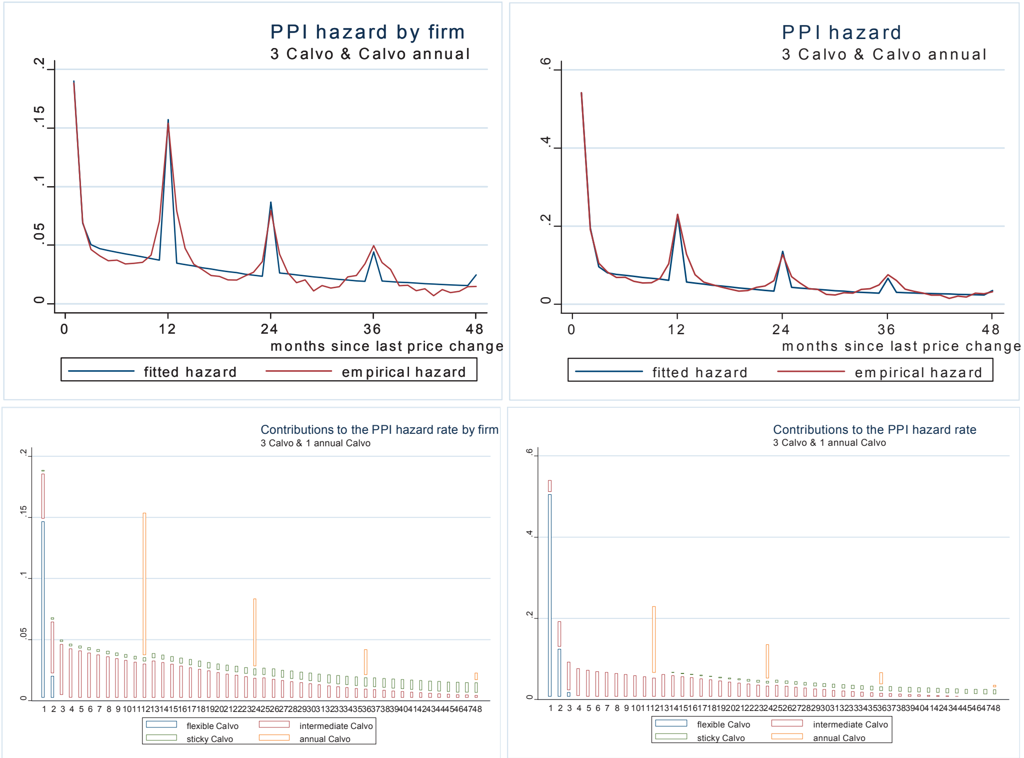
Price setting models	standard + 1 annual Calvo						standard Calvo					
	firms			spells			firms			spells		
	Weight	Calvo Parameter	Mean duration	Weight	Calvo Parameter	Mean duration	Weight	Calvo Parameter	Mean duration	Weight	Calvo Parameter	Mean duration
Flexible Calvo	16.5%	0.11	1.1	56.9%	0.11	1.1	15.4%	0.08	1.1	56.7%	0.11	1.1
	(0,38%)	(0,01)	(0,02)	(0,14%)	(0,00)	(0,00)	(0,37%)	(0,01)	(0,01)	(0,14%)	(0,00)	(0,00)
Intermediate Calvo	48.2%	0.92	12.3	32.3%	0.89	9.2	60.8%	0.93	13.8	37.8%	0.90	10.5
	(1,60%)	(0,00)	(0,48)	(0,28%)	(0,00)	(0,11)	(1,77%)	(0,00)	(0,47)	(0,31%)	(0,00)	(0,11)
									9.2			6.9
Sticky Calvo	26.9%	0.99	91.4	6.8%	0.98	50.1	23.8%	0.99	105.9	5.5%	0.98	56.0
	(1,68%)	(0,00)	(8,86)	(0,29%)	(0,00)	(2,12)	(1,87%)	(0,00)	(13,82)	(0,33%)	(0,00)	(3,28)
Annual Calvo	8.5%	0.28	16.8	4.0%	0.21	15.2						
	(0,27%)	(0,02)	(0,03)	(0,06%)	(0,01)	(0,01)						
Log likelihood		-60621			-448639			-61229			-451117	
Number of observations		26965			244864			26965			244864	
Joint significance Wald test		20.15			2479.23			68.98			2553.07	
p- value		0.00			0.00			0.00			0.00	

To obtain the optimal number of different groups of Calvo agents in the producer price data, we compute the values for the two model selection criteria explained in the methodological section. As table 1 shows, according to both AIC and BIC, it is optimal to estimate a model composed of 3 types of standard Calvo agents, plus 1 group of annual Calvo agents.

The results of estimating our benchmark specification, which includes three different groups of Calvo agents plus one group of annual Calvo firms, can be seen in the first and second columns of table 2. In addition, in the third and fourth columns of this table we present results for a basic specification in which there are no annual Calvo agents. In the case of both specifications, we report the results of estimating two different samples: one with all the price spells included in the PPI sample (we refer to this as "spells' sample") and another including only one spell (randomly selected) per firm in the sample (we refer to this as "firms' sample"). As indicated in section 4.1, using the firms' sample corrects the over-representation of price spells with short durations. In all cases, all parameters are individually and jointly significant. Moreover, as Figure 7 shows, these parsimonious models fit the overall hazard function extremely well.

The results in table 2 indicate that the Calvo parameters (and the implied durations) are fairly similar across samples and specifications. In all cases we find that the three types of standard Calvo models can be characterized as follows: one group of flexible price setters with average price durations of 1.1 months, one group of intermediate price setters with average price durations

Figure 7. Producer Prices: Hazard and contribution to hazard



between 9 and 14 months and one group of sticky price setters with average price durations over 3 years[24]. In addition, the group of annual Calvo price setters has average price durations close to the intermediate standard Calvo group, between 15 and 17 months. As expected, the estimated weights of each group vary greatly across the two samples. In the case of the firms' sample the largest group is the intermediate Calvo group (48-60%), followed by the sticky Calvo (24-27%) and the flexible Calvo (15-16%) groups. On the other hand, in the case of the sample with all spells the largest group is the flexible Calvo (57%), followed by the intermediate Calvo (32-37%) and the sticky Calvo (6-7%). This result is not surprising given that shorter spells tend to be highly over-represented in the spells sample since they occur more often. In both samples, the annual Calvo price setters are the smallest group (4-9%).

The results just mentioned are based on the share of spells or firms in the sample. However, it might be more informative to know the share of the value of production in the aggregate economy of each group. Using a maximum a posteriori (MAP) rule, we have assigned each firm to a specific

[24] Alternatively, since the duration distribution is asymmetric, it is also interesting to look at the median duration of prices. We find shorter median durations for all the groups: the intermediate Calvo agents (6.1-9.2 months), sticky Calvo group (34.4-73.1 months), flexible Calvo group (0.3 months) and annual Calvo group (12 months).

Table 3. Price setting models by PPI main industrial groupings (using PPI weights)

Price setting Model	non-durables food	non-durables non- food	durables	intermediate	energy	capital	All groups
Flexible Calvo	3.5%	1.3%	1.0%	8.8%	6.1%	1.3%	21.9%
Intermediate Calvo	8.6%	7.0%	7.2%	17.6%	4.2%	5.9%	50.5%
Sticky Calvo	3.0%	3.1%	3.7%	7.2%	0.4%	3.5%	20.9%
Annual Calvo	1.0%	1.1%	1.1%	2.0%	0.3%	1.2%	6.8%
Share of PPI	16.1%	12.4%	13.1%	35.5%	11.1%	11.8%	100.0%

type of pricing rule. Then table 3 reports, for the firm sample, the distribution of firms weighted according to the PPI weights across the main PPI components and the different pricing rules.

The most important group in terms of the share of the value of production in the aggregate economy is that of intermediate Calvo agents. This group represents around 50% of the production value in the economy and is characterised by mean and median durations of 12.3 and 8.7 months[25]. All different types of goods are represented in this group, although the share of energy products is particularly low. The second most important group corresponds to flexible Calvo agents, which represent 22% of the PPI and have an average duration slightly above 1 month. The contribution of energy goods and, to a lesser extent, other intermediate goods is particularly relevant, whereas the share of capital, durables and non-durables non-food goods is quite moderate. In turn, the share of sticky Calvo agents is only slightly below that of flexible ones, although estimated durations are very high. Producers of capital and consumer durable goods tend to use this pricing rule, which is hardly used in energy branches. Finally, the share of agents using annual pricing is only 7% and the corresponding duration is slightly less than one year and a half. This type of behaviour is particularly frequent for producers of capital and consumption durables goods.

These production weighted shares may be compared with the firm and spell shares shown in table 2. As expected, the share in terms of price spells of flexible Calvo agents is much higher than in terms of firms (weighted or not using production weights). The shares in terms of price spells of sticky Calvo agents is much lower than in terms of firms, which in turn, over-represent the share of firms weighted in terms of production value.

The bottom part of figure 7 presents the contributions of the different types of agents to the hazard of the benchmark model. As can be seen, the downward slope of the hazard is, to a large

[25] These are the mean and median durations derived from estimates in terms of number of firms, as shown in table 2.

Table 4. Consumer Prices: selection of the number of different Calvo agents

Consumer prices								
	standard + 1 annual Calvo				standard Calvo			
number of models	firms		spells		firms		spells	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
1	75704	75711	894119	894129	73253	73255	891611	891614
2	76654	76664	787640	787656	68925	68931	789456	789465
3	67275	67290	778891	778914	68132	68142	781875	781891
4	67810	67829	781125	781154	68177	68192	786004	786026
5	67825	67848	779982	780017	68203	68222	791049	791078

extent, explained by the aggregation of these Calvo agents: the weight of intermediate-Calvo price setters relative to sticky-Calvo price setters is decreasing, being negligible for large horizons. The existence of highly flexible Calvo price setters is accounted for by the very-flexible Calvo contracts. Finally, the models that consider annual Calvo agents show that their share is modest, although very important in explaining the spikes at 12, 24, 36,... months.

4.3 Results for consumer price data

This section examines the relevance of the three stylised facts in explaining the empirical hazard for Spanish consumer price data. The dataset on which we compute the hazard function contains over 1.1 million price records for a 9 year period (1993-2001) and covers around 70% of the expenditure of the CPI basket. Energy products are not covered in this database. This dataset is also employed in Álvarez and Hernando (2004), where a detailed analysis can be found.

Like in the producer price case, we start by finding out the optimal number of different groups of Calvo agents found in the data. Table 4 shows that, according to the two model selection criteria used, it is optimal to estimate a model composed of 3 types of standard Calvo agents, plus 1 group of annual Calvo agents. This is similar to the PPI case.

The results of estimating the finite mixture model for our benchmark specification, which includes three different groups of Calvo agents plus one group of annual Calvo firms, can be seen in the

Table 5. Consumer Prices: estimation of price setting models

Price setting models	standard + 1 annual Calvo						standard Calvo					
	firms			spells			firms			spells		
	Weight	Calvo Parameter	Mean duration	Weight	Calvo Parameter	Mean duration	Weight	Calvo Parameter	Mean duration	Weight	Calvo Parameter	Mean duration
Flexible Calvo	22.1% (0,62%)	0.20 (0,02)	1.3 (0,03)	47.5% (0,28%)	0.24 (0,00)	1.3 (0,01)	21.6% (0,68%)	0.19 (0,02)	1.2 (0,03)	48.3% (0,26%)	0.24 (0,00)	1.3 (0,00)
Intermediate Calvo	50.4% (1,44%)	0.90 (0,00)	10.3 (0,41)	39.5% (0,31%)	0.84 (0,00)	6.4 (0,08)	59.9% (1,64%)	0.92 (0,00)	11.8 (0,43)	41.9% (0,35%)	0.86 (0,00)	7.3 (0,09)
Sticky Calvo	20.1% (1,52%)	0.98 (0,00)	58.4 (4,12)	9.8% (0,30%)	0.96 (0,00)	27.9 (0,56)	18.5% (1,75%)	0.98 (0,00)	62.6 (5,45)	9.7% (0,37%)	0.96 (0,00)	28.2 (0,64)
Annual Calvo	7.4% (0,34%)	0.42 (0,02)	20.5 (0,06)	3.2% (0,06%)	0.30 (0,01)	17.2 (0,02)						
Log likelihood		-33630			-389439			-34061			-390932	
Number of observations		12494			179673			12494			179673	
Joint significance Wald test		65.75			2058.42			86.84			2792.25	
p- value		0.00			0.00			0.00			0.00	

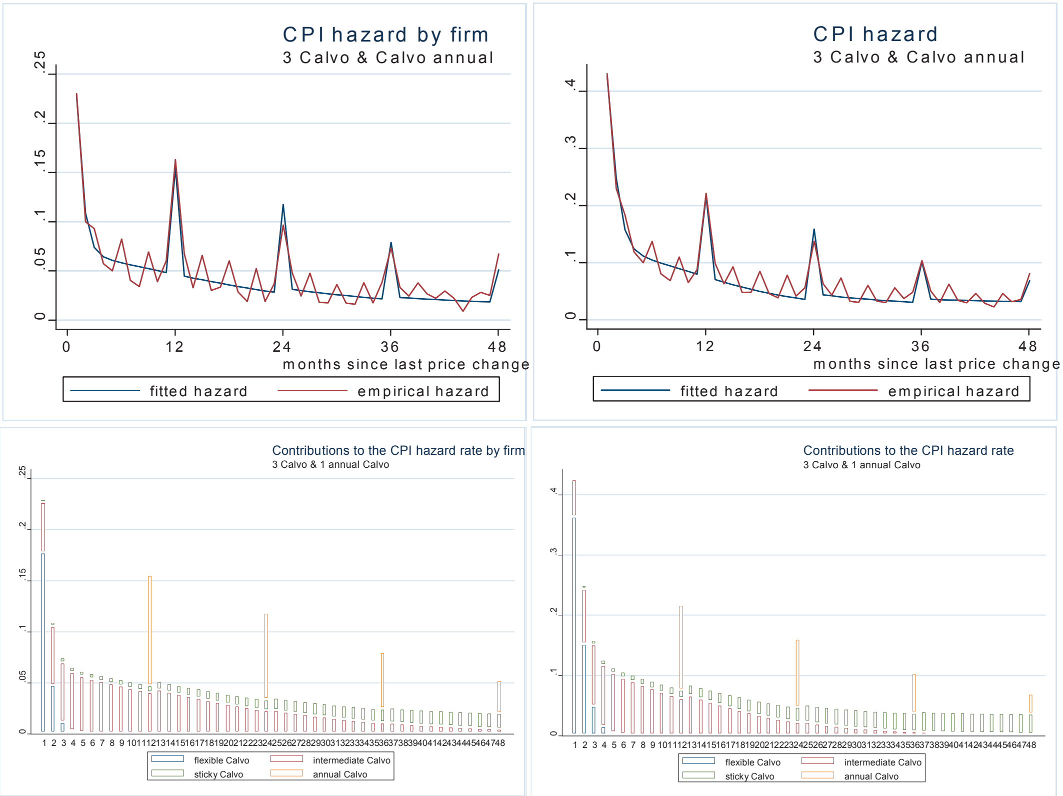
first and second columns of table 5. In addition, in the third and fourth columns of this table we present results for a basic specification in which annual Calvo agents are not considered. In all cases, all parameters are individually and jointly significant. Moreover, as Figure 8 shows, these parsimonious models fit the overall hazard function extremely well[26]. The results are very similar to the PPI case and also very similar across samples and specifications. The three types of standard Calvo models can be characterized as follows: one group of flexible price setters with average price durations of 1.2-1.3 months , one group of intermediate price setters with average price durations between 6 and 12 months and one group of sticky price setters with average price durations between 27 and 63 months. In addition, the group of annual calvo price setters have average price durations of around a year and a half[27]. That is, we find on average that CPI prices have shorter mean durations than PPI prices for the intermediate and sticky groups of Calvo agents, but slightly longer for the flexible and annual Calvo agents.

The estimated weights of each group have a slightly different ordering in the firms sample than for the PPI case: the largest group is the intermediate Calvo group (50-60%), followed by the flexible Calvo (22%) and the sticky Calvo (19-20%). The ordering is the same for the spells sample: the largest group is the flexible Calvo (48%), followed by the intermediate Calvo (40-42%) and the sticky Calvo (10%). In both samples, the annual Calvo price setters are the smallest group (3-9%),

[26] Note the quarterly spikes of the estimated hazard function. These are explained by the fact that some prices are collected on a quarterly basis.

[27] Alternatively, since the duration distribution is asymmetric, it is also interesting to look at the median duration of prices. We find shorter median durations for the different agents: intermediate Calvo agents (4.1-7.8 months), sticky Calvo groups (19.0-43.0 months), flexible Calvo group (0.4-0.5 months) and annual Calvo group (12 months).

Figure 8. Consumer Prices: Hazard and contribution to hazard



marginally smaller than in the PPI data.

Table 6 reports, for the firms' sample, the distribution of firms across the main CPI components and the different pricing rules, weighted according to the CPI weights. Like in the PPI case, the most important group in terms of share of household consumption is the intermediate Calvo agents, with 57% of household expenditure. All different types of goods are represented in this group, although the share of non energy industrial goods is particularly high and that of unprocessed food particularly low. Flexible Calvo pricing rules are very common among retailers selling unprocessed food and, to a lesser extent, processed food. Indeed, to deal with the high frequency of price changes many statistical institutes collect unprocessed food prices more than once a month. Similarly, a significant fraction of sticky Calvo agents is found among retailers of non energy industrial goods and services. Finally, annual Calvo price setters are also significantly present in services and non energy industrial goods.

These expenditure weighted shares may be compared with firm and spell shares. The weight in terms of firms and spells of intermediate and annual Calvo price setters is lower than in household

Table 6. Price setting models by CPI main componentes (using CPI weights)

Price setting Model	unprocessed food	processed food	non energy	services	all components
Flexible Calvo	6.8%	4.4%	1.4%	0.8%	13.4%
Intermediate Calvo	5.7%	12.4%	22.0%	16.4%	56.6%
Sticky Calvo	0.2%	1.5%	7.2%	9.0%	17.9%
Annual Calvo	0.1%	0.5%	4.8%	6.8%	12.1%
Share of CPI	12.8%	18.8%	35.4%	33.0%	100.0%

consumption. On the contrary, the share of flexible Calvo agents is higher in terms of spells and firms than in terms of total expenditure.

The bottom part of figure 8 presents the contributions of the different types of agents to the hazard of the benchmark model. The downward slope of the hazard corresponds mostly to the behaviour of these Calvo agents: the weight of intermediate Calvo price setters relative to sticky Calvo price setters is decreasing and is negligible for large horizons. The existence of highly flexible retailers is explained by Calvo agents with very short durations. Finally, the models that consider annual Calvo agents show that their share in the economy is relatively modest, although it is very important in explaining the spikes at 12, 24, 36,... months.

5 Conclusions

In this paper we show that the common empirical finding that hazard functions for price changes are decreasing can be reconciled with standard models of price setting behaviour by allowing for the existence of heterogeneous price setters. This idea is formalised by analysing the consequences for the aggregate hazard rate of the coexistence of firms with different pricing rules. In particular, we derive analytically the form of this heterogeneity effect for the most commonly used pricing rules and find that the aggregate hazard is (nearly always) decreasing.

Results are illustrated using Spanish producer and consumer price data. A parsimonious approach is taken, assuming that the aggregate economy is composed of several Calvo agents with different average price durations. Specifically, we estimate a finite mixture of Calvo models considering 1, 2, 3, 4 and 5 groups and then choose the optimal model according to several model selection criteria. We find that a very accurate representation of individual data is obtained by considering just 4 groups of agents: one group of flexible Calvo agents -with average price duration slightly over 1 month-, one group of intermediate Calvo agents -with average price duration around 10 months- and one group of sticky Calvo agents -with average price duration over 3 years- plus an annual Calvo process -with average price duration around a year and a half.

In terms of the relative size of the groups, the largest is the intermediate Calvo group, accounting for around 50% of the production value in the case of the PPI and 57% of consumer's expenditure in the case of the CPI. The flexible and sticky Calvo groups are roughly similar in size in terms of the share of PPI (slightly above 20%). In the case of CPI, the share of flexible Calvo agents (13%) is lower than the share of the sticky Calvo group (18%). Finally, the annual Calvo group is the smallest one, accounting for 7% of PPI and 12% of CPI. An analysis of the composition of the groups in terms of the different types of goods and services provides interesting results. Specifically, we observe that the flexible pricing rule is used mostly by producers of energy and intermediate goods and by retailers of food products; the intermediate rule is common among all producers and retailers, although to a lesser extent among energy producers and retailers of unprocessed food; and the sticky and annual Calvo pricing rules are mainly used by producers of capital and consumer durable goods and by retailers of non-energy goods and services.

A Appendix: Continuous time case.

In continuous time, the derivative of the aggregate hazard is given by

$$\frac{\partial h(k)}{\partial k} = \frac{\partial h^1(k)}{\partial k}\beta(k) + \frac{\partial h^2(k)}{\partial k}\left[1-\beta(k)\right] + \frac{\partial \beta(k)}{\partial k}h^1(k) + \frac{\partial\left[1-\beta(k)\right]}{\partial k}h^2(k) \tag{6}$$

The derivative of the weight on the hazard of the first group is equal to

$$\frac{\partial \beta(k)}{\partial k} = -\beta(k)\left[1-\beta(k)\right]\left[h^1(k) - h^2(k)\right] \tag{7}$$

Substituting equation (7) into equation (6), we get the derivative of the hazard with respect to k

$$\begin{aligned} \frac{\partial h(k)}{\partial k} &= \frac{\partial h^1(k)}{\partial k}\beta(k) + \frac{\partial h^2(k)}{\partial k}\left[1-\beta(k)\right] + H(k) \\ \text{where } H(k) &= -\beta(k)\left[1-\beta(k)\right]\left[h^1(k) - h^2(k)\right]^2 \leq 0 \end{aligned} \tag{8}$$

That is, the derivative of the aggregate hazard is a convex linear combination of the derivatives of the individual hazards plus a heterogeneity effect $H(k)$, which is never positive. This effect disappears if there is no heterogeneity ($h^1(k) = h^2(k)$) or if, for a given k, there are no more firms belonging to one group ($\beta(k) = 0$) or $\beta(k) = 1$). This corresponds to the well known fact in the duration analysis literature that uncontrolled heterogeneity biases estimated hazard functions towards negative duration dependence.

A necessary and sufficient condition for the derivative to be negative is that the heterogeneity effect ($H(k)$) is larger than the weighted sum of the derivatives of the individual hazards:

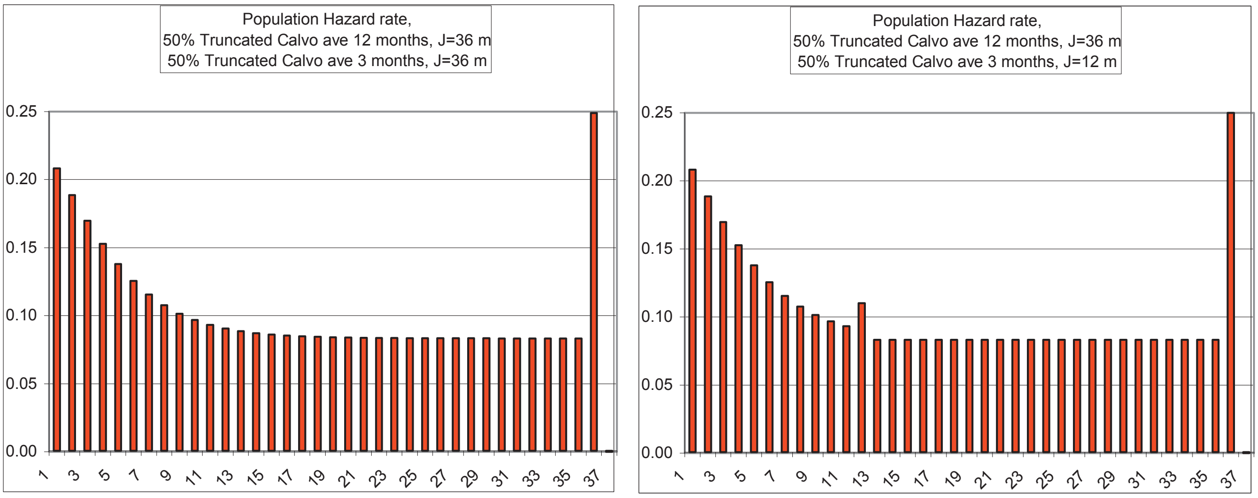
$$\frac{\partial h(k)}{\partial k} \leq 0 \qquad iff \qquad \frac{\partial h^1(k)}{\partial k}\beta(k) + \frac{\partial h^2(k)}{\partial k}\left[1-\beta(k)\right] \leq |H(k)| \tag{9}$$

B Appendix: Truncated Calvo agents

Alternatively, we could assume that the population of firms is composed of two groups, each one of them setting prices according to a different truncated Calvo mechanism. In this case, there are three possible scenarios: 1) both groups have different Calvo parameters of the probability of not changing prices before the truncation occurs, but equal period of truncation; 2) both groups have equal Calvo parameters but different truncations; 3) each group has a different Calvo parameter and truncation point.

Case 1. $\left(\boldsymbol{J}^1 = \boldsymbol{J}^2 = J\right)$ & $(\theta_1 \neq \boldsymbol{\theta}_2)$: The Calvo parameter θ_i is different, but the truncation point J is the same for both groups

Figure A.1. Cases 1 and 2: Hazard of 2 groups with Truncated Calvo price setting



$$\text{density function: } f^i(k) = \begin{cases} (1-\theta_i)\,\theta_i^{k-1} & for & k=1,...,J-1 \\ \theta_i^{J-1} & for & k=J \\ 0 & for & k>J \end{cases}$$

$$\text{survival function: } S^i(k) = \begin{cases} \theta_i^{k-1} & for & k=1,...,J \\ 0 & for & k>J \end{cases}$$

$$\text{hazard: } h^i(k) = \begin{cases} (1-\theta_i) & for & k=1,...,J-1 \\ 1 & for & k=J \end{cases}$$

$$\text{aggregate hazard: } h(k) = \begin{cases} \beta(k)(1-\theta_1)+[1-\beta(k)]\,(1-\theta_2) & for & k=1,...,J-1 \\ \\ 1 & for & k=J \end{cases}$$

and the change in the aggregate hazard as k changes

$$\frac{\Delta h(k)}{\Delta k} = \begin{cases} -\frac{\beta(k)[1-\beta(k)]}{1-\delta(k)\Delta k}\,(\theta_1-\theta_2)^2 \leq 0 & for & k=1,...,J-1 \\ 1-\frac{\beta(k)[1-\beta(k)]}{1-\delta(k)\Delta k}\,(\theta_1-\theta_2)^2 > 0 & for & k=J \\ -1 & for & k=J+1 \\ 0 & for & k>J+1 \end{cases}$$

That is, in this case the aggregate hazard will be decreasing for all k, except for the last period (J), when it will jump up to one.

Case 2. $\left(\boldsymbol{J}^1 > \boldsymbol{J}^2\right)$ & $(\theta_1 > \boldsymbol{\theta}_2)$: The calvo parameter θ_i and the truncation point J_i are different for both groups

$$\text{aggregate hazard: } h(k) = \begin{cases} \beta(k)(1-\theta_1) + [1-\beta(k)]\,(1-\theta_2) & for & k = 1, ..., J_2 - 1 \\ \beta(k)(1-\theta_1) + [1-\beta(k)] & for & k = J_2 \\ (1-\theta_1) & for & k = J_2 + 1, ..., J_1 - 1 \\ 1 & for & k = J_1 \end{cases}$$

and the change in the aggregate hazard as k changes

$$\frac{\Delta h(k)}{\Delta k} = \begin{cases} -\frac{\beta(k)[1-\beta(k)]}{1-\delta(k)\Delta k}\,(\theta_1-\theta_2)^2 < 0 & for & k = 1, ..., J_2 - 1 \\ [1-\beta(k)]\,\theta_2 > 0 & for & k = J_2 \\ -\,[1-\beta(k)]\,\theta_1 < 0 & for & k = J_2 + 1 \\ 0 & for & J_1 > k > J_2 + 1 \\ 1 & for & k = J_1 \\ -1 & for & k = J_1 + 1 \\ 0 & for & k > J_1 + 1 \end{cases}$$

The aggregate hazard in this case is decreasing for the first $(J_2 - 1)$ periods and constant for the periods $(J_2 + 2$ until $J_1 - 1)$. In addition, the aggregate hazard will jump up in periods of truncation, that is in periods J_1 and J_2. This can be seen in figure A.1.

Case 3: $\left(\boldsymbol{J}^1 > \boldsymbol{J}^2\right)$ & $(\theta_1 = \boldsymbol{\theta}_2 = \theta)$:

$$\text{aggregate hazard: } h(k) = \begin{cases} (1-\theta) & for & k = 0, 1, ..., J_2 - 1 \\ \lambda(1-\theta) + (1-\lambda) = 1 - \lambda\theta & for & k = J_2 \\ (1-\theta) & for & k = J_2 + 1, ..., J_1 - 1 \\ 1 & for & k = J_1 \end{cases}$$

and the change in the aggregate hazard as k changes

$$\frac{\Delta h(k)}{\Delta k} = \begin{cases} 0 & for & k = 0, 1, ..., J_2 - 1 \\ 1 - \lambda\theta > 0 & for & k = J_2 \\ (1-\theta) - (1-\lambda\theta) = -\theta\,(1-\lambda) < 0 & for & k = J_2 + 1 \\ 0 & for & J_1 > k > J_2 + 1 \\ 1 > 0 & for & k = J_1 \\ -1 < 0 & for & k = J_1 + 1 \\ 0 & for & k > J_1 + 1 \end{cases}$$

In this case, the aggregate hazard will be constant everywhere, except for the truncation period of each group, when it will jump up.

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