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Abstract

In this paper we present a methodology designed to estimate the future path of the interest payments of central government. The basic idea is to represent in a compact way the joint dynamics of debt liabilities and interest payments as a function of four elements: the initial outstanding amounts of debt, the expected primary funding needs, the expected yield curves and the expected issuance strategy to be followed by the government. The procedure is amenable to scenario-based simulation and produces a detailed representation of the debt term structure. We provide results for the period 2015-2025.

Keywords: interest payments, yield curve, forward rates, debt dynamics.

JEL Classification: E43, E44, E47, E63.

Resumen

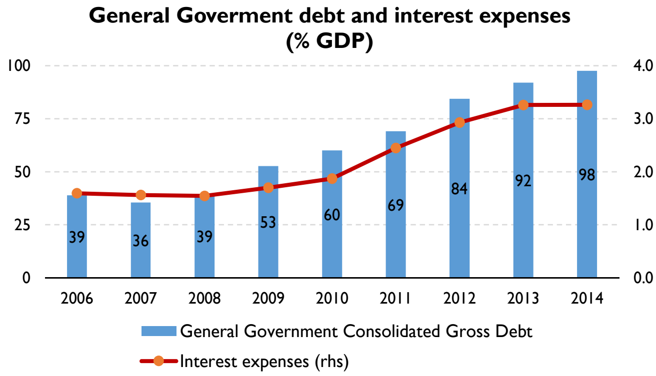
Este artículo presenta una nueva metodología para estimar el gasto financiero de la Administración Central. La idea subyacente es la de representar desagregadamente la dinámica de la deuda y del gasto por intereses a partir de cuatro elementos: el saldo vivo de la deuda pública inicial, las expectativas de capacidad/necesidad de financiación primaria, los tipos de interés *forward* y la estrategia de emisión de la Administración Central. Este procedimiento proporciona una representación detallada de la estructura de vencimientos de los distintos tipos de instrumentos de deuda y es susceptible de ser empleado para la simulación de escenarios.

Palabras clave: gasto por intereses, curva de tipos de interés, tipos de interés *forward,* dinámica de la deuda pública.

Códigos JEL: E43, E44, E47, E63.

1 Introduction

Since the start of the economic and financial crisis, the level of government debt in Spain has dramatically risen from 36% in 2007 to 98% at the end of 2014. In parallel to this, the interest burden has become one of the most important government expenditure items (exceeding 3% of GDP) and, therefore, central to the monitoring of budget stability and debt sustainability (see Chart).



Source: AMECO

Thus, a thorough monitoring of the trend of the expected interest burden, both on a cash basis and in national accounts terms, becomes very relevant, also with the aim of exploring the existence of room for maneuver for Government expenditure in other areas.

Against this background, we developed a toolkit to forecast the interest burden evolution. This paper presents the methodological development and, as an application of it, an estimate of interest expenses for the period from 2015 to 2025.

The basic idea behind the methodology is to represent in a compact way the joint dynamics of debt liabilities and interest payments as a function of five elements: the initial outstanding amounts of debt (i.e. “debt portfolio”) and their associated payment streams, the expected path of the primary funding needs, the expected yield curves and the expected issuance strategy by the issuer.

The illustrative 2015-2025 exercise requires initial calculations that can be grouped into two categories or stages, each of them related to a distinctive output of the methodology. In the first stage we calculate the interest burden of the pre-existing debt portfolio of the central Government, using the specific features of each individual security outstanding at the beginning of the period to build the interest cash payments and the underlying interest rate in accrual terms.

In the second stage, we derive estimates of the expected interest expenses on future gross financing needs. For that, we make use of the forward yield curve for estimating the cost of financing, and we obtain a path for the expected gross financing needs based on the redemption schedule derived from our dynamic equations and estimates on primary funding needs. Both outputs of the numerical exercise, initial interest cost and future cost, are recorded in cash and accrued terms to make it comparable with the State Budget (cash) and National Accounts figures (accrued).

Modeling interest payments is not only a key ingredient of the monitoring of the central Government budget and fiscal conditions but also has relevant implications for debt management purposes, see for example Denmarks Nationalbank (2014), Bolder (2008) or Bolder and Deeley (2011). However, the focus of our approach is aimed at the general implications of macroeconomic and budgetary projections on the interest burden rather than on the quantification of alternative financial scenarios (e.g. yield curve scenarios) on the interest burden and its cost and risk profile.

The paper has two main parts. The first one is devoted to general methodological issues and comprises sections two and three. The second part deals with the basic inputs of our modeling approach and comprises sections four to seven. Exposing the structure of the paper in greater detail, section 2 summarizes the conceptual framework behind the recording of interests in cash basis and in national accounts. In section 3 we describe the dynamics of debt in our modeling framework. Section 4 details the sources, methods and assumptions taken to estimate the set of initial outstanding amounts and yields of government liabilities bearing interests for our numerical exercise. Section 5 describes the use of the forward interest rates to estimate the expected cost of future issuances Section 6 is devoted to the assumptions behind the government issuance strategies. Section 7 covers the establishment of paths of primary funding needs, which is the contemporary impulse for the debt dynamics. Complete numerical results are presented for the period 2015-2025 in section 8. Finally, section 9 concludes.

2 Cash and accrual recording

To calculate interest expenses two criteria can be followed: cash recording, consistent with budget figures, or accrual recording, in line with national accounts standards. The methodological difference between accrual and cash recording is the timing of recognition of the interest expenses. The cash method accounts for interest expenses only when the money is paid out, whereas accrual recording accounts for the interest expenses in a smooth way over the whole life of the financial instrument reflecting the building-up of the economic liability.

2.1 Cash recording

In cash recording interest expenses are accounted for in the moment they are paid. In the case of zero coupon bonds, implicit interest payments are recorded at maturity. For bonds with coupons, interest expenses amounts to the coupons actually paid and are recorded in the moment that they are paid. Loans are treated in a similar manner: interest payments are reflected when the cash payments are made. Hence, knowing the cash flow structure of each security is sufficient for calculating and temporary allocating the interest expenses.

When bonds are issued at premium (above nominal) -by product of coupons exceeding market rates- interest expenses in cash terms are over-estimated compared to national accounting over the whole life of the debt instrument as the premium fees are considered, in budgetary term, as non-financial revenues (and not as a reduction in interest expenditure).

2.2 Accrual recording

In National Accounts interest expenses are recorded on an accrual basis continuously in time as the corresponding liability is arising. Moreover, the European standards for national accounts prescribes the recording on a compound interest basis using the rate prevailing at inception, and following a principle of reinvestment of the interest accrued but not paid in the corresponding debt instrument (see the Box 1 at the end of this section).

In analytical terms the national account requirements implies that the interests recorded in each period *t*, I_t, depend on the yield at inception, r_0, and the outstanding amount at the beginning of the period, S_{t-1}, -which should include interests accrued and not paid- according to the rule $\mathrm{I_t} = \mathrm{r_0} * \mathrm{S_{t-1}}$, r_0 expressing yield per period t, if there is no coupon or other cash payments during the period.

In more general terms, if there is a cash payment P_t at a fraction α_t of the length of the period *t*, the interests accrued would then be:

$$I_t = S_{t-1}[(1 + r_0)^{\alpha_t} - 1] + [S_{t-1}(1 + r_0)^{\alpha_t} - P_t]\,[(1 + r_0)^{1-\alpha_t} - 1] \qquad [1]$$

where the two right-hand side additive members capture the accruals before and after the payment respectively. The accruals after the payment ($[\mathrm{S_{t-1}}(1 + \mathrm{r_0})^{\alpha_t} - \mathrm{P_t}][(1 + \mathrm{r_0})^{1-\alpha_t} - 1]$) result from applying the interest rate r_0 on an outstanding amount that has been increased over the stock at the beginning of the period (S_{t-1}) by the interest accrued before the payment ($S_{t-1}[(1+r_0)^{\alpha t} - 1]$) and reduced by the amount of cash effectively paid (P_t). Note that if $\mathrm{P_t} = 0$ then [1] collapses into $\mathrm{I_t} = \mathrm{r_0} * \mathrm{S_{t-1}}$. Equation [1] can be easily generalized to any number of cash payments during the period.

Taking $S_t = [S_{t-1}(1 + r_0)^{\alpha_t}\text{-}P_t](1 + r_0)^{1\text{-}\alpha_t}$ (which derives from the rule of reinvestment of accruals), [1] can be expressed in the convenient way:

$$I_t = S_t - S_{t-1} + P_t \qquad [2]$$

where S_t can in turn be derived, calculating iteratively, forwardly from

$$S_t = \frac{S_{t+1}}{1+r_0} + \frac{P_{t+1}}{(1+r_0)^{\alpha_t}},$$

as

$$S_t = \sum_{i=t+1}^{n} \frac{P_i}{(1+r_0)^{i-t-1+\alpha_i}} \, ,$$

n being the period in which the last payment is made; i.e, S_t is the net present value (NPV) discounted with the rate at inception r_0 of all future cash payments.

[1] and [2] tell us that to derive a full set of accruals for t=0…T it is necessary to have at our disposal an initial stock S_0, the associated yields at issuance (r_0) and the stream of cash flows (P_t).

BOX 1: National Accounts conventions

According to the European system of national and regional accounts in the European Union (ESA 2010), interests are to be recorded on an accrual basis ("as accruing continuously over time to the creditor on the amount of principal outstanding"), and, if not paid, as reinvested in an asset of the creditor vis-à-vis the debtor (ESA 2010 4.50), preferably under the same financial instrument on which they accrue (5.242). Moreover, for the government sector they are to be recorded under the "debtor approach": "based on the rate or yield prevailing at the time of creation of the financial instrument" (20.180). Although there is no clear prescription in the ESA, the Manual on Government Deficit and Debt (MDD), which complements the ESA 2010 by clarifying the statistical treatment of government related issues, recommends the recording of interests on a compound interest basis (MDD 10).The following table shows the difference between interest payments recorded on a cash and accrual basis based in actual data for the Spanish State during the period 2009-2015:

Spanish State. Interest payments (EUR millions)			
Year	Cash basis	Accrual basis	Difference
2009	17.652	16.359	1.293
2010	19.641	18.157	1.484
2011	22.195	22.403	-208
2012	26.059	25.694	365
2013	28.410	28.797	-387
2014	31.818	30.826	992
2015	31.744	29.488	2.256

It is also interesting the role of the cash-accrual adjustment attributed to the difference between the redemption price and the issue price as, according to the ESA 2010, it has to be distributed over the years to the maturity of the bond.

Due to the recent decline of interest rates during 2014 and 2015, the cash-accrual adjustment resulting from the re-issuance of bonds above par has become substantial and is one of the main factors behind the deficit-debt adjustment in Spain, as well as the national accounts adjustment of interest payments. The following table compares for 2014 and 2015 the amount of the premium fees with the cash-accrual adjustment, according to the actual data published for the Spanish State:

Year	Premium fees	Cash-accrual adjustment
2014	8.454	-6.012
2015	12.259	-9.070

3 Modeling debt dynamics

In this section, we explain the procedure used to project the debt portfolio and its corresponding flow of interest payments stratified by residual maturity. The procedure considers a discrete state space representation of both elements that provide a compact representation of the joint dynamics of debt and interest payments.

The quarterly evolution of the debt portfolio is driven by its intrinsic dynamics (determined by its motion to maturity) and an impulse factor related to the new flow of debt. The equation is:

$$B_t = F B_{t-1} + \Delta B_t \qquad [3]$$

Being:

- B_t is a kx1 vector that represents the outstanding debt at the end of period t at the different i=1…k residual maturities. The first element represents the debt that matures immediately (and has to be refinanced).
- F is a square k-dimensional binary matrix that embeds the motion in residual maturity of a bond from period t to t+1; i.e F is of the form.

$$F_{kxk} = \begin{bmatrix} 0_{(k-1)x1} & I_{(k-1)x(k-1)} \\ 0 & 0_{1x(k-1)} \end{bmatrix}$$

- ΔB_t is a kx1 vector that contains the issuance of new debt (gross financing needs) at the various maturities i=1…k in period t.

The new debt is issued to finance redemptions of existing debt, coupon payments and primary funding needs:

$$\Delta B_t = w\,(B_{t-1,1} + \rho C_{t-1} + x_t) \qquad [4]$$

Being:

- w is a kx1 vector that defines the issuance strategy (see section 6).
- $B_{t-1,1}$ contains the debt refinanced in period t (first element of the vector B_{t-1}).
- C_{t-1} is a kx1 vector that comprises the coupons attached to the outstanding amounts at the end of period t-1 contained in vector B_{t-1} for the i=1…k residual maturities.
- ρ is a 1xk binary vector that signals the residual maturities at the end of t-1 that pay coupon in period t (for quarterly data -of annually paying par bonds- as in this exercise, this is a vector with 1 in its components multiples of 4 and 0 otherwise).

- x_t is the primary funding needs: operations that have to be financed without relation to the existing debt portfolio (see section 7).

The issuance vector w is constrained: $w_i \geq 0 \ \forall i$ and $\sum_i w_i = 1$.

Independently of the composition of the current debt portfolio, we assume that the new debt is only formed by fixed income bonds issued at par and, hence, the evolution of the coupons mirrors the evolution of the debt portfolio:

$$C_t = F C_{t-1} + y_t \bullet \Delta B_t \qquad [5]$$

The simplification that future emissions will be made at par can be justified as a neutral approach. The difficulty to know in advance the specific conditions that will prevail in the sovereign debt markets during time horizon of the model (10 years) makes virtually impossible to model so far ahead the preference for specific cash-flow profiles, i.e., the decisions whether to issue at par, above par or below par.

Where:

- y_t is the kx1 vector of yields of the newly issued bonds in t at different maturities i=1…k. (see section 5 for the derivation of this vector from the market forward rates).The fixed coupon is defined as the element-by-element (•) product of y_t and ΔB_t.

The interests accrued in period t depend on the frequency of the coupon relative to the frequency of the time series in the model:

$$I_t = \beta C_{t-1} \qquad [6]$$

Where:

- ß is a scalar capturing the relative frequencies. For quarterly data of annually paying par bonds, ß =1/4 (ignoring differences in the length of the quarters).

Assuming an initial debt portfolio with its corresponding coupons B_0 and C_0 (see section 4), the model [3]-[6] can be iterated from t=1 to t=T to generate the corresponding paths for debt, coupons and interest accrued.

4 Initial debt portfolio

The first step of our methodology requires setting the initial stock of public debt outstanding. For our numerical exercise an initial stock of debt at the beginning of 2015 has been prepared for the Central Government mainly using the security by security information about the outstanding amount of the negotiable State debt or T-bonds (*Bonos y Obligaciones del Estado*) at the end of 2014, provided by the Spanish Treasury, and completed with the issuances of the first quarter of 2015 and shorter-term securities. Additional information for the remaining subsectors of the Central Government sector and for loans has been obtained from the Bank of Spain and other sources. A detailed description of the instruments that make up the portfolio is provided in Box 3.

The information from the Treasury covers the details singled out above necessary to estimate cash payments and accrued interest. In particular, it includes the relevant data on the coupon payments (rate, frequency, date of payment, fixed or variable nature), the dates of issuance and maturity, and the outstanding amount at the end of 2014. All this information is provided for every issued tranche of a particular bond.

In addition, data for zero coupon short-term securities (*Letras del Tesoro* or T-bills) was taken from information of issuances published in the web page of the Treasury. The information used for relevant T-bills was the average yield, allotted amount and the issuing and maturity dates. The Spanish Treasury also provides information on an aggregated basis about the outstanding amount, average maturity and average interest of the State debt other than Treasury bills and bonds.

All this information enables to construct monthly cash flows for each of the securities or loans with fixed coupon rates. For variable coupons rates, which are linked to inflation, a smooth convergence to 2% inflation rate (the ECB policy objective) by 2018 has been assumed. For coupon payments linked to the EURIBOR, EONIA or other market rates, the particular conditions of each issuance, such as spread and reference indices, have been considered.

While the monthly cash payments prepared as described allows us to calculate directly the interests on a cash basis, the estimation of accruals requires the additional calculation of individual rates at issuance (the r_0 in the discussion in section 2). For that, we calculate the corresponding *annual* internal rate of return (IRR)[1] of the cash flows for each individual security or loan (or synthetic loan constructed from aggregated data)[2]. A detailed numerical example is provided in Box 2. This particular type of calculation of the initial rates enables, using equation [2], to replicate exactly the accruals recorded by the Treasury[3]. Note that equation [2] on the basis of annual IRR_j can be applied to periods of any frequency, monthly, quarterly or annual, but it always entails that the interest are compounded on an annual basis. As said, this is the computation basis followed by the Treasury.

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1. i.e. the annual rate that makes the NPV equal to zero at inception ($0 = \sum_{i=0}^{n} \frac{\sum_{k=1}^{12} \frac{P_{k,i}}{(1+IRR_j)^{k/12}}}{(1+IRR_j)^{i}}$) where Pk,i is the cash payment in month k of year i after issuance, which can be zero, and n is the year of the last payment.

2. We actually calculate IRRj on daily payments, rather than monthly payments, setting always the day of payment on the 15th of every month irrespective of the actual payment date. That means that the discount rate is not always $\frac{P_{k,i}}{(1+IRR_j)^{i+k/12}}$ for every month, but slightly different depending on the number of days of each month.

3. For a security j for any period t, if the corresponding S_t and S_{t-1} are calculated as the annual NPV of the monthly payments due after the end of t and t-1 with IRR_j as annual discount rate, and P_t being the sum of all payments due in period t.

BOX 2: The role of the Internal Rate of Return to estimate accrued interest

In the table included at the end of this box a numerical example is provided in order to illustrate how the Internal Rate of Return (IRR) is used to estimate accrued interest. We assume a five year bond issued above par on 12/03/2016, with a premium of 5%, nominal value of 500, coupon rate of 2.4% to be paid each March 12th, and date of redemption fixed on 12/03/2021.

The annual IIR is 1.36%, that is, the annual rate that makes the Net Present Value (NPV) of all cash flows of the bond (P_t) – shown in the second column- equal to zero at inception. The NPVs (S_t) after each payment or at the end of each year, calculated using the IIR of 1.36% as discount rate, are shown in the third column of the table.

As shown in section 2, accrued interest can be derived according to equation:

$$I_t = S_{t-1}[(1+r_0)^{\alpha_t} - 1] + [S_{t-1}(1+r_0)^{\alpha_t} - P_t]\,[(1+r_0)^{1-\alpha_t} - 1] \qquad [1]$$

Where r_0 is the interest rate at inception -the *annual* IRR of 1.36% in our example-, α_t is the fraction of the length of the (annual) period *t* where a payment is made, and the two right-hand side additive members capture the accruals before and after the payment respectively.

The entries in the fourth column of the table below correspond to interest accrued since the previous date. As the table entries refer to dates of cash payments (or end of year), no intra-period payment takes place, and only the first of the two additive terms is operative, according to the rule:

$$I_t = S_{t-1}[(1+r_0)^{\alpha_t} - 1] \qquad [1b],$$

where α_t is the fraction of the years elapsed since the previous period and $\alpha_{t-1} + \alpha_t = 1$

The last column of the table has entries for the end of year dates and uses equation [2] for the calculation of accrued interest during the year:

$$I_t = S_t - S_{t-1} + P_t \qquad [2]$$

As discussed in section 2, both calculation approaches deliver the same result for the annual accruals.

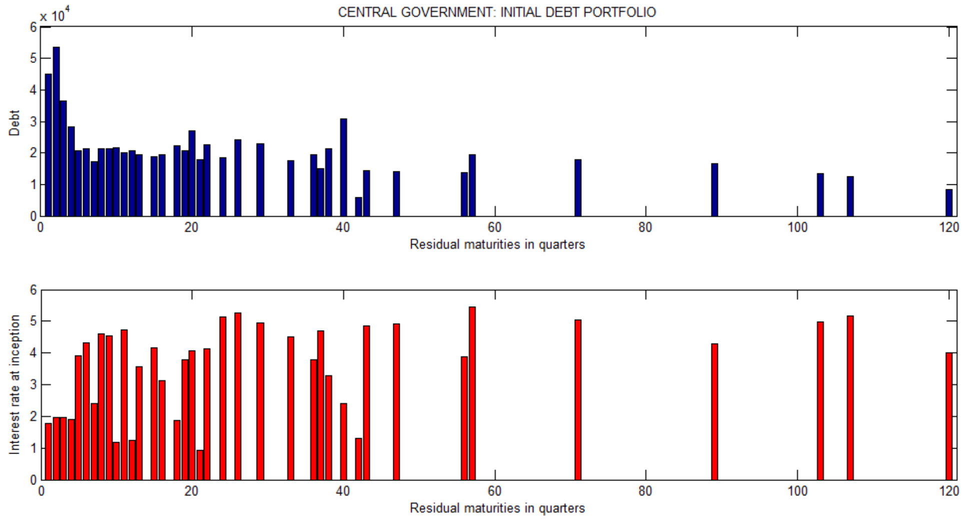
Date	Cash-flows (P_t)	NPV (S_t)	Interest accrued from previous date (using equation 1b)	Interest accrued in the year (using equation 2)
12/03/2016	-525	525.00		
31/12/2016	0	530.74	5.74	5.74
12/03/2017	12	520.13	1.39	
31/12/2017	0	525.81	5.68	7.08
12/03/2018	12	515.19	1.38	
31/12/2018	0	520.82	5.63	7.01
12/03/2019	12	510.19	1.37	
31/12/2019	0	515.77	5.57	6.94
12/03/2020	12	505.14	1.37	
31/12/2020	0	510.66	5.52	6.89
12/03/2021	512	0.00	1.34	
31/12/2021	0	0.00	0.00	1.34
TOTAL	35		35	35

Furthermore, the outstanding amounts have been stratified according to the number of quarters to redemption or residual maturity. To assign IIRs to the strata, the weighted average across the several IRR_j of the j elements in each strata i has been calculated (to obtain an average IRR_i per residual maturity i).

The simplifications above enables us to construct the initial conditions for the dynamic modelling: two vectors of dimension kx1, where k is the residual maturity in quarters, B_0 and C_0, containing respectively the initial nominal value and the corresponding coupon payment for every i =1… k residual maturity, where $C_{0,I} = IRR_i\, B_{0,I}$.

Note that vector C_0 does not correspond with the actual coupon payments of the initial securities, but it rather represents a re-basement of the payments as if the securities had all been issued at par at the calculated IRRs. This aligns with the simplification in the dynamic modelling of reissuance at par (see section 3).

The following graph shows the initial debt portfolio of the central Government and its corresponding interest rate profile:



Source: Authors calculations.

BOX 3: Budgetary information

The annual General State Budget includes a detailed annex in which the interest burden on the debt portfolio and on loans are accounted on a cash basis. Aggregate information on interest on an accrual basis according to the ESA 2010 is also available.

Specifically, the General State Budget includes in chapter 3 details on a cash basis of the expected interest payments by type of debt:

- Treasury bonds. The General State Budget identifies each issue of T-Bonds with a rate and a date. Foreign currency bonds and inflation-linked bonds are also detailed. This item accounts for 91.5% of the total interest expenditure in the 2015 budget.
- Treasury Bills. The T-Bills interests are less than 2% of the total interest expenditure.
- Loans. The Budget identifies on a loan-by-loan basis the interests due to Schuldschein loans, those granted by the European Investment Bank, the European Stability Mechanism (being the main part of this item) and other loans. It accounts for around 3% of the total interest expenditure.
- Debt assumptions from loans to public entities, like the Spanish National Railway Network (RENFE) or the Administrator Railway Infrastructure (ADIF), granted by the European Investment Bank and assumed by the Treasury. Since July 2014 part of the financial mechanisms for the regions called “Fondo de Pago a Proveedores” (FFPP), which were banking loans[4], is also included. Those interests account for around 3.5% of the total interest expenditure.

In order to assess the accuracy of the results and the reliability of our estimate of the initial debt portfolio, a comparison has been made with the data from chapter 3 of the initial State budget for 2015 described above. Combining again as explained in this section the security-by-security information on outstanding amounts and coupon payments regarding Treasury bonds, an estimate of the cash interests linked to the Treasury bills and, finally, an estimate on the interests paid on a cash basis regarding loans granted to the State, an homogeneous comparison has been made with the amounts of the initial State budget.

While the initial 2015 State budget included an amount of 35.490 EUR millions for interest payments on cash basis, our estimate amounted to 31.267 EUR millions. The difference of 4.223 could be partially explained by a prudent buffer fund. The comparison on an accrual basis shows a smaller difference. While the initial State budget provides an amount of 31.650 EUR millions, our estimate is lower 29.175, therefore the difference between both figures is 2.475. However, this buffer in cash interest payments forecast is not reflected in a similar buffer in the forecast for accrued interest (without which there is no “true” buffer as far as budgetary stability objectives are concerned).

4. Law 13/2014 (July, 14th) that transforms the Financing Fund for Supplier Payments (Fondo para la financiación del pago a proveedores).

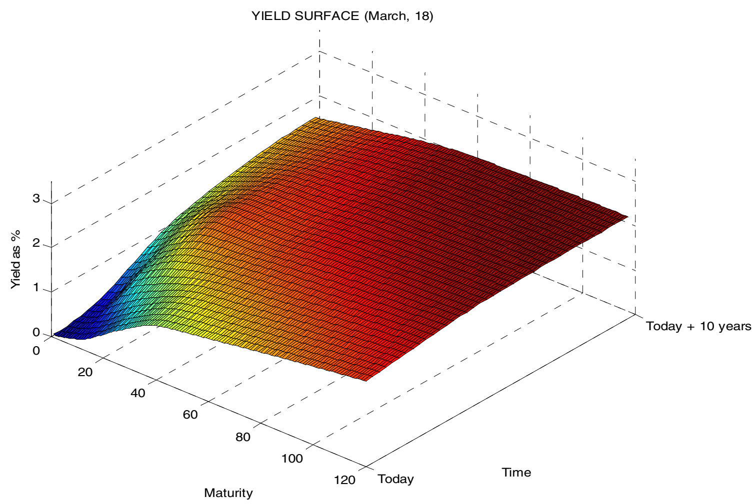
5 Forward interest rates

To estimate the interest payments for the future gross financing needs of the Central Government, there is a need to assess its financing needs (related both to refinancing and new financing needs) and its issuing strategy, but also the expected interest rates at which that new government debt will be issued.

To estimate such rates in different points in time, we have used the information on implied market forward rates, the expected future yield on the Spanish government bonds, based on trading market data. Forward rates are usually presented in two ways. First, as (market expectations on) the path of future interest rates on bonds for the same maturity. Second, as the relationship between yield and a bond's maturity at some point in time (spot or future). This two presentations are put together in the "market forward matrix" where the path of future interest rates on bonds with the same maturity are placed in columns (or rows) and the relationship between the yield and a bond's maturity is in rows (columns).

For this exercise, the forward matrix used was based on market data as of March, 18th 2015. The precise date was chosen once all issuance of the first quarter were completed.

To get enough data granularity, interpolation by means of cubic splines has been applied on both dimensions of the forward matrix (maturities and calendar time) to obtain a yield surface:



Source: Authors calculations based on Bloomberg data.

Note that forward rates are based on arbitrage-free rates for dealing today at rates that are effective at some point of time in the future. As such, forward rates are a type of market view on where interest rates should be in the future; forward rates do not necessary have predictive power of future interest rates and, in fact, empirical analysis show that this has been historically low.

6 Issuance strategy

For the time horizon of the exercise we have distinguished two distinctive periods. For 2015 the assumed strategy follows the Spanish Treasury published guidelines. For 2016-2018 it has been assumed that results in maintaining the maturity structure, i.e., the same share of bonds at each maturity, prevailing at the end of 2015.

For 2015 we follow the guidelines included in its Issuance Strategy Report[5] that includes gross and net financing plans and a tentative issuance calendar. Briefly, the Treasury funding needs are, both in gross and in net terms, very similar to those of 2014, although the Central Government is also offering regional and local governments additional funding tools, through which the Treasury would take over most of their market financing (see section 7 on the impact of this policy on the future primary funding needs).

- Total gross issuance will be around 239.369 € bn. Medium- and long-term instruments (T-bonds) gross funding will be around 141.996 € bn and short-term T-bills will be 97.373 € bn.

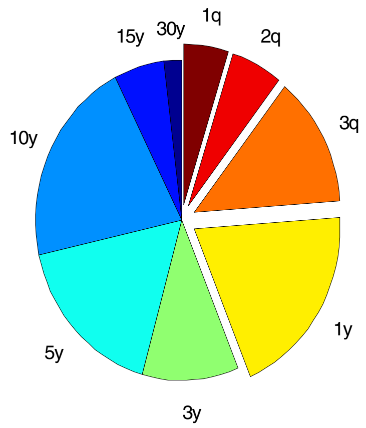
- Net funding target is 55 € bn. Medium and long-term net funding is set in 50 € bn and net issuance of short-term T-bills in 5 € bn.

- On the securities issued (i) T-bills issuance schedule and maturity pattern is not expected to change substantially compared to 2014 (regular auctions of 3 to 12 months securities) although the maturity structure will slightly move towards longer maturities; (ii) nominal fixed-coupon T-bonds auctions are l also not expected to change in schedule, though the maturity structure could change; and (iii) inflation-linked T-bonds will be reinforced and incorporated into regular auctions.

Note that the Treasury might also conduct special auctions, resort to bank syndication and issue debt through private placements, in which specific securities are sold directly to investors.

5. http://www.tesoro.es/sites/default/files/150112_Estrategia_EN.pdf

Based on this plans, the issuance strategy assumed for our numerical simulations is represented in the following pie chart:



Source: Authors calculations.

7 Primary funding

The debt dynamics are ultimately driven by the evolution of the primary funding needs. In this way, its state space representation [3]-[6] can be considered as an amplification filter of the basic impulses contained in the primary funding.

We have considered a simple model that assumes that the primary funding is a time-varying linear function of the nominal GDP:

$$x_t = \delta_t P_t Y_t \qquad [7]$$

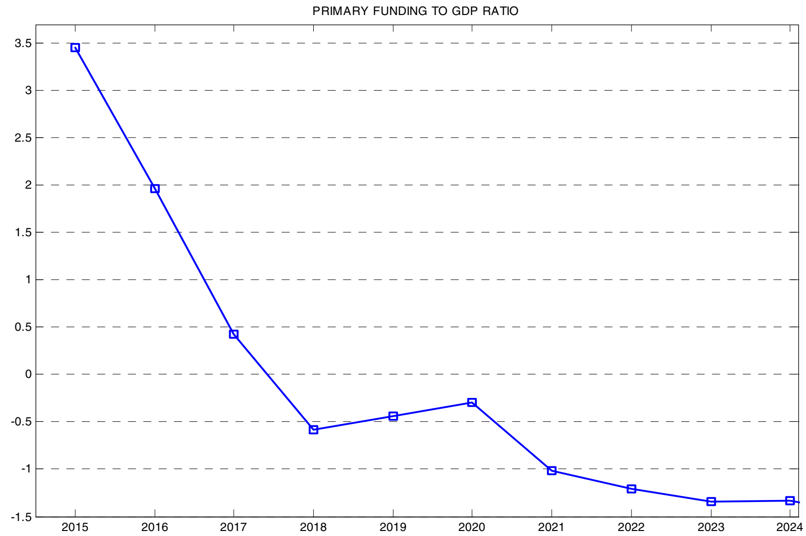
Equation [7] provides a simple way to generate complex stochastic scenarios combining alternative paths for the δ ratio (fiscal dimension), the price level P (inflation dimension) and the real GDP Y (growth dimension).

On the one hand, the inflation and growth dimension can be linked to an explicit macroeconomic model, granting a well-rooted economic support for the simulations. On the other hand, the δ ratio is a simplified, reduced-form attempt to capture the complex mapping from macro conditions (nominal GDP) to primary funding. We have encapsulated in one variable the interactions of the expenditure and income side of the Government's budget as well as Government's financial investment activities, simplifying many gears and levers in order to achieve a compact representation amenable for scenario-based simulation.

From 2016 onwards, we have considered that the primary funding is derived from the macroeconomic scenarios internally computed by AIReF. This parametric way to introduce alternative paths for the primary funding is simple and transparent and allows us to assess the consistency of other assumptions, e.g. those contained in the Stability Program.

The future annual primary funding path includes, together with an assumed primary deficit dynamic, an estimate of the stock-flow adjustments associated with the progressive financing support offered by the Central Government to the regional Governments[6]. The annual profile resulting from these estimates is:

6. Since 2012 the Central Government has been offering the Regional Governments financing lines in order to reduce their cost of finance in a context of excessive risk premium and/or lack of access to financial markets.

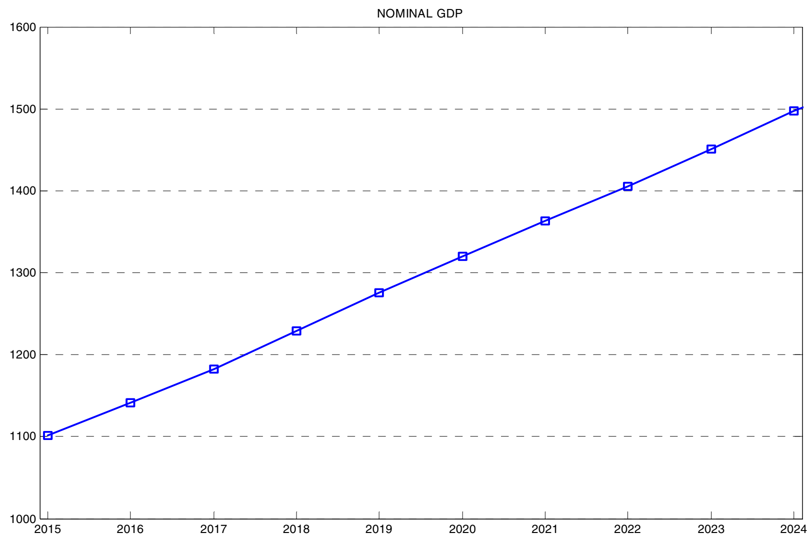


Source: Authors calculations based on AIReF's scenarios.

Note that this ratio becomes negative from 2018 onwards. In this way, the ultimate source of debt issuance will eventually disappear. This sign reversal opens different possibilities for the issuer, which range from stopping new issuance to embracing an active program of debt repurchases aimed at reducing the debt level and its corresponding interest burden.

We have assumed that new issuance stops when the primary funding requirements become negative, while existing debt continues to be refinanced. This simple schedule provides a benchmark for more sophisticated strategies.

A given path for the annual nominal GDP would allow us to map the ratio into an annual nominal time series. This series can then be interpolated by means of a univariate temporal disaggregation procedure to be properly plugged in our quarterly debt simulation engine. The next figure plots the assumed annual nominal GDP profile we have used in our calculations.



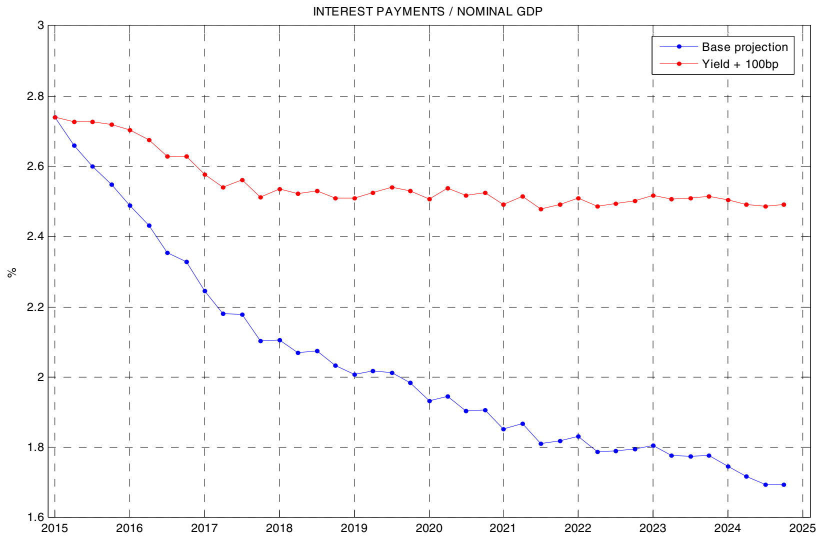
Source: Authors calculations.

8 Numerical results

The final output of our methodological approach is a quarterly path for the interest payments linked to the combination of an exogenous path for the primary funding needs, a yield surface based on the forward rates, an initial debt portfolio and an issuance strategy.

We provide here some numerical results related to a complete computation starting at the second quarter of 2015. The time scale of the model is quarterly and the maturities range from 1 to 120 quarters (0.25 to 30 years). The inputs for the numerical results have been described in the previous sections.

The following figure depicts two ten-year-ahead quarterly paths of interest accrued, one resulting from using all the above mentioned inputs and another one linked to a 100 basis points (bp) parallel shift of the yield surface[7]:



Source: Authors calculations.

The base projection draws a steady reduction of the interest burden with respect to nominal GDP, ending at around 1.7% in 2024. However, this result is quite sensitive to movements in the yield surface. The 100 bp shift generates a much slower decline of the interest burden, ending at around 2.5% in 2024, despite the sign reversal of the primary funding needs common to both projections. Refinancing the new debt issued at higher interest rates has thus a noticeable accumulative impact. In the long run, this impact increases the interest payments over nominal GDP ratio in about one point.

7. All the computations have been coded in MATLAB. Apart from avoiding messy spreadsheet calculations, those functions provide an easy way to perform scenario-based simulations and risk analysis.

9 Conclusions

In this paper we have outlined a fully-fledged methodology to estimate the evolution of interest payments as a result of its basic determinants: initial debt portfolio, primary funding requirements, expected yield surface and issuance strategy. This methodology can be considered in a stand-alone way or embedded in a more complex system (e.g. as a component of a structural macro model).

We apply the methodology to produce an estimate of interest burden for the Spanish Central Government for the period 2015-2025. The methodology can be extended and improved in various ways. For instance, a more sophisticated issuance strategy when the primary funding becomes negative may provide an interesting complement to the inertial "no pumping" scheme used in this paper. Similarly, a comprehensive macroeconomic scenario analysis, exploring alternative combinations of growth, inflation and primary funding requirements, can offer a better understanding of the dynamics of debt and interest payments and its sensitivity to alternative assumptions.

Risk analysis deserves a special mention. The exposure to alternative yield scenarios is a key ingredient in any modern debt management procedure. Thus, by assuming that the yield curve evolves in its three basic components (level, slope and curvature) we can simulate alternative yield surfaces and compute the probability distribution of the present value of the future payment cash-flow. This probability distribution will enable us to compute location measures (mean/median cost) as well as risk measures (cost at risk, expected shortfall cost, variance).

Finally, using the issuance strategy as an instrument variable, a cost-risk frontier can be generated and a search for efficient strategies can be implemented.

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