

**INTERNATIONAL TRADE
POLICY TOWARDS MONOPOLY
AND OLIGOPOLY**

2009

Praveen Kujal and Juan Ruiz

**Documentos de Trabajo
N.º 0901**

BANCO DE ESPAÑA
Eurosistema



INTERNATIONAL TRADE POLICY TOWARDS MONOPOLY AND OLIGOPOLY

INTERNATIONAL TRADE POLICY TOWARDS MONOPOLY AND OLIGOPOLY ^(*)

Praveen Kujal

UNIVERSIDAD CARLOS III DE MADRID

Juan Ruiz ^(**)

BANCO DE ESPAÑA

(*) We thank participants at the ETSG conference for their comments. Kujal acknowledges financial support from the Spanish Ministry of Education and Science (Secretaría de Estado de Universidades e Investigación and grant SEJ2005-08633/ECON). The opinions and analyses herein are the responsibility of the authors and, therefore, do not necessarily coincide with those of the Banco de España or the Eurosystem.

(**) Corresponding author. Address: Banco de España, Dirección General Adjunta de Asuntos Internacionales, Alcalá 48, 28014 Madrid, Spain. Tel.: +34 91 338 4334, Fax: +34 91 338 6212, e-mail: jruiz@bde.es.

The Working Paper Series seeks to disseminate original research in economics and finance. All papers have been anonymously refereed. By publishing these papers, the Banco de España aims to contribute to economic analysis and, in particular, to knowledge of the Spanish economy and its international environment.

The opinions and analyses in the Working Paper Series are the responsibility of the authors and, therefore, do not necessarily coincide with those of the Banco de España or the Eurosystem.

The Banco de España disseminates its main reports and most of its publications via the INTERNET at the following website: <http://www.bde.es>.

Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

© BANCO DE ESPAÑA, Madrid, 2009

ISSN: 0213-2710 (print)

ISSN: 1579-8666 (on line)

Depósito legal: M. 8916-2009

Unidad de Publicaciones, Banco de España

Abstract

This paper highlights the importance of product differentiation and endogenous R&D in determining the optimal R&D policy, in a model where investment in cost reducing R&D is committed before firms compete in a differentiated-goods third-country export market. R&D is always taxed in oligopolies for high degrees of product differentiation. For lower degrees of product differentiation the duopoly is subsidized or the government remains inactive. In contrast, the monopoly is always subsidized. The government with a duopoly may be active or inactive depending on the degree of product differentiation. Thus, we may observe a reversal in the sign of the optimal R&D policy if the degree of product differentiation changes or, alternatively, if there is a change in the number of firms. Similar qualitative results hold if trade policy uses output subsidies, instead of R&D promotion.

Keywords: product differentiation, strategic trade policy, policy reversals, R&D subsidies, monopoly, duopoly.

JEL classification: F12, F13, L13.

1 Introduction

This paper highlights the importance of product differentiation and endogenous sunk costs in international trade policy. In our model firms first invest in R&D and then compete in a differentiated goods market in a third country. The results for R&D subsidies (for the well studied domestic monopoly) do not extend when the domestic and/or foreign market has a duopoly and products are differentiated. They are robust for domestic monopolies, but not so in three contexts: First, when the firms spend on cost reducing R&D before the market competition stage; second, when the product market is differentiated; third, when there is a duopoly in the home or foreign market. Depending on these, the optimal R&D subsidy could be positive, negative, or governments may even choose inaction as an optimal policy.

The model of two domestic monopolies competing in a third market (Brander and Spencer, 1985) has been criticized for want of robustness. The main result from this model is that unilateral trade policy is beneficial and bilateral policy has the characteristics of a prisoners' dilemma. Several authors have argued that imposing output subsidies is not a robust policy in the context of this model (for example, see Eaton and Grossman, 1986 and Dixit, 1984). Further, Helpman and Krugman (1994) conjecture that a reasonable policy should be a tax under both Cournot and Bertrand competition. An exception to this is the recent study by Kujal and Ruiz (2007) who demonstrate that output subsidies are robust to changes in market competition if a prior R&D stage is included in a model with differentiated goods¹. On the other hand, it is generally believed that R&D subsidies are far more robust than output subsidies².

In this paper it is shown that a monopoly's R&D is *always subsidized*. However, this optimal policy changes when there is duopoly in the exporting industry. A duopoly's R&D is *always taxed* for high degrees of product differentiation. This qualitative outcome is independent of the number of firms in the other exporting country. Industrial policy thus depends on the market structure with a monopoly emerging as a special case.

Why R&D is taxed for a domestic oligopoly is easy to understand. It is to be noted that the profit shifting effect in the third market model applies only

¹They show that for sufficiently cost effective R&D governments subsidize exports independently of the mode of competition. This result holds under both output and price competition for linear demands and constant marginal costs. Note that Brander (1995) shows that the results from the strategic trade literature can be extended to models with product differentiation. These models, however, have no prior R&D stage.

²Bagwell and Staiger (1994) show that R&D subsidy towards monopolies is robust to the nature of market competition.

to domestic monopolies. A subsidy towards a domestic monopoly shifts its reaction function to the Stackelberg (leader) point, thus increasing its profits. This, however, does not happen if there is oligopoly in the domestic market. A subsidy to both firms shifts both reaction functions out. However, due to competition among themselves in the third market, they are producing too much from the perspective of joint profit maximization. The advantage of unilaterally achieving the Stackelberg point under domestic monopoly is thus lost under duopoly. The optimal policy in this case is thus a tax for a sufficiently low degree of product differentiation. The optimal tax effectively restricts excessive competition among domestic exporters.

Similar results are obtained for output subsidies. A monopoly is subsidized and a duopoly is taxed for higher degrees of product differentiation. The only difference is that the multiple equilibria observed under R&D subsidies for the intermediate degree of product differentiation are not observed under output subsidies. The results obtained here show few differences between R&D and output subsidies if the degree of product differentiation is high. Besides, the results are robust to price or quantity competition.

By including the R&D stage prior to market competition, one can capture the fundamental aspect of entry barriers in oligopolistic industries. The prior R&D stage captures investment in the long run that has strategic value for firms (see Grossman, 1988; Sutton, 1992; and Herguera, Kujal and Petrakis, 2000 and 2002). As argued by Grossman (1988), firm investment in quality or innovation has commitment value and should have a marked effect not only on market competition but also on the choice of trade policy instruments. This aspect of modeling oligopolies has not been studied fully in international trade.

In fact, the results obtained here hinge on this assumption. The optimal policy in this case depends upon the net effect of the two stages and not solely on the strategic complementarity or substitutability of the variables in the market competition stage. Kujal and Ruiz (2007) highlight the importance of firm investment in a prior R&D stage. They show that by doing so the classic policy reversal result is not observed. In their model, the sign of the optimal output subsidy depends on the net effect of the export subsidy on R&D and the market competition stage. This is unlike the model with no R&D where the sign of the strategic trade policy depends only on the strategic complementarity or substitutability of the variables chosen by firms in the market competition stage. Under R&D and Cournot competition, unilateral export subsidy increases welfare through its effect both on R&D and output. This means that governments want to subsidize exports (Spencer and Brander, 1983). Under Bertrand competition, however, the two effects have the opposite sign. If R&D is sufficiently cost-effective, then it will be rather elastic with

respect to an export subsidy. The high elasticity of R&D makes the effect of the output subsidy on the R&D stage stronger than the effect on the price competition stage. In this case, governments subsidize output under Bertrand competition. Conversely, if R&D is not sufficiently cost-effective, then the effect of output subsidy on the price competition stage dominates the effect on the R&D stage and hence the optimal policy under Bertrand competition is to tax output.

The results obtained here contrast with the general belief that R&D subsidies are more robust than output subsidies and with those of Dixit (1984) for the case of output subsidies. Dixit contends that the policy instrument *does not change* if the number of domestic and foreign firms increases proportionally across the exporting countries³. That is, if n_f is the number of foreign firms, and n_h the number of domestic firms, the sign of the equilibrium subsidy is equal to the sign of $n_f + 1 - n_h$. It is to be noted that the optimal policy depends upon the relative distribution of foreign and domestic firms. When $n_f \geq n_h$ the optimal policy is always a subsidy or otherwise a tax. As in the case of R&D subsidies, and unlike Dixit (1984), it is shown here that a proportional increase in the number of firms at home and abroad *alters* the optimal trade policy instrument for both domestic and foreign governments. Further, oligopolies are always taxed for high levels of product differentiation.

Our results are along the lines suggested by Helpman and Krugman (1994). They argue that the best policy is the one that maximizes welfare and argue, “*..that the case for export subsidies is very fragile indeed (p. 102).*” Even for duopolies, the results are along similar lines, but product differentiation also plays an important role in determining the sign of the optimal policy.

The remainder of the paper is structured as follows. In Section 2, the specific model under free trade is solved. In section 3, government incentives to tax or subsidize R&D under Cournot competition in a third market are analyzed. In Section 4 we briefly discuss results for output subsidies. Section 5 concludes.

2 Free trade

For this study, a third-country model with one or two firms located in two different countries is used. Firms produce a differentiated good, which they sell in a third country. Denote by n_h the number of firms in the home country and by n_f the number in the foreign country, and let $n = n_h + n_f$. Further, there is a competitive numeraire sector. Firms operate under constant returns to scale

³Also see Bhagwati et al.(1998, p. 397). Brander (1995) shows that the results hold for a third market model.

and initially have the same marginal costs of production c . They can invest $\frac{\Delta^2}{2}$ in a cost saving technology prior to engaging in market competition and reduce their marginal cost by Δ . All firms face symmetric demand functions. Consumers maximize utility,

$$U(x_1, x_2, \dots, x_n) = a \left(\sum_{i=1}^n x_i \right) - \frac{1}{2} \left(\sum_{i=1}^n x_i^2 + 2\gamma \left(\sum_{i \neq j} x_i x_j \right) \right) + m$$

with m representing money. The parameter $\gamma \in [0, 1]$ measures the degree of product differentiation, with lower values of γ indicating more differentiated products and x_i the quantity consumed of good i . Resulting inverse demand is $p_i = a - x_i - \gamma(\sum_{h \neq i} x_h)$. If firm i is in the home country ($i \in H$), then,

$$x_i(\bar{\mathbf{p}}) = \frac{a(1-\gamma) - p_i(1 + \gamma(n_h + n_f - 2)) + \gamma \left(\sum_{j \in \{H-i\}} p_{jh} + \sum_{j \in F} p_{jf} \right)}{(1-\gamma)(1 + (n_h + n_f - 1)\gamma)}$$

where x_i is the output produced by firm $i \in H$, p_h and p_f are the prices charged for the home and foreign varieties of the good, respectively, and

$$\bar{\mathbf{p}} = \{p_{1h}, p_{2h}, \dots, p_{n_h h}, p_{1f}, p_{2f}, \dots, p_{n_f f}\}$$

is the vector of prices.⁴ Firms play a two-stage game. In stage one, they simultaneously decide how much to invest in cost-reducing R&D (Δ_i) and in stage two, given the decreased unit cost, they simultaneously compete in quantities. Thus, firms can use R&D strategically to improve their position in the subsequent market competition stage, because investment in R&D has commitment value. We look for the subgame perfect equilibria of the game.

2.1 Quantity competition

In the quantity competition stage, firm i chooses x_i to maximize profits, given inverse demand, $p_i = a - x_i - \gamma(\sum_{j \in \{H \cup F - i\}} x_j)$, and unit costs, $(c - \Delta_i)$. Firm i solves:

$$\max_{x_i} \left[\left(a - x_i - \gamma \left(\sum_{j \in \{H \cup F - i\}} x_j \right) - (c - \Delta_i) \right) x_i - \frac{\Delta_i^2}{2} \right]$$

⁴Note that there are n_h varieties of the home good and n_f varieties of the foreign good, each with a (potentially) different price.

with $x_{j \in \{H \cup F - i\}}$ and Δ_i taken as given. Each firm's reaction function is thus given by

$$x_i(\mathbf{x}_{-i}) = \frac{1}{2} \left(a - c + \Delta_i - \gamma \left(\sum_{j \in \{H \cup F - i\}} x_j \right) \right), \text{ for all } i \in \{H \cup F\}$$

The slope of each reaction function is negative, and decreases in the degree of product differentiation. The intersection of the $n (= n_h + n_f)$ reaction functions gives equilibrium quantities $\mathbf{x} = \{x_{1h}, x_{2h}, \dots, x_{n_h h}, x_{1f}, x_{2f}, \dots, x_{n_f f}\}$, each chosen given the output of the other firm. Equilibrium output and profits (as a function of first-stage R&D expenditures) are:

$$\begin{aligned} \hat{x}_i(\mathbf{\Delta}; \gamma) &= \frac{(a - c)(2 - \gamma) + (2 + \gamma(n_f + n_h - 2))\Delta_i - \gamma \left(\sum_{j \in \{H \cup F - i\}} \Delta_j \right)}{(2 - \gamma)(2 + (n_f + n_h - 1)\gamma)} \\ \hat{\pi}_i(\mathbf{\Delta}; \gamma) &= \left[\frac{(a - c)(2 - \gamma) + (2 + \gamma(n_f + n_h - 2))\Delta_i - \gamma \left(\sum_{j \in \{H \cup F - i\}} \Delta_j \right)}{(2 - \gamma)(2 + (n_f + n_h - 1)\gamma)} \right]^2 \\ &\quad - \frac{\Delta_i^2}{2}. \end{aligned} \tag{1}$$

2.2 R&D Stage

In the R&D stage firm i , given $\mathbf{\Delta}_{-i}$, chooses Δ_i to maximize its profits (defined above). Reaction Functions in R&D expenditures are given by

$$\Delta_i(\mathbf{\Delta}_{-i}) = \frac{2[2 + \gamma(n - 2)] \left((a - c)(2 - \gamma) - \gamma \left(\sum_{j \in \{H \cup F - i\}} \Delta_j \right) \right)}{8 + \gamma[(4 - \gamma^2)\gamma + n^2\gamma[2 - (4 - \gamma)\gamma] - 16] - 2n[(4 - \gamma)(2 - \gamma)\gamma - 4]}$$

where $n = n_h + n_f$. Using symmetry, one can easily solve the system of n reaction functions to derive the equilibrium level of R&D spending for each firm:

$$\Delta^*(\gamma) = \frac{2(a - c)[2 + (n - 2)\gamma]}{4 - \gamma[8 - 6n - 2\gamma(n - 3)(n - 1) + \gamma^2(n - 1)^2]},$$

Replacing the expression above in (1) we obtain firms' equilibrium profits.

$$\pi^*(\gamma) = (a - c)^2 \frac{((a - c)(2 - \gamma)[2 + (n - 1)\gamma])^2 - 2((a - c)[2 + (n - 2)\gamma])^2}{(4 - \gamma[8 - 6n - 2\gamma(n - 3)(n - 1) + \gamma^2(n - 1)^2])^2}.$$

Note that a firm has greater incentives to invest in cost-reducing R&D under Cournot competition than under a pure cost-minimizing strategy. This is due to the *positive* strategic effect of R&D on profits.

3 R&D subsidies

The R&D subsidies towards a monopoly and duopoly under output competition are studied in this section⁵. We continue to assume firms in two countries that sell a differentiated good in a third market, and introduce a stage, previous to R&D choice, where governments in exporting countries simultaneously decide to engage or not in active R&D policy. Governments that have decided to engage in active policy then simultaneously commit to a subsidy (positive or negative) to R&D. Given the policy announcement of both governments, firms choose the profit maximizing level of R&D in the second stage. In the final stage firms compete in quantities.

The results for R&D policy for bilateral duopolies under Cournot competition are presented first. This allows one to introduce the basic model with R&D subsidies. After this, the case of domestic monopolies is presented. Finally, we study the asymmetric case of a monopoly in one country and a duopoly in the other.⁶

3.1 Bilateral duopolies

First we present the results where only one government pursues an active R&D policy, while the other remains inactive. After this, the results of the case in which both countries choose an R&D subsidy are presented, and then the equilibrium choice of R&D subsidies characterized. Starting with the case in which only the home government pursues an active R&D policy, the domestic (h) and the foreign (f) firms' profit maximization problem is solved in the quantity competition stage:

⁵Results for Bertrand competition are available in Kujal and Ruiz (2003). The qualitative results regarding government policy do not change with respect to Cournot competition.

⁶Notice that in a two-stage game, one cannot make the convenient symmetry assumption on output that is conventional in one-stage games. One needs to explicitly solve for the reaction functions for each firm and then solve the problem in the R&D stage. It has not been possible in this study to find explicit solutions for the case of many firms in each country owing to analytical complexity. Hence, only the problem of domestic duopolies is solved.

$$\max_{x_h} \left[(a - x_h - \gamma \left(\sum_{j \in \{H \cup F - i\}} x_j \right) - (c - \Delta_h)) \right] x_h - (1 - z_h) \frac{\Delta_h^2}{2}$$

$$\max_{x_f} \left[(a - x_f - \gamma \left(\sum_{j \in \{H \cup F - i\}} x_j \right) - (c - \Delta_f)) \right] x_f - \frac{\Delta_f^2}{2}.$$

where z_h is the subsidy to R&D expenditures. From the first order conditions, we obtain the reaction functions

$$x_i(\mathbf{x}_{-i}) = \frac{1}{2} \left(a - c + \Delta_i - \gamma \left(\sum_{j \in \{H \cup F - i\}} x_j \right) \right), \text{ for all } i \in \{H \cup F\} \quad (2)$$

Note that the reaction functions under R&D subsidy are the same as under free trade. This is because only the R&D subsidy enters the first order conditions in the final stage. As before, under free trade, the intersection of the $n_h + n_f$ reaction functions gives the vector of equilibrium quantities, each chosen given the output of the other firm. It is easy to see in Eq. (2) that the derivative $\frac{dx_i}{dx_{-i}}$ decreases in γ and becomes zero for $\gamma = 0$. This shows that, as goods become more differentiated, the output shifting effect of the increase in own production is smaller. Given that the transfer of profits from foreign to domestic firms, induced by a subsidy, depends on the output shifting effect, the incentive to subsidize decreases as γ gets smaller. This simple intuition (which also applies to the case of bilateral subsidies) shows that R&D subsidies cannot be independent of the degree of product differentiation.⁷

Solving the system of equations implicit in Eq. (2), the equilibrium output can be obtained as a function of first-stage R&D expenditures:

$$\hat{x}_i(\Delta; \gamma) = \frac{(a - c)(2 - \gamma) + (2 + \gamma(n_f + n_h - 2))\Delta_i - \gamma \left(\sum_{j \in \{H \cup F - i\}} \Delta_j \right)}{(2 - \gamma)(2 + (n_f + n_h - 1)\gamma)}. \quad (3)$$

Turning now to the R&D stage, equilibrium quantities are substituted into firm's profits to solve for the equilibrium R&D under unilateral subsidies. The following reaction functions are obtained for the domestic and foreign firms, respectively.

⁷It is to be noted that, besides the profit shifting effect, the government also has an incentive to limit excess competition between firms (Helpman and Krugman, 1994). This point will be discussed later in the paper.

$$\Delta_{hi}(\Delta_{-hi}) = \frac{4(1+\gamma)((a-c)(2-\gamma) - \gamma(\Delta_{hj} + \Delta_{f1} + \Delta_{f2}))}{8 - z_h(2-\gamma)^2(2+3\gamma)^2 - \gamma(16 - \gamma(16 + 3(8-3\gamma)\gamma))} \quad (4)$$

$$\Delta_{fi}(\Delta_{-fi}) = \frac{4(1+\gamma)((a-c)(2-\gamma) - \gamma(\Delta_{h1} + \Delta_{h2} + \Delta_{fj}))}{8 - \gamma(16 - \gamma(16 + 3(8-3\gamma)\gamma))}, i \neq j, i, j = 1, 2. \quad (5)$$

This gives the following equilibrium R&D for the domestic and the foreign firms.

$$\Delta_h^*(\gamma) = \frac{4(1+\gamma)(a-c)(-4 + (10-3\gamma)\gamma^2)}{A(\gamma)}$$

$$\Delta_f^*(\gamma) = \frac{4(1+\gamma)(a-c)(-4 + (10-3\gamma)\gamma^2 + z_h(2-\gamma)^2(2+3\gamma))}{A(\gamma)}$$

where, $A(\gamma) = (4 + \gamma^2(-10 + 3\gamma))(-4 + \gamma(-16 + 3\gamma(-2 + 3\gamma))) - z_h(-2 + \gamma)^2(2 + 3\gamma)(8 + \gamma(20 + 3\gamma(-4 + \gamma(-8 + 3\gamma))))$.

Finally, in the first stage, equilibrium R&D is substituted into the definition of total welfare for exporting country i thus:

$$W = \sum_{i=1}^2 (\pi_i^*(\Delta_i^*) - z_h \frac{\Delta_i^{*2}(\gamma; z_h)}{2}). \quad (6)$$

Using the foregoing expressions the welfare-maximizing unilateral tax is given by

$$z_h^* = -\frac{\gamma(8 + \gamma(4 + \gamma(-28 + 3\gamma(-8 + 3\gamma(-8 + 3\gamma)))))}{\gamma(2-\gamma)(2+3\gamma)(-4 + \gamma(2+\gamma)(-4+3\gamma))}.$$

It is worth noting here that the optimal subsidy is independent of market size $(a-c)$ and marginal costs. We focus only on the levels of product differentiation that provide interior solutions. In the case of unilateral R&D subsidies this would be true for ⁸ $\gamma < 0.665703$. Figure 1 (left) depicts the optimal R&D subsidy z_h^* for different degrees of product differentiation (γ). From the figure it is clear that the optimal policy depends on the degree of product differentiation. In particular, R&D is taxed for high levels of product differentiation (i.e. $\gamma < 0.514708$) and subsidized for low levels of product differentiation. Thus, a model that abstracts from product differentiation and focuses only on the special case of a (local) monopoly (i.e. low γ) may not be a reliable guide for policy making.

⁸This range ensures that output, R&D and welfare are positive.

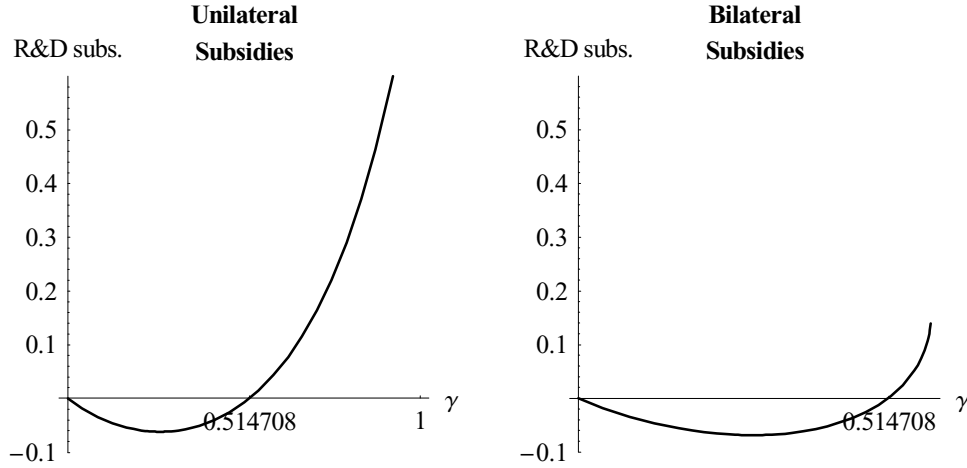


Figure 1: R&D subsidies in the case of two duopolies. Left: Unilateral subsidy when the foreign government commits to remain inactive. Right: Bilateral subsidies when both governments are active.

It is clear that a unilateral R&D policy always increases welfare over free trade because the country always has the option to set a zero subsidy. Welfare of the country that decides against engaging itself in R&D policy (henceforth the inactive country) increases if the unilateral policy of the rival government is to tax. The tax reduces the production of the taxing country and increases the profits of the exporter in the inactive country. In the case of a unilateral subsidy the effects are reversed.

Turning now to the case where both countries commit to using R&D policies (bilateral subsidies), the firm's problem in the quantity competition stage is solved

$$\max_{x_i} \left[(a - x_i - \gamma \left(\sum_{j \in \{H \cup F - i\}} x_j \right) - (c - \Delta_i)) x_i - (1 - z_i) \frac{\Delta_i^2}{2} \right]$$

and obtain output reaction functions obtained as in Eq. (2), which translate into equilibrium outputs as in (3). Substituting equilibrium quantities into the

equation describing profits we obtain,

$$\hat{\pi}_i(\mathbf{\Delta};\gamma) = \left[\frac{(a-c)(2-\gamma) + (2+\gamma(n_f+n_h-2))\Delta_i - \gamma \left(\sum_{j \in \{H \cup F - i\}} \Delta_j \right)}{(2-\gamma)(2+(n_f+n_h-1)\gamma)} \right]^2 - (1-z_i) \frac{\Delta_i^2}{2}.$$

Maximizing $\hat{\pi}_i(\mathbf{\Delta};\gamma)$ with respect to Δ_i , we obtain the reaction functions for the domestic and foreign firms. These are exactly the same as in Eq. (4) for the home firms (and analogously for the foreign firms, which now subsidize a fraction z_f of R&D expenditures). The intersection of the $n(=n_h+n_f)$ reaction functions gives the equilibrium R&D expenditures as a function of γ . Given the R&D level chosen by the firms, government i chooses z_i to maximize total welfare in Eq. (6). This gives the equilibrium bilateral R&D subsidies thus:

$$z_i^* = \frac{2+2\gamma-3\gamma^2 - (1+\gamma)\sqrt{((2-(4-\gamma)\gamma)(2+3\gamma(4+3\gamma)))}}{((-2+\gamma)(2+\gamma)(2+3\gamma)}$$

Again, the focus is on $0 < \gamma < 0.586505$ to obtain interior solutions. Figure 1 (right) shows that if both countries engage in active policy, they tax R&D for high degrees of product differentiation ($\gamma < 0.514708$); otherwise, they subsidize it. As in the case of unilateral subsidies, equilibrium R&D values can be substituted in Eq. (6) to obtain total welfare under bilateral R&D subsidies.

Proposition 1 describes the equilibrium of the R&D policy game for bilateral duopolies. R&D is taxed for high degrees of product differentiation and unilaterally subsidized for low degrees of differentiation.

Proposition 1 (R&D subsidies for bilateral duopolies) *Restricting attention to interior solutions ($\gamma < 0.586505$), the equilibrium of the policy game with two exporters in each country is as follows:*

- For $\gamma < 0.514708$ both countries tax R&D.
- For $0.514708 < \gamma < 0.586505$ there are two equilibria. In each equilibrium, one country subsidizes R&D while the other does not engage in active trade policy.

Proof: It is argued earlier that if a foreign government is not engaged in active policy, then the home government always prefers to be unilaterally active. If the other country is active, then the home country is active if $\gamma < 0.514708$ (as welfare is bigger than remaining unilaterally inactive), but inactive otherwise

($0.514708 < \gamma < 0.586505$) (see figure. 2). As the policy game is symmetric, there are two equilibria for the latter range. This proves the structure of the equilibria claimed in the proposition.

As regards the sign of the active policy, it can be seen that, under bilateral policy ($\gamma < 0.514708$), both countries tax (see figure 1, right). For $0.514708 < \gamma < 0.585998$, the equilibrium involves a unilateral policy and the active country imposes a subsidy (see figure. 1, left). ■

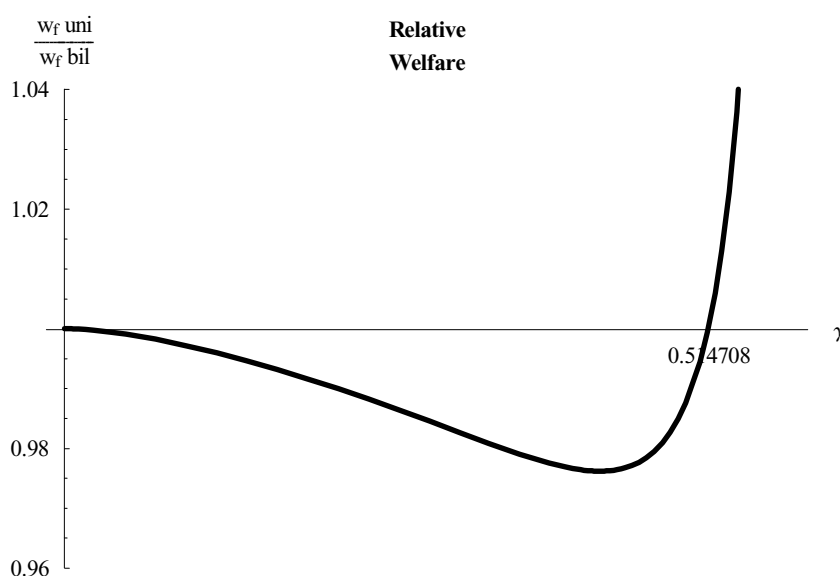


Figure 2: R&D subsidies for bilateral duopolies: Welfare for a unilaterally inactive government ($w_f \text{ uni}$) vs. welfare for a bilateral subsidy ($w_f \text{ bil}$).

As suggested by Helpman and Krugman (1994), firms are taxed for bilateral duopolies. We, however, find that this holds good only for high degrees of product differentiation. On the other hand, a policy reversal (to a unilateral R&D subsidy) is observed for lower degrees of product differentiation.

The effect of an R&D policy on R&D investment and its consequent effect on output (and hence prices and profits) helps one in understanding the conflicting effects at play. Under bilateral duopolies, the tax on R&D decreases R&D expenditure of all firms in both markets. This softens overinvestment in R&D as a result of which the firms' output decreases and the prices and profits increase. Thus, a cooperative solution would involve both governments taxing

R&D. Contrarily, it is well known that unilateral incentives to subsidize R&D do exist as they help domestic firms achieve the Stackelberg leader position. This effect increases output and transfers profits from foreign firms.

However, in the case of a domestic duopoly, the government also wants to curtail excessive competition among domestic firms that export to the third market. A standard result from Cournot markets, with more than one firm at home, is that firms produce too much in terms of joint profits. Therefore, to ameliorate this *negative domestic externality* the government chooses to tax the domestic oligopoly. Helpman and Krugman (1994) advance a similar view by arguing that a tax achieves tacit collusion between the firms as it reduces R&D and output, and increases prices and profits. It is shown here that under high degrees of product differentiation this domestic restraint effect dominates the profit shifting effect.

Graphically, under domestic duopoly, R&D subsidy shifts the reaction function of all the domestic firms. If the output of a domestic firm is on the horizontal axis and the aggregate output of all the others (including the other domestic firms) on the vertical axis, then it can be seen that the subsidy shifts the reaction function of not only the domestic firm but also of all other firms in the market. With both reaction functions shifting out it is easy to see that lower profits are achieved. Hence, the profit transfer effect of a subsidy on duopolies is outweighed by the negative externality of excessive competition among domestic firms. As a result, regardless of the relative distribution of firms, duopolies are always taxed for high degrees of product differentiation. However, as the degree of product differentiation decreases, the profit shifting effect dominates and governments feel encouraged to subsidize R&D.

The case of a monopoly in each country will now be discussed to show how the optimal R&D policy is modified.

3.2 Bilateral monopolies

We briefly present the results for a monopoly in each country. The focus is on the degree of product differentiation for which an interior solution is obtained under an R&D subsidy, i.e. $\gamma < 0.663916$ under unilateral policy and $\gamma < 0.585998$ under bilateral policy.⁹ The following Proposition 2 shows that, with two monopolies, R&D is subsidized. However, the decision to subsidize R&D (as opposed to remain inactive) depends on product differentiation. If the degree of product differentiation is high, both countries subsidize R&D; if it is low two equilibria exist in which *only one* country subsidizes.

⁹These restrictions ensure that output and R&D investment are positive.

Proposition 2 (R&D subsidies for bilateral monopolies) *Restricting to interior solutions ($\gamma < 0.585998$), and when there is one exporter in each country, the equilibrium of the policy game when is as follows:*

- For $\gamma < 0.427853$, both countries subsidize R&D.
- For $0.427853 < \gamma < 0.585998$, there are two equilibria. In each equilibrium, one country subsidizes R&D while the other does not engage in active trade policy.

Proof: As argued in the previous section, the government always has an incentive to engage in active trade policy if the other country is inactive. The active government subsidizes R&D (figure 3, left) and if both countries engage in active policy, then both of them subsidize R&D (figure 3, right). For $\gamma < 0.427853$, both countries subsidize, because welfare under bilateral subsidies is bigger than welfare obtained by remaining inactive unilaterally (figure 4). However, for $\gamma > 0.427853$, one government prefers to remain inactive if the other commits to engage itself in R&D policy. ■

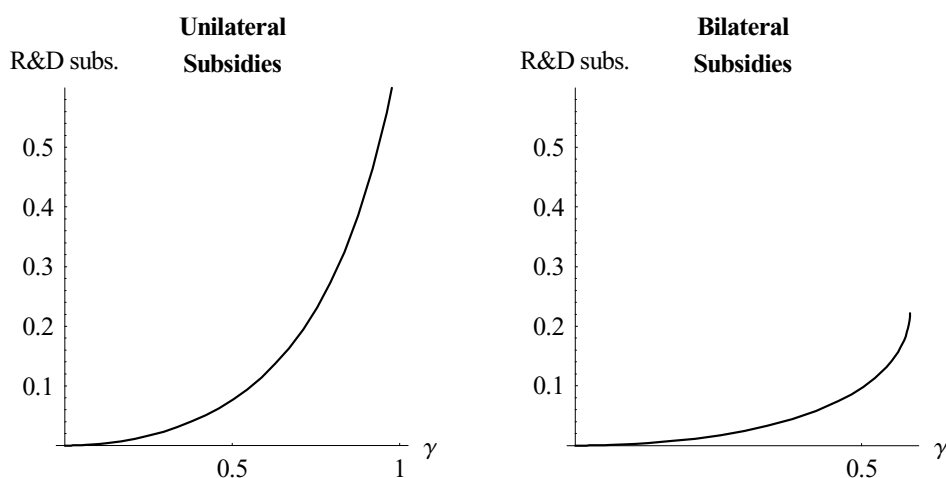


Figure 3: R&D subsidies in the case of two monopolies. Left: Unilateral subsidy when the foreign government remains inactive. Right: Bilateral subsidies when both governments are active.

It is worth noting that, contrary to Spencer and Brander (1983) and Bagwell and Staiger (1994), governments do not always subsidize R&D for domestic

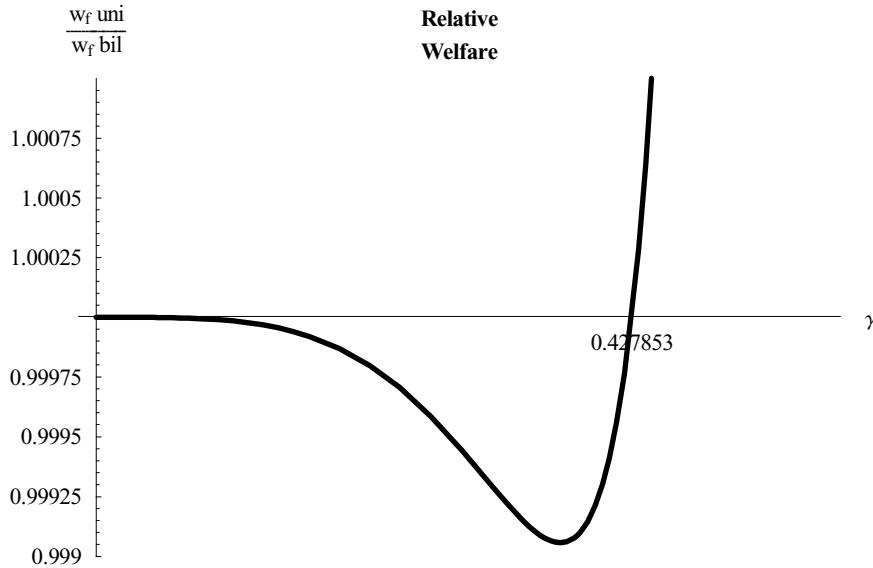


Figure 4: R&D subsidies for bilateral monopolies: Welfare for a unilaterally inactive government ($w_f uni$) vs. welfare for a bilateral subsidy ($w_f bil$).

monopolies. We find that bilateral subsidies to R&D (instead of one government committing not to intervene) are observed only for sufficiently small γ .

3.3 Monopoly versus a duopoly

We now analyze the case in which the home country (h) has a monopoly and the foreign country (f) a duopoly. Focusing on interior solutions, the analysis would be restricted to $\gamma < 0.627557$ for unilateral, and $\gamma < 0.587535$ for bilateral policy. The following proposition shows that the government with duopoly either taxes R&D or remains inactive. The government with monopoly, on the other hand, either subsidizes R&D or remains inactive¹⁰:

Proposition 3 (R&D subsidies for a monopoly vs. a duopoly) *Restricting to interior solutions, that is, $\gamma < 0.587535$ and when there is a home monopoly ($n_h = 1$) and a foreign duopoly ($n_f = 2$), the equilibrium of the policy game is as follows:*

- For $\gamma < 0.273545$, both countries engage in active R&D policies: the

¹⁰It can be verified that a domestic monopoly is always subsidized when the number of foreign firms increases.

foreign country (with duopoly) taxes R&D while the home country (with monopoly) subsidizes it.

- *For $0.273545 < \gamma < 0.531453$, the foreign government taxes R&D while the home government remains inactive.*
- *For $0.531453 < \gamma < 0.587535$, there would be two equilibria:*
 - *Equilibrium I: The foreign government taxes R&D and the home government is inactive.*
 - *Equilibrium II: The foreign government is inactive and the home government subsidizes R&D.*

Proof: As argued earlier, if a government does not engage itself in active policy then the other government prefers to be active. The country with monopoly always subsidizes R&D (figure 5, left) and the one with duopoly taxes R&D for $\gamma < 0.611472$, and subsidizes it otherwise (figure 5, right).

What would be the best response of the other active country? If the country with monopoly is active, then the government with duopoly wants to be active too for $\gamma < 0.531453$, and on other instances inactive. This can be seen in figure 6 (left), which shows that welfare under bilateral subsidies is bigger than welfare obtained by remaining inactive unilaterally for low γ . Similarly, if the country with two firms is active, then the country with monopoly wants to be active if $\gamma < 0.273545$, and inactive otherwise (see figure 6, right).

Putting together these reaction functions, one can obtain the structure of the equilibrium in the proposition. In the case of a bilateral active policy ($\gamma < 0.273545$), the country with one firm subsidizes R&D (see figure 7, left) while the one with two firms taxes it (figure 7, right). ■

Thus, a country with monopoly subsidizes R&D, while the one with duopoly taxes it. However, for some degrees of product differentiation, both may not be active at the same time in equilibrium. This reaffirms that policy choice cannot be independent of the degree of product differentiation, market structure and the nature of (endogenous) sunk costs.

The results in this section show that the sign of the optimal R&D policy is sensitive to both the distribution of firms in the two exporting countries and to the degree of product differentiation, though in a non-linear way in some cases. Propositions 1 and 3 show that R&D by a duopoly is taxed for high degrees of product differentiation and subsidized for lower degrees of differentiation. In contrast, a monopoly is always subsidized. This result is in stark contrast to that of Dixit (1984) who finds that the optimal output subsidy does not change if the relative number of firms remains the same. One may observe a reversal

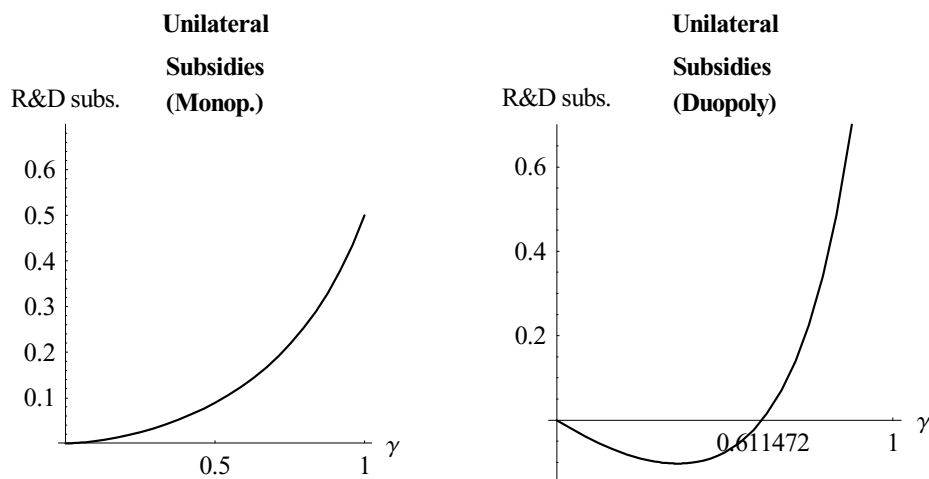


Figure 5: Unilateral R&D subsidies in the case of monopoly vs a duopoly. Left: unilateral subsidy for a monopolist when the foreign government commits to remain inactive. Right: unilateral subsidy for a duopolist.

in the sign of the optimal R&D policy if the degree of product differentiation changes (keeping the number of firms unaltered in each exporting country) or, alternatively, if there is a change in the number of firms (holding fixed the degree of product differentiation).

4 Output subsidies

As a straightforward extension of the model in the previous section, one can study the choice of output subsidies. In particular, assume an output subsidy s_h that reduces marginal costs. Thus, the profit maximization problem of the firm is given by:

$$\max_{x_h} \left[(a - x_h - \gamma \left(\sum_{j \in \{H \cup F - i\}} x_j \right) - (c - s_h - \Delta_h)) \right] x_h - \frac{\Delta_h^2}{2}$$

Following similar steps as in the previous section¹¹, one can derive the equilibrium output subsidy. The main difference from R&D policy is that multiple equilibria do not exist with output subsidies. When both countries have a monopoly, the optimal policy (in the range where interior solutions

¹¹Details of the optimal choice of output subsidies can be seen in Kujal and Ruiz (2003).

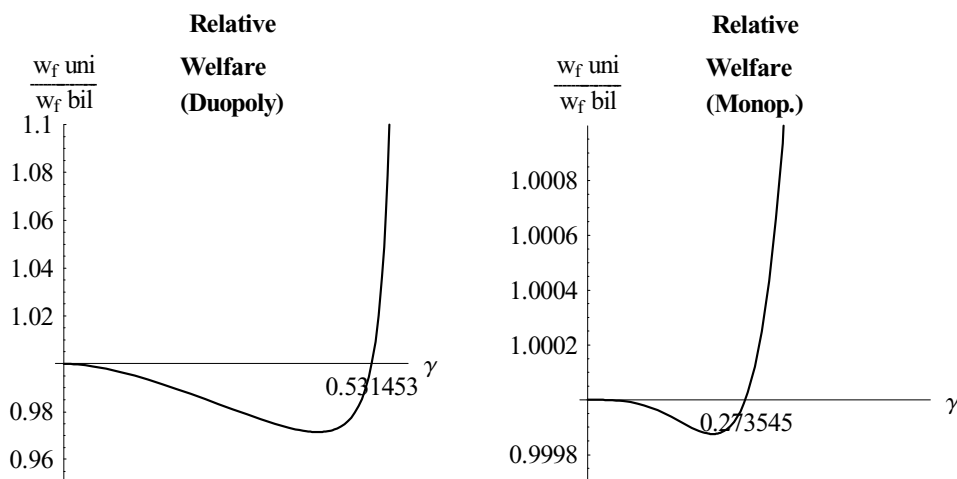


Figure 6: Monopoly vs a duopoly: Welfare for a unilaterally inactive government $w_f uni$ vs. for a bilateral subsidy $w_f bil$. Left: government with a duopoly. Right: government with a monopoly.

exist) is always an output subsidy, as emphasized by Kujal and Ruiz (2007). However, it is to be noted that R&D is assumed to be fully effective at cost reduction, in what constitutes a special case of a more general model¹².

In the case where there are duopolies in each country, the optimal policy for both governments is taxing if the degree of product differentiation is high, and subsidizing if differentiation is for an intermediate range of γ . Interestingly, the optimal policy choice under this scenario, according to Dixit (1984), would *always* be a subsidy, because in his model the sign of the policy is the same as $n_f + 1 - n_h > 0$. Moreover, unlike Helpman and Krugman (1994), our results suggest that taxing may not always be an optimal policy, because governments would optimally subsidize for low product differentiation. The reason is similar to the case of R&D subsidies: for high degrees of product differentiation, the profit shifting effect is small and the negative externality of excessive competition between the two domestic firms dominates. That is why oligopolies are taxed. However, for an intermediate range of product differentiation the profit shifting effect dominates the negative (output) externality and hence firms are subsidized.

¹²In Kujal and Ruiz (2007), it is shown that policy reversals may still be observed for low effectiveness of R&D at decreasing marginal costs. When R&D is fully effective, the special case in this paper, then no policy reversal is observed even when moving to Bertrand competition.

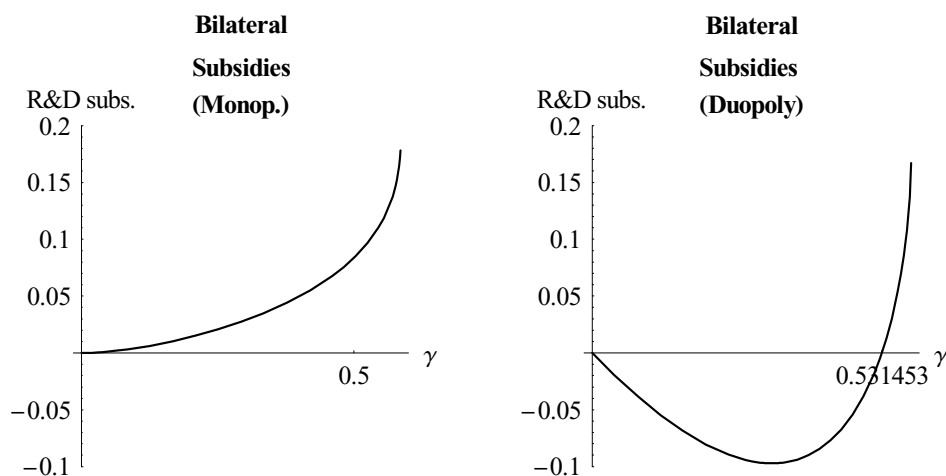


Figure 7: Bilateral R&D subsidies in the case of monopoly vs a duopoly. Left: subsidy for a monopolist. Right: subsidy for a duopolist.

As in the R&D case, for a duopoly in one country and a monopoly in the other, the duopoly is taxed when the degree of product differentiation is high. On the other hand, the government with two firms remains *inactive* for intermediate values of γ (since $n_f + 1 - n_h = 0$ an inactive government is always the optimal policy in Dixit, 1984), but subsidizes output for high values of γ . Independent of the number of foreign firms a domestic monopoly is always subsidized in a manner similar to Dixit (1984).

Our results show that the use of output subsidies yields the same qualitative predictions as for R&D subsidies: there could be reversals in the sign of the optimal policy depending on the degree of product differentiation or the number of firms in each country.

5 Conclusion

This study has shown that the optimal R&D and output policy is not only sensitive to the distribution of firms in the two exporting countries, but also to the degree of product differentiation. Bilateral duopolies are taxed for R&D (or output) for high degrees of product differentiation and subsidized for lower degrees of differentiation. This is in contrast to the finding of Dixit (1984) that the optimal output subsidy does not change if the relative number of firms remains the same. Second, increasing the number of firms, in one, or both the

countries, may change the sign of the optimal policy in a non-monotonic way. A monopoly's R&D is always subsidized, whereas a duopoly is always taxed for high degrees of product differentiation. Countries may, however, tax, subsidize, or remain inactive for lower degrees of product differentiation. R&D is always taxed for a duopoly when the other country has a monopoly. Therefore, what matters in policy choice are the distribution of firms and the degree of product differentiation.

A government with a domestic duopoly does not have unilateral incentives to subsidize R&D (or output) owing to the negative externality between domestic firms. An R&D (or output) subsidy shifts the reaction function of *all* domestic firms out. Instead of achieving the Stackelberg leader point, a lower level of profits is attained by both domestic firms. For low degrees of product differentiation, a government may be inactive or even revert policy and subsidize R&D.

Further, trade policy depends on the number of firms that are present inside and outside a country. In contrast to Dixit (1984), it is shown here that the optimal policy does not depend only on the relative asymmetry in the distribution of firms. Interestingly, in some cases the equilibrium involves a government remaining inactive, even though its welfare is reduced with respect to free trade. Thus, retaliation may not always be observed.

References

- BAGWELL, K., and R. STAIGER(1994). “The Sensitivity of Strategic and Corrective R&D Policy in Oligopolistic Industries”, *Journal of International Economics*, 36, pp. 133-150.
- BHAGWATI, J., A. PANAGARIYA and T. N. SRINIVASAN (1998). *Lectures on International Trade*, Boston: MIT Press.
- BRANDER, J. (1995). “Strategic Trade Policy”, in Gene Grossman and Ken Rogoff (eds.), *Handbook of International Economics*, Elsevier Science, pp. 1395-1455.
- BRANDER, J., and B. SPENCER (1985). “Export Subsidies and International Market Share Rivalry”, *Journal of International Economics*, 18, pp. 83-100.
- DIXIT, A. (1984). “International Trade Policy for Oligopolistic Industries”, *The Economic Journal*, 94, pp. 1-16..
- EATON, J., and G. GROSSMAN (1986). “Optimal Trade and Industrial Policy under Oligopoly”, *Quarterly Journal of Economics*, 101, pp. 381-406.
- GROSSMAN, G. (1988). “Strategic Export Promotion: A Critique”, in Paul Krugman (ed.), *Strategic Trade Policy and the New International Economics*, Boston: MIT Press.
- HELPMAN, E., and P. KRUGMAN (1994). *Trade Policy and Market Structure*, Boston: MIT Press.
- HERGUERA, Í., P. KUJAL and E. PETRAKIS (2000). “Quantity Restrictions and Endogenous Quality Choice”, *International Journal of Industrial Organization*, 18, pp. 1259-1277.
- (2002). “Tariffs, Quality Reversals and Exit in Vertically Differentiated Industries”, *Journal of International Economics*, 58, pp. 467-492.
- KUJAL, P., and J. M. RUIZ (2003). *International Trade Policy towards Monopolies and Oligopolies*, Economics Working Paper Archive. <http://ideas.repec.org/p/wpa/wuwpit/0302002.html>.
- (2007). “Cost Effectiveness of R&D and Strategic Trade Policy”, *The B.E. Journal of Economic Analysis & Policy*, 7 Iss. 1 (Topics), Article 21.
- SPENCER, B., and J. BRANDER (1983). “International R&D Rivalry and Industrial Strategy”, *Review of Economic Studies*, 50, pp. 707-722.
- SUTTON, J. (1992). *Sunk Costs and Market Structure*, Boston: MIT Press.

BANCO DE ESPAÑA PUBLICATIONS

WORKING PAPERS¹

- 0721 CLAUDIA CANALS, XAVIER GABAIX, JOSEP M. VILARRUBIA AND DAVID WEINSTEIN: Trade patterns, trade balances and idiosyncratic shocks.
- 0722 MARTÍN VALLCORBA AND JAVIER DELGADO: Determinantes de la morosidad bancaria en una economía dolarizada. El caso uruguayo.
- 0723 ANTÓN NÁKOV AND ANDREA PESCATORI: Inflation-output gap trade-off with a dominant oil supplier.
- 0724 JUAN AYUSO, JUAN F. JIMENO AND ERNESTO VILLANUEVA: The effects of the introduction of tax incentives on retirement savings.
- 0725 DONATO MASCIANDARO, MARÍA J. NIETO AND HENRIETTE PRAST: Financial governance of banking supervision.
- 0726 LUIS GUTIÉRREZ DE ROZAS: Testing for competition in the Spanish banking industry: The Panzar-Rosse approach revisited.
- 0727 LUCÍA CUADRO SÁEZ, MARCEL FRATZSCHER AND CHRISTIAN THIMANN: The transmission of emerging market shocks to global equity markets.
- 0728 AGUSTÍN MARAVALL AND ANA DEL RÍO: Temporal aggregation, systematic sampling, and the Hodrick Prescott filter.
- 0729 LUIS J. ÁLVAREZ: What do micro price data tell us on the validity of the New Keynesian Phillips Curve?
- 0730 ALFREDO MARTÍN-OLIVER AND VICENTE SALAS-FUMÁS: How do intangible assets create economic value? An application to banks.
- 0731 REBECA JIMÉNEZ-RODRÍGUEZ: The industrial impact of oil price shocks: Evidence from the industries of six OECD countries.
- 0732 PILAR CUADRADO, AITOR LACUESTA, JOSÉ MARÍA MARTÍNEZ AND EDUARDO PÉREZ: El futuro de la tasa de actividad española: un enfoque generacional.
- 0733 PALOMA ACEVEDO, ENRIQUE ALBEROLA AND CARMEN BROTO: Local debt expansion... vulnerability reduction? An assessment for six crises-prone countries.
- 0734 PEDRO ALBARRÁN, RAQUEL CARRASCO AND MAITE MARTÍNEZ-GRANADO: Inequality for wage earners and self-employed: Evidence from panel data.
- 0735 ANTÓN NÁKOV AND ANDREA PESCATORI: Oil and the Great Moderation.
- 0736 MICHIEL VAN LEUVENSTEIJN, JACOB A. BIKKER, ADRIAN VAN RIXTEL AND CHRISTOFFER KOK SØRENSEN: A new approach to measuring competition in the loan markets of the euro area.
- 0737 MARIO GARCÍA-FERREIRA AND ERNESTO VILLANUEVA: Employment risk and household formation: Evidence from differences in firing costs.
- 0738 LAURA HOSPIDO: Modelling heterogeneity and dynamics in the volatility of individual wages.
- 0739 PALOMA LÓPEZ-GARCÍA, SERGIO PUENTE AND ÁNGEL LUIS GÓMEZ: Firm productivity dynamics in Spain.
- 0740 ALFREDO MARTÍN-OLIVER AND VICENTE SALAS-FUMÁS: The output and profit contribution of information technology and advertising investments in banks.
- 0741 ÓSCAR ARCE: Price determinacy under non-Ricardian fiscal strategies.
- 0801 ENRIQUE BENITO: Size, growth and bank dynamics.
- 0802 RICARDO GIMENO AND JOSÉ MANUEL MARQUÉS: Uncertainty and the price of risk in a nominal convergence process.
- 0803 ISABEL ARGIMÓN AND PABLO HERNÁNDEZ DE COS: Los determinantes de los saldos presupuestarios de las Comunidades Autónomas.
- 0804 OLYMPIA BOVER: Wealth inequality and household structure: US vs. Spain.
- 0805 JAVIER ANDRÉS, J. DAVID LÓPEZ-SALIDO AND EDWARD NELSON: Money and the natural rate of interest: structural estimates for the United States and the euro area.
- 0806 CARLOS THOMAS: Search frictions, real rigidities and inflation dynamics.
- 0807 MAXIMO CAMACHO AND GABRIEL PEREZ-QUIROS: Introducing the EURO-STING: Short Term INDicator of Euro Area Growth.
- 0808 RUBÉN SEGURA-CAYUELA AND JOSEP M. VILARRUBIA: The effect of foreign service on trade volumes and trade partners.
- 0809 AITOR ERCE: A structural model of sovereign debt issuance: assessing the role of financial factors.
- 0810 ALICIA GARCÍA-HERRERO AND JUAN M. RUIZ: Do trade and financial linkages foster business cycle synchronization in a small economy?
- 0811 RUBÉN SEGURA-CAYUELA AND JOSEP M. VILARRUBIA: Uncertainty and entry into export markets.
- 0812 CARMEN BROTO AND ESTHER RUIZ: Testing for conditional heteroscedasticity in the components of inflation.
- 0813 JUAN J. DOLADO, MARCEL JANSEN AND JUAN F. JIMENO: On the job search in a model with heterogeneous jobs and workers.
- 0814 SAMUEL BENTOLILA, JUAN J. DOLADO AND JUAN F. JIMENO: Does immigration affect the Phillips curve? Some evidence for Spain.
- 0815 ÓSCAR J. ARCE AND J. DAVID LÓPEZ-SALIDO: Housing bubbles.
- 0816 GABRIEL JIMÉNEZ, VICENTE SALAS-FUMÁS AND JESÚS SAURINA: Organizational distance and use of collateral for business loans.

1. Previously published Working Papers are listed in the Banco de España publications catalogue.

- 0817 CARMEN BROTO, JAVIER DÍAZ-CASSOU AND AITOR ERCE-DOMÍNGUEZ: Measuring and explaining the volatility of capital flows towards emerging countries.
- 0818 CARLOS THOMAS AND FRANCESCO ZANETTI: Labor market reform and price stability: an application to the Euro Area.
- 0819 DAVID G. MAYES, MARÍA J. NIETO AND LARRY D. WALL: Multiple safety net regulators and agency problems in the EU: Is Prompt Corrective Action partly the solution?
- 0820 CARMEN MARTÍNEZ-CARRASCAL AND ANNALISA FERRANDO: The impact of financial position on investment: an analysis for non-financial corporations in the euro area.
- 0821 GABRIEL JIMÉNEZ, JOSÉ A. LÓPEZ AND JESÚS SAURINA: Empirical analysis of corporate credit lines.
- 0822 RAMÓN MARÍA-DOLORES: Exchange rate pass-through in new Member States and candidate countries of the EU.
- 0823 IGNACIO HERNANDO, MARÍA J. NIETO AND LARRY D. WALL: Determinants of domestic and cross-border bank acquisitions in the European Union.
- 0824 JAMES COSTAIN AND ANTÓN NÁKOV: Price adjustments in a general model of state-dependent pricing.
- 0825 ALFREDO MARTÍN-OLIVER, VICENTE SALAS-FUMÁS AND JESÚS SAURINA: Search cost and price dispersion in vertically related markets: the case of bank loans and deposits.
- 0826 CARMEN BROTO: Inflation targeting in Latin America: Empirical analysis using GARCH models.
- 0827 RAMÓN MARÍA-DOLORES AND JESÚS VAZQUEZ: Term structure and the estimated monetary policy rule in the eurozone.
- 0828 MICHIEL VAN LEUVENSTEIJN, CHRISTOFFER KOK SØRENSEN, JACOB A. BIKKER AND ADRIAN VAN RIXTEL: Impact of bank competition on the interest rate pass-through in the euro area.
- 0829 CRISTINA BARCELÓ: The impact of alternative imputation methods on the measurement of income and wealth: Evidence from the Spanish survey of household finances.
- 0830 JAVIER ANDRÉS AND ÓSCAR ARCE: Banking competition, housing prices and macroeconomic stability.
- 0831 JAMES COSTAIN AND ANTÓN NÁKOV: Dynamics of the price distribution in a general model of state-dependent pricing.
- 0832 JUAN A. ROJAS: Social Security reform with imperfect substitution between less and more experienced workers.
- 0833 GABRIEL JIMÉNEZ, STEVEN ONGENA, JOSÉ LUIS PEYDRÓ AND JESÚS SAURINA: Hazardous times for monetary policy: What do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking?
- 0834 ENRIQUE ALBEROLA AND JOSÉ MARÍA SERENA: Sovereign external assets and the resilience of global imbalances.
- 0835 AITOR LACUESTA, SERGIO PUENTE AND PILAR CUADRADO: Omitted variables in the measure of a labour quality index: the case of Spain.
- 0836 CHIARA COLUZZI, ANNALISA FERRANDO AND CARMEN MARTÍNEZ-CARRASCAL: Financing obstacles and growth: An analysis for euro area non-financial corporations.
- 0837 ÓSCAR ARCE, JOSÉ MANUEL CAMPA AND ÁNGEL GAVILÁN: Asymmetric collateral requirements and output composition.
- 0838 ÁNGEL GAVILÁN AND JUAN A. ROJAS: Solving Portfolio Problems with the Smolyak-Parameterized Expectations Algorithm.
- 0901 PRAVEEN KUJAL AND JUAN RUIZ: International trade policy towards monopoly and oligopoly.

BANCO DE ESPAÑA
Eurosistema

Unidad de Publicaciones
Alcalá, 522; 28027 Madrid
Telephone +34 91 338 6363. Fax +34 91 338 6488
e-mail: publicaciones@bde.es
www.bde.es